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ABSTRACT

This guide represents the final experimental version of a pilot project conducted in the Unites States between 1973 and 1976. The ideas and manner of presentation are based on the works of Georges and Frederique Papy. They are recognized as having introduced colored arrow drawings ("papygrams") and models of our numeration system (the Papy "minicomputer") into the teaching of mathematics at the elementary and secondary level in Belgium. This program follows the "spiral approach." Suggestions are given for acquiring materials for lessons. This guide is divided into the Language of Strings and Arrows, Geometry and Measurement, and the Appendix, which contains the String Game. The five workbooks in this volume are intended for individual work. The answer key for the workbooks is included with the guide. A chart is given for the teacher to keep track of each student's progress. (Author/SA)

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CSMP Mathematics for the Upper Primary Grades
Part I

Comprehensive School Mathematics Program

CEMREL, Inc.

Final Experimental Version

2-33301

Teacher's Guide

The Languages of Strings and Arrows
Geometry and Measurement
Workbooks

U.S. DEPARTMENT OF HEALTH,
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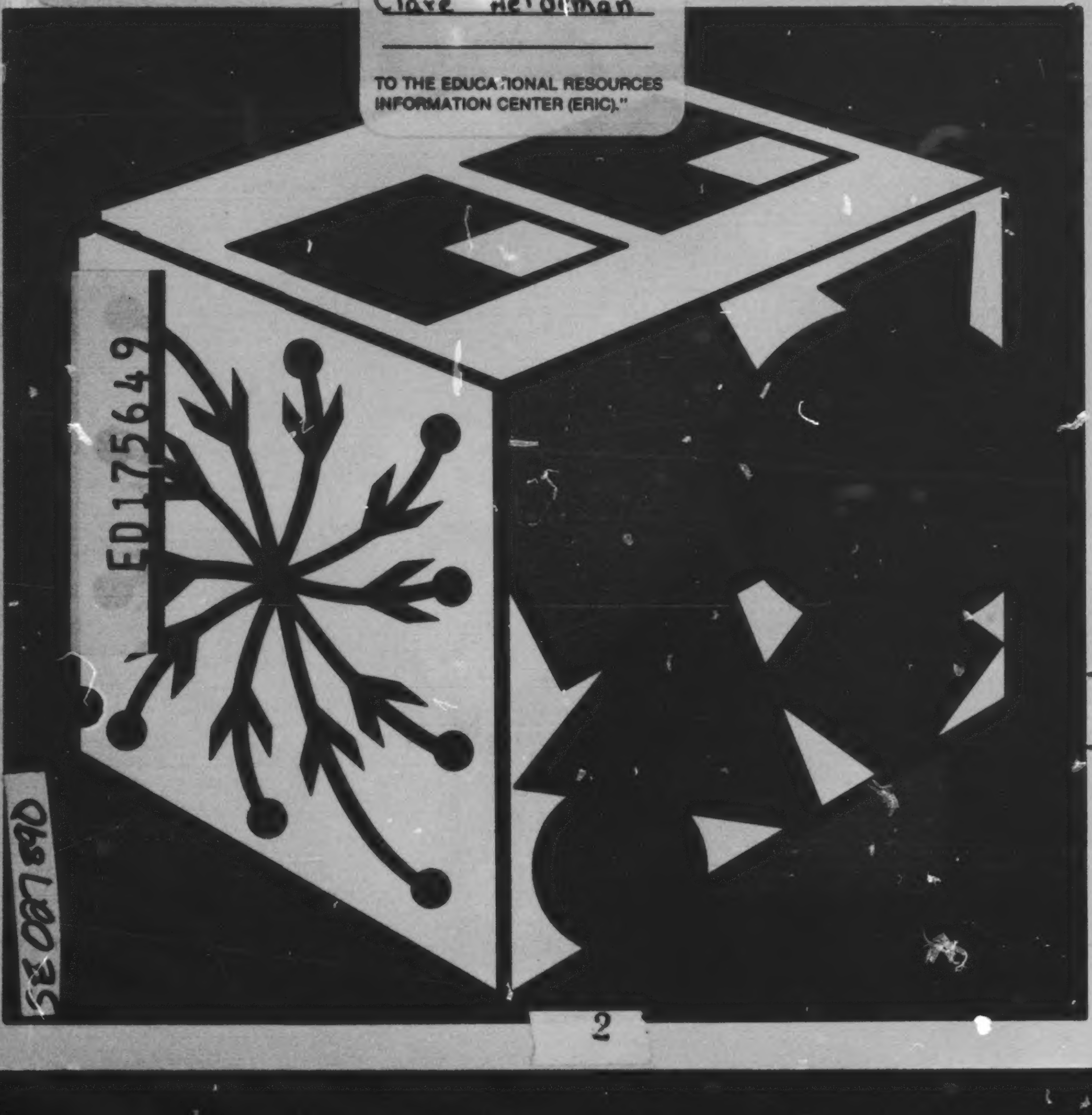

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Comprehensive School Mathematics Program

CSMP Mathematics
for the
Upper Primary Grades
Part I

The Languages of Strings and Arrows

CEMREL, Inc.

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THE LANGUAGES OF STRINGS AND ARROWS

INTRODUCTION

The language of strings is the nonverbal language of sets, that is, collections of objects put together in some way. The language of arrows is the nonverbal language of relations among objects. These languages permeate the CSMP content strands and are used, separately or together, to present an unending variety of numerical and nonnumerical situations.

Classification: The Language of Strings

As the word implies, classifying means putting things into classes, or as the mathematician says: sets. If the mathematics of sets has any place in an elementary school mathematics curriculum, it is precisely because it can help students to understand and use the ideas of classification.

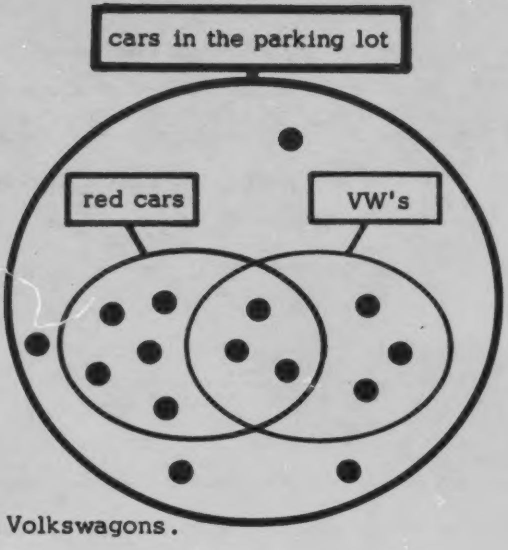
The basic idea is simple: Given a set S and any object x, either x belongs to S (x is in S) or x does not belong to S (x is not in S). We represent this simple act of sorting -- "in" or "out" -- neatly by using string pictures to illustrate "in" and "out" in a dramatic way. Objects to be sorted are represented by dots and the sets into which they are sorted are represented by drawing "strings" around dots. A dot inside (the region delineated by) a set's string is for an object in the set and a dot outside a set's string is for an object not in the set. (See for example Lesson L2 Where's My Nose?.)

This language of strings and dots provides a precise (and nonverbal) way of recording and communicating thoughts about classification. The abilities to classify, to reason about classification, and to extract information from a classification are important skills for everyday life, for intellectual activity, in general, and for the pursuit and understanding of mathematics, in particular. Ordinarily, we are not expected to deal with such problems until our command of our natural language is fully developed and until we face classification problems in adult life. It is not that an ability to classify requires special intelligence, but that it requires a special mode of thinking. The unique quality of the language of strings is that it provides a nonverbal language that is particularly suited to the mode of thinking involved in classification. It frees young minds to think logically and creatively about classes and to report their thinking long before they have extensive verbal skills.

In this strand we present situations and ask carefully phrased questions to continue to advance skills in classification, always remembering that the skills grow out of such experiences. To be able to draw strings and dots is not an objective in itself; to develop the mode of thinking involved is the objective. Thus it is important for us to construct the situations carefully. The sets into which we ask students to classify objects must be determined by well-defined attributes; otherwise, the students have the added problem of deciding whether a certain object does or does not have a certain attribute. (See also the comments at the beginning of Lesson L3 Set Membership.)

For this reason we make extensive use of A-Blocks (Attribute Blocks) and sets of numbers in designing classification situations. Students can immediately say whether or not a block is red, whether or not it is a square, and so on. They know whether or not a given number is less than 15, whether or not it is odd, and so on.

One reason for classifying objects is to count the objects that have a certain attribute. Suppose for example, in a parking lot there are eight red cars and six Volkswagons. If four cars in the lot are neither red nor Volkswagons, must there be 18 cars in the lot? Suppose we count the cars in the parking lot and find there are exactly 15. Is this possible? A string picture immediately settles this apparent paradox.



There are three red Volkswagons.

Relations: The Language of Arrows

Relations are interesting and important to us in our everyday lives, in our careers, in school, and in scientific pursuits. We are always trying to establish, explore, and understand relations. In mathematics it is the same; to study mathematics means to study the relations among mathematical objects like numbers or geometric shapes. The tools we use to understand the everyday world are useful to understand the world of mathematics. Conversely, the tools we develop to help us think about mathematical things often serve us in nonmathematical situations.

A serious study of anything requires a language for representing the things under investigation. The language of arrows provides an apt language for studying and talking about relations. Arrow diagrams are a handy graphical representation of a relation, somewhat the same way that a blueprint is a handy graphical representation of a house. By means of arrow diagrams, we can represent important facts about a given relation in a simple, suggestive, pictorial way -- usually more conveniently than the same information could be presented in words. (See for example the Lesson L1 Sending Letters.)

The convenience of arrow diagrams has important pedagogical consequences for introducing children to the study of relations in the early grades. A child can "read" -- and also draw -- an arrow diagram of a relation long before he or she can read or present the same information in words. The difficulty of presenting certain ideas to children lies not in their intellectual inability to grasp the ideas; rather, the limitations are often mechanical, motor problems of writing or reading about the ideas in conventional systems. Arrow diagrams have all the virtues of a good notation: they present information in a clear, natural way; they are attractive, colorful things to look at; they are easy and fun for children to draw. Another educational bonus occurs when arrow diagrams are used imaginatively. A story or problem may be captured by an arrow diagram, and at the same time, the resulting arrow diagram may suggest other, similar situations. This allows students to call upon previous experience or to expand their creativity.

One of the purposes of this strand is to exploit the power of the language of arrows to help children think logically about relations. Again it must be remembered that our goals concern the thinking process and not the mechanism. We resist making the ability to draw prescribed arrows an objective in itself, just as we resist viewing an arrow diagram as just another format for drill problems in arithmetic.

The general aim of The Languages of Strings and Arrows strand is to suggest situations that are inherently interesting and thought provoking, and to give children modes of thinking and appropriate languages with which they can organize, classify, and analyze. In addition to a varied assortment of lessons concerning sets and relations, this strand includes lessons involving systematic methods for solving combinatorial (counting) problems; methods for collecting, recording and interpreting data in "real life" situations; probabilistic thinking; and networks.

L1 Sending Letters

CAPSULE LESSON SUMMARY

Present an arrow picture where the dots are for children and the arrows are for "sent a letter to ". Collectively, discuss some of the many observations that can be made about this picture. Individually, draw similar arrow pictures.

MATERIALS

Teacher: Colored chalk

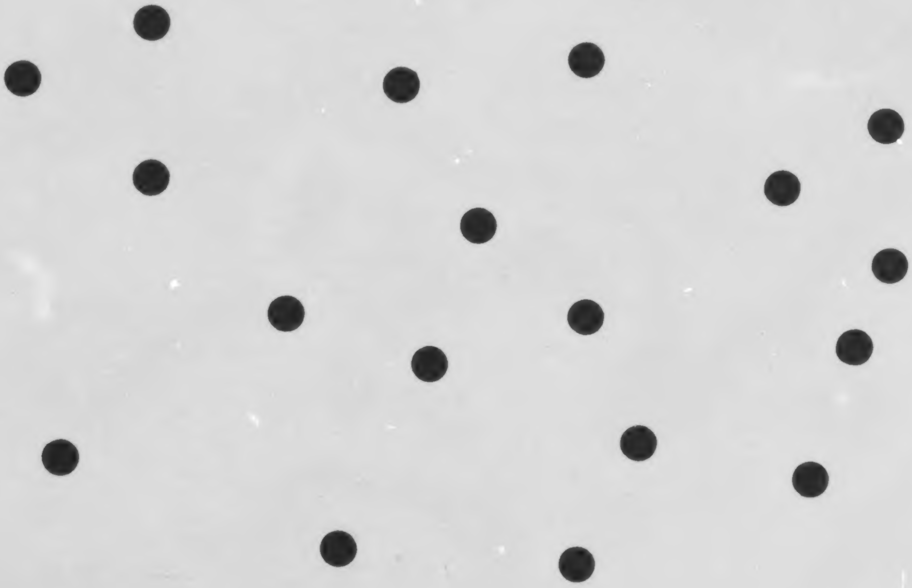
Student: Unlined paper; colored pencils

DESCRIPTION OF LESSON

Begin the lesson with a brief exchange about writing letters to friends.

T: I have a friend, Nick, who has just started second grade at another school. He gave me an arrow picture that he had drawn himself. Nick's picture is very interesting and I would like to show it to you. I will draw it on the board so everyone can see.

Draw eighteen well spaced dots on the board.



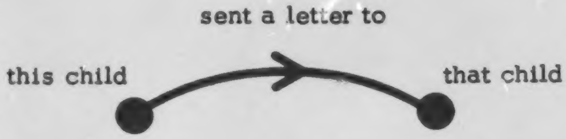
L 1

T: These are the children in Nick's second grade class. How many children are there in Nick's class?

Call on two or three students one at a time to go to the board and count the dots, pointing to each dot as they count. The class should agree that there are eighteen children in Nick's class.

T: Nick told me that during the summer vacation some of the children in his class wrote letters to other children in the class. He drew red arrows to show this.

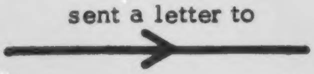
On another part of the board, draw this picture.



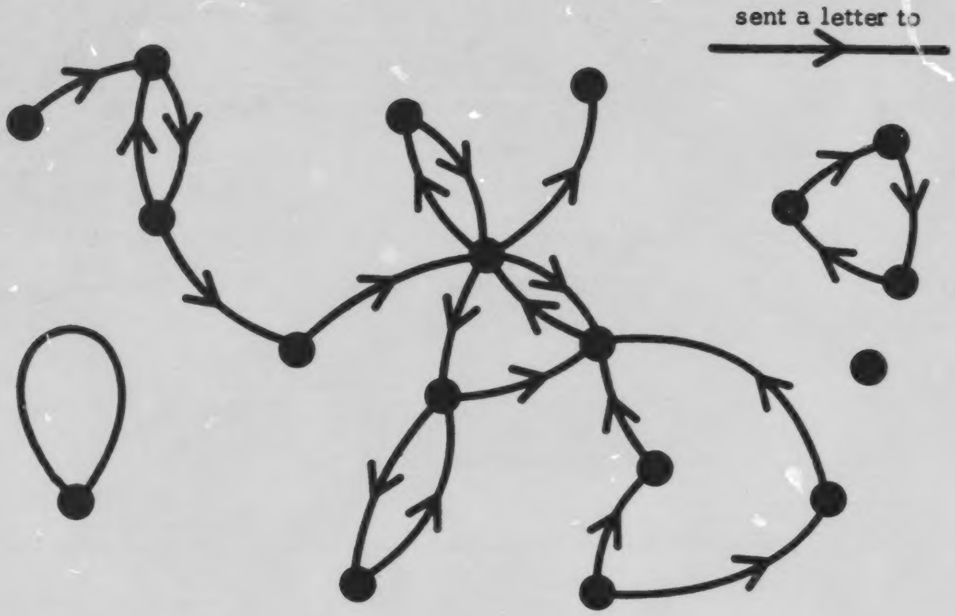
Ask a student to read the message of the red arrow as you point to the starting dot, trace the arrow, and then point to the ending dot.

S: This child sent a letter to that child.

Erase this picture and draw a key arrow near the eighteen dots.



T: Nick's picture has many red arrows so it will take me a couple minutes to finish the drawing on the board. I would like to surprise you, so everyone cover your eyes while I complete the picture.



T: You may open your eyes now. Look at the picture on the board. Think carefully about this picture for a moment without saying anything.

After about a minute, ask the class what they think about Nick's picture. Let students comment spontaneously, even if their remarks are not directly related to the meaning of the arrows. For example, the following comments might be made.

S: It's a very complicated picture.

S: There are lots of arrows.

S: It looks like a dancer.

S: I see a child who sent a lot of letters.

S: I see a child who received a lot of letters.

S: I see someone who didn't receive any letters.

When a comment like one of the last three here is made, ask the student to point to the dot at which he or she is looking and to explain. Be sure that arrows which start at a dot are counted as letters sent and that arrows which end at a dot are counted as letters received.

Let the discussion continue as long as new observations are being made. When the students stop making comments, ask some specific questions; for example,

T: Nick told me that he is the child who sent the most letters. Where is Nick? How many letters did he send? (Four)

Ask a student to point to the dot for Nick and to trace the four arrows for the letters Nick sent. Label the dot.

T: Can you find a child who sent just one letter and received just one letter?

There are many, so allow several students to point to different dots at which just one red arrow starts and just one red arrow ends.

T: Nick told me that his friend Jenna received the most letters. Where is Jenna. How many letters did she receive? (Four)

Ask a student to point to the dot for Jenna and to trace the four arrows for the letters Jenna received. Label the dot.

T (pointing to the dot with a loop): What do you think about this child?

S: That child sent a letter to herself (himself).

T: Find some children who exchanged letters; that is, sent letters to each other.

There are several such pairs of children so allow students to find as many as they can.

T: Nick told me that there is a new pupil in his class. This child sent no letters and received no letters. Where is the new pupil in this picture?

Ask a student to point to the appropriate dot.

T: In the story told by Nick's picture, how many letters were sent in all? (22) How many letters were received? (22)

If you could be one of the children in Nick's class, who would you like to be? Explain why.

Students will react differently to such a question; encourage them to express themselves.

Distribute unlined paper and colored pencils. Ask the students to draw a picture for their own story about sending letters. One way to get them started is to ask them to draw dots for themselves (label it) and then to draw dots for some of their friends. Now ask them to draw some red arrows as they wish which tell a story about sending letters.

You will receive many different pictures. Show some of them to the whole class and let other students comment briefly. You may wish to display some pictures on a bulletin board.

L 2

L2 Where's My Nose?

CAPSULE LESSON SUMMARY

Progressively present a detective story about finding where certain things are in a picture of fourteen dots. Use colored strings for clues.

MATERIALS

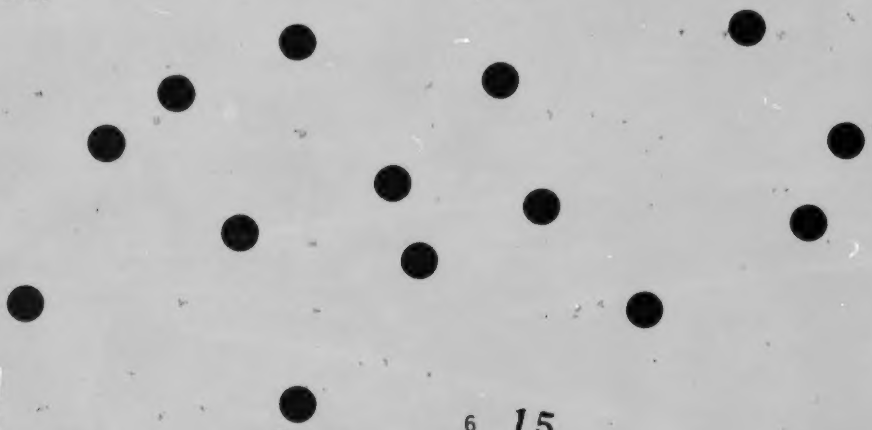
Teacher: "Where's My Nose" storybook; colored chalk
Student: "Where's My Nose" storybook

NOTE: A classroom set of materials for CSMP Mathematics for the Upper Primary Grades, Part I contains 15 copies of the storybook WHERE'S MY NOSE. This number should allow one copy of the storybook for every two students. If you so desire, additional copies can be ordered so that your students may have their own storybook to read or take home.

DESCRIPTION OF LESSON

We suggest that you tell your class the story of WHERE'S MY NOSE? holding up your copy of the book at crucial moments so that your students can see the pictures. Then, when you have finished telling the story and discussing the incidental questions, distribute copies of the storybook, pairing the children off as the need arises.

Before you start the class, draw fourteen dots well spread out across your board.



T: Today I'm going to tell you a story about my friend "Nick" and his grandmother. Nick loved his grandmother very much and used to visit her often. She lived in the country with her charming black poodle and her three lovely cats. Whenever Nick came to see her, she made up very interesting games for them to play together. She knew that Nick wanted to be a detective when he grew up so she often invented detective games for him.

One day Nick went to visit his grandmother. She was very happy to see him and he was happy to be with her again. He had been wondering for several days what new game she would have for him when he arrived. He was very excited when she told him that she had prepared a detective game for him. She gave him a piece of paper with lots of dots on it just like the picture you see on the board.

Nick was puzzled by this picture because he had been expecting to receive a detective game from his grandmother. But she told him to be patient and that he would soon be surprised. First she said he should count the dots. How many dots are there?

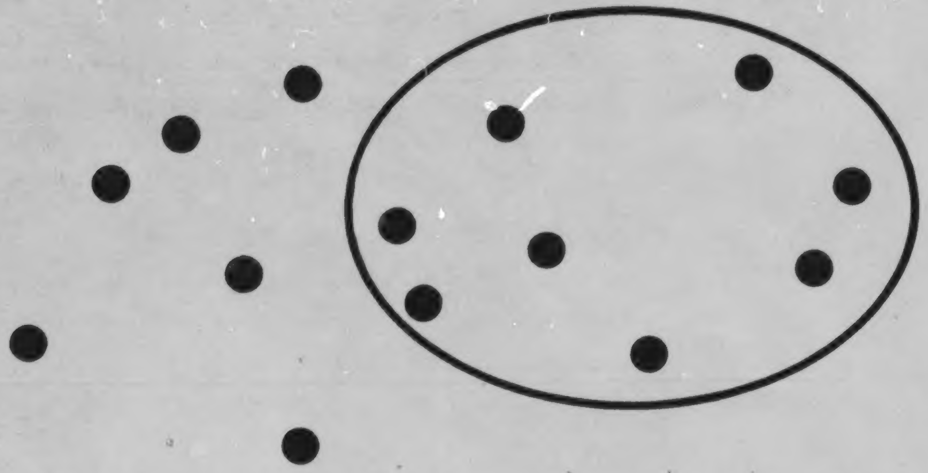
S: Fourteen.

Ask a student to point to the dots and count aloud with the class watching to be sure all of the dots are counted just once.

T: She told him the dots were for three rabbits, her charming black poodle, her three lovely cats, three bugs, a squirrel, herself, Nick, and Nick's nose. Then she asked him if he could identify the dots (i.e., tell her which dots were for which objects). Of course it was not possible for Nick to do so. His grandmother told him he was going to have a chance to be a detective. She would give him some clues so that he could identify each dot. First she drew this red string.

L 2

Draw a red string around eight of the dots you have drawn on the board.



T: She told him that the dots inside the red string are for animals with four legs. Nick thought for a moment and then shouted that the dot for him must be outside the red string. Why was he right?

S: Yes, because he has only two legs.

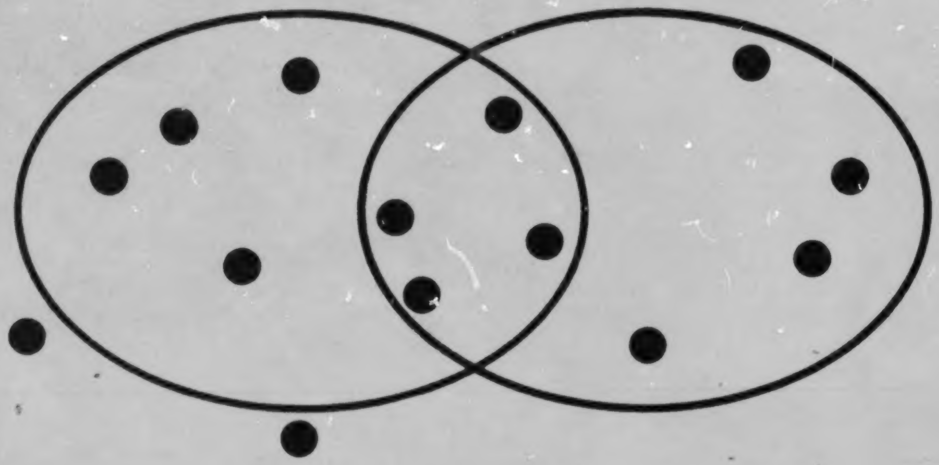
T: What about the other dots outside the red string?

S: One is for Nick's nose, one is for Nick's grandmother, and the other three are for the three bugs (because bugs have six legs).

T: Nick's grandmother then asked if it was possible at this time to say any more about the dots. Perhaps it is now possible to locate Nick's nose. Nick thought and thought but could not think of any additional information from this clue, so he asked his grandmother to give him a second clue.

Draw a blue string as in the next illustration. Be careful that there are eight dots in the blue string and four of these are also in the red string.

T: His grandmother then drew this blue string and told him that the blue string was for all things which are able to climb trees.



T: Nick was a great tree climber so he surely was in the blue string. What do you think about those two dots that are outside of both strings? Nick was sure he knew something about them.

S: One is for Nick's nose and one is for Nick's grandmother.

T: How are you sure?

S: Well, Nick's nose can't climb a tree all by itself and Nick's grandmother said she was no longer able to climb trees. That's all that's left (meaning that all the other objects are already accounted for as being animals with four legs or things able to climb trees or both).

T: But which is which?

S: It's still not possible to tell. We need another clue.

L 2

T: Nick's grandmother reminded Nick that her poodle is not able to climb trees. Can you now locate her poodle?

S: It must be one of these four dots (pointing to the dots inside the red string but outside the blue string), but it's not possible to tell which.

T: How about the three bugs?

S: Three of these four (pointing to the dots inside the blue string but outside the red one).

T: Why?

S: They don't have four legs so they're outside the red string, but they can climb trees so they're inside the blue one.

T: How about the three lovely cats? Everyone knows that cats have four legs. So what question do we need to ask?

S: "Do they climb trees?"

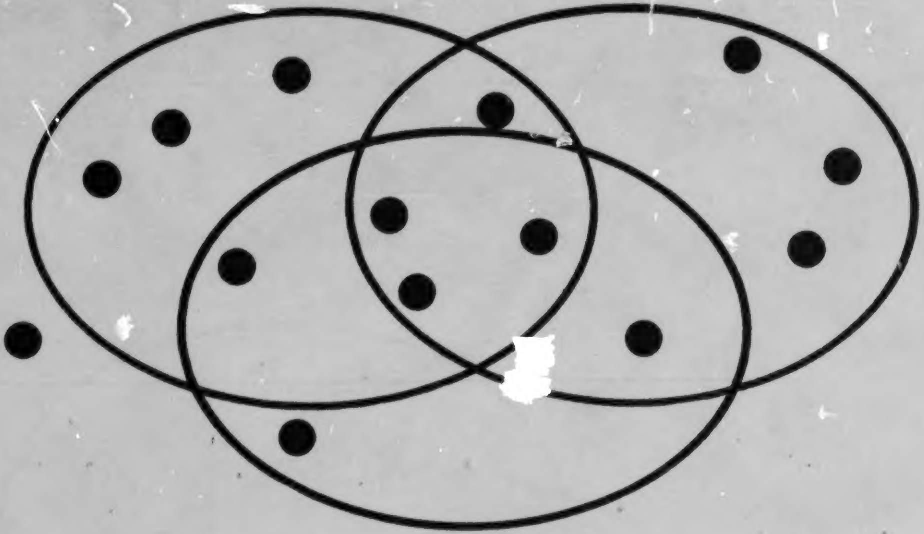
T: Yes, they do.

S: So they are here (pointing to the four dots in the "middle"; i.e., inside both the red and blue strings). Three of these four dots must be the cats.

T: Now Nick's grandmother was ready to give the final clue. After this clue all the dots will be known. Let's see how good a detective you are.

Draw a green string as in the next illustration. Be careful that you have the correct number of dots in each region of your picture.

T: She drew this green string and told Nick it was for all things that like to eat meat.



T: Nick became very excited. He loved meat, so now he could find the dot for himself. Where is Nick?

S: There (pointing to the only dot inside both the green and blue strings but outside the red one).

T: Explain your answer.

S: Nick likes meat and can climb trees and does not have four legs. So Nick is in the green string and also in the blue string, but he is outside the red string. This has to be Nick because it's the only dot there.

Label the dot "Nick".

T: Very good. Where is Nick's grandmother?

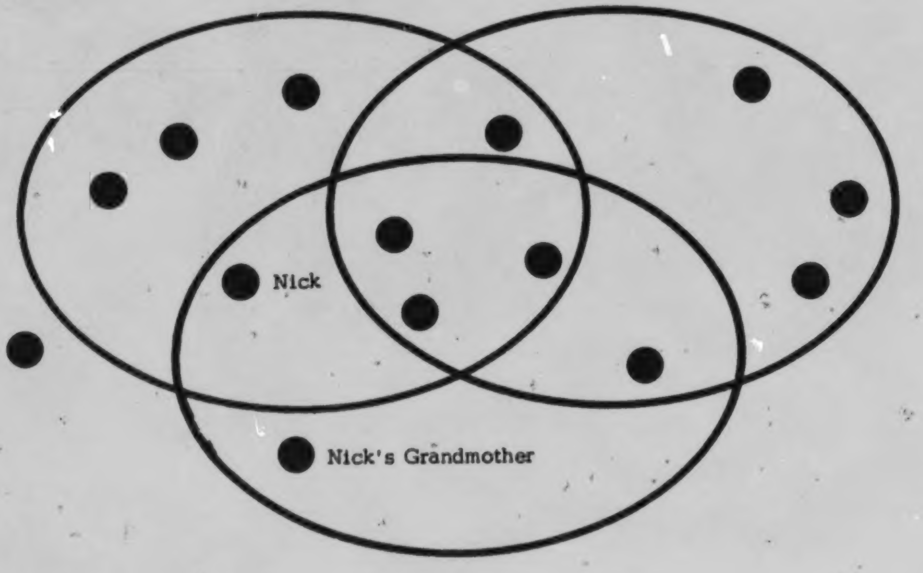
S: She must be there (pointing to the dot inside the green string but outside both the red and blue strings).

T: Why?

S: Because she likes meat but can't climb trees and doesn't have four legs. And there is only one dot inside the green string and outside both the red and blue strings.

T: Good thinking. I will label this dot so we will remember it's Nick's grandmother.

You should now have two of the fourteen dots labeled.

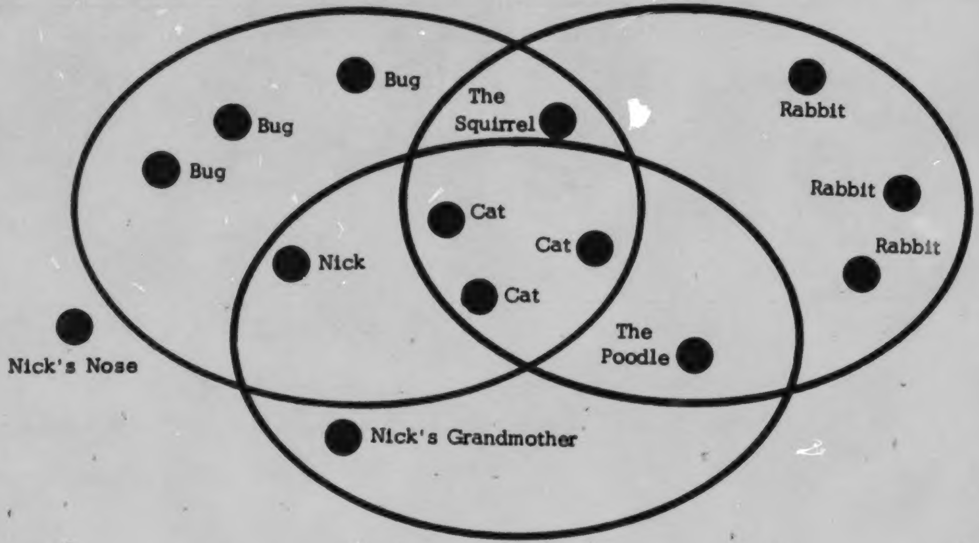


T: The three lovely cats also love meat as most cats do. Who can locate the three cats?

S: There (pointing to the three dots in the middle), because we already know cats have four legs and can climb trees.

T: And the poodle certainly likes meat. So now you should be able to finish the problem completely. Where are the rabbits, the squirrel, the poodle, the bugs, and Nick's nose?

Choose one student to locate the nose, a second to find the poodle, a third to indicate the bugs, a fourth for the rabbits, and a fifth for the squirrel. Ask each student to explain his or her answer. After a correct answer, label the appropriate dots. The final picture will have all the dots labeled.


Bug
Bug
Bug
The Squirrel
Rabbit
Rabbit
Rabbit
Cat
Cat
Cat
Nick
Nick's Nose
The Poodle
Nick's Grandmother

T: Nick was very proud of his successful detective work and you should be proud of yourselves also. That was a problem which could only be solved by clever detectives. What did you think of the story?

Distribute copies of the storybook and let the students look at the pictures and read the story again on their own.

L 3

L3 Set Membership

CAPSULE LESSON SUMMARY

Locate dots for students in a three string picture and identify which students a certain dot could be for. Determine where a person is in a string picture by asking appropriate questions about the person.

MATERIALS

Teacher: Colored chalk

Student: Worksheets L3* and **; pencil

DESCRIPTION OF LESSON

Before starting the lesson, look over the attire of the students in your class this day. Try to determine labels for three overlapping strings so that there is a student represented in each of the eight regions (see the next illustration). You might consider some of the following possibilities:

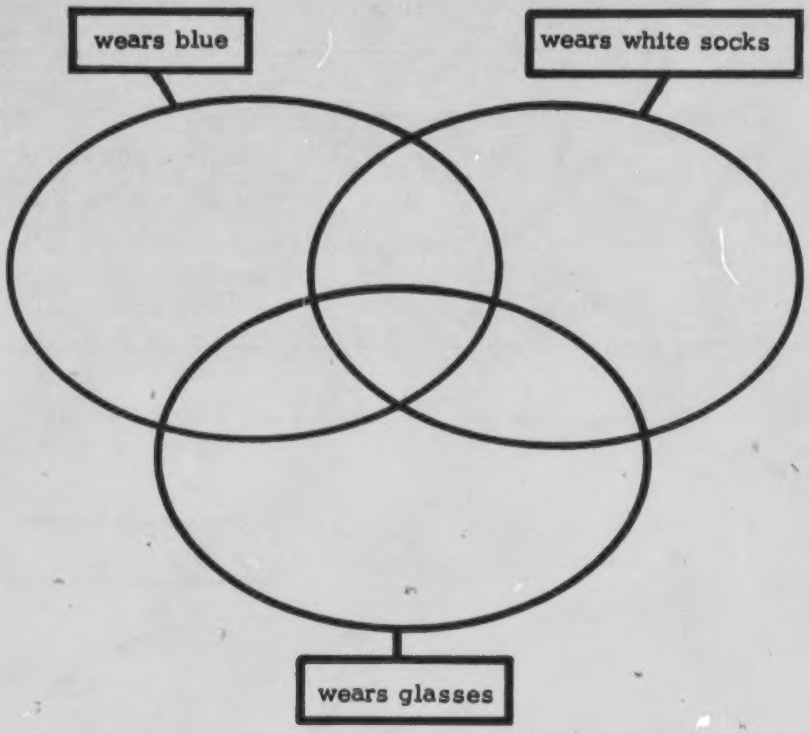
- people who are wearing something blue (red, yellow, green, etc.).
- people who are wearing glasses.
- people who are wearing white socks.
- people who are wearing sweaters.
- people who are wearing sneakers.
- people who are wearing a belt.

This lesson is described with the first three possibilities above.

Exercise 1

Draw and label three different colored strings.

T: The blue string is for those people who are wearing some blue. The red string is for those people who are wearing white socks. The green string is for those people who are wearing glasses now.



T: Who would like to show us where they are in this picture? Laura, come up and stand on this chair so everyone can see you. Is Laura wearing some blue?

S: Yes, blue stripes on her blouse.

T: Is Laura wearing white socks?

S: No.

T: Is Laura wearing glasses?

S: No.

L 3

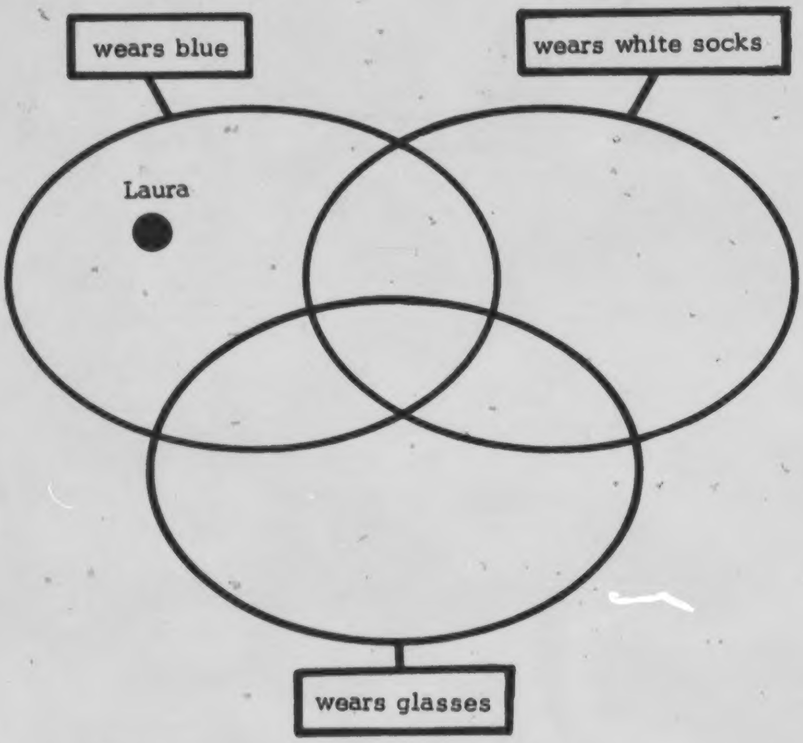
T: Laura, draw a dot for yourself in our picture.

Suppose this student needs some help locating a dot for herself. You should use large sweeping motions with your open hand to indicate the interior of the strings.

T: Laura is wearing blue, so is she inside or outside of the blue string? (Inside) Laura is not wearing white socks, so is she inside or outside of the red string? (Outside) Laura is not wearing glasses, so is she inside or outside of the green string? (Outside)

Help the student draw a dot for herself in the correct place.

T: Let's write Laura's name by her dot so we can remember that this dot is for Laura.

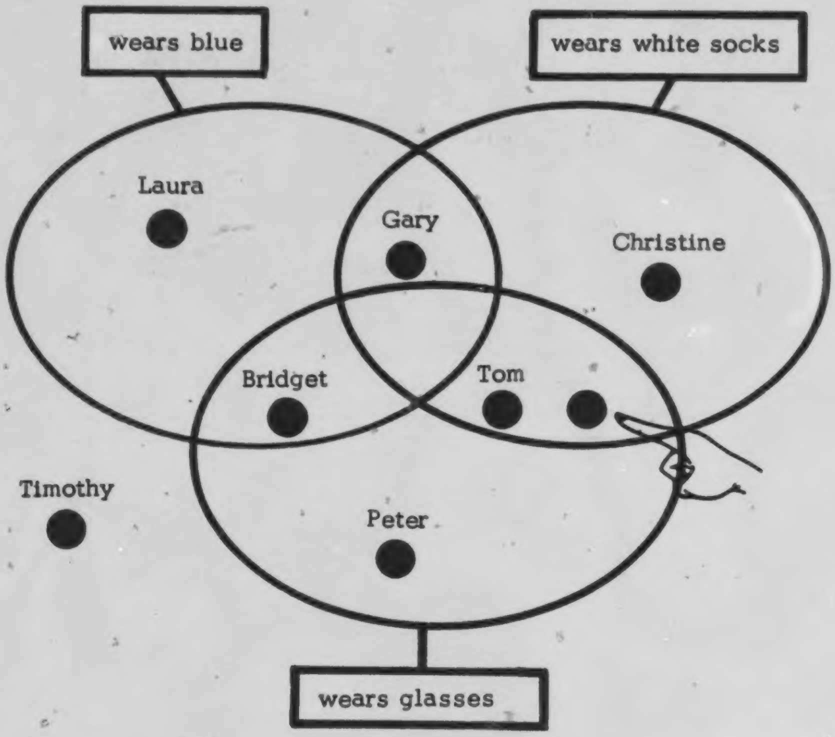


Call on several other students to locate themselves in the picture. Each time direct the student to stand on the chair, ask the three important questions, and then ask the student to draw his or her dot. Try to select students whose dots go in the various regions, including the region outside of all three strings.

Exercise 2

Leave all the dots drawn thus far in the picture. Draw a new dot, making sure the class knows which dot you have just drawn.

T: I am thinking of somebody in this class and the dot for that person is here.


wears blue
wears white socks
Laura
Gary
Christine
Bridget
Tom
Timothy
Peter
wears glasses

T: Whom could this dot be for?

S: Kenneth.

L 3

In this situation, students might answer "Tom" where Tom's dot is already in the picture. Remind them that Tom's dot has been labeled and this dot should be for someone else.

T: Stand up Kenneth. Is Kenneth wearing some blue?

S: No.

T: So a dot for Kenneth should be outside the blue string.
Is Kenneth wearing white socks?

S: Yes.

T: Then a dot for Kenneth should be inside the red string.
Is Kenneth wearing glasses?

S: No.

T: Then a dot for Kenneth should be outside the green string; this dot is inside the green string.

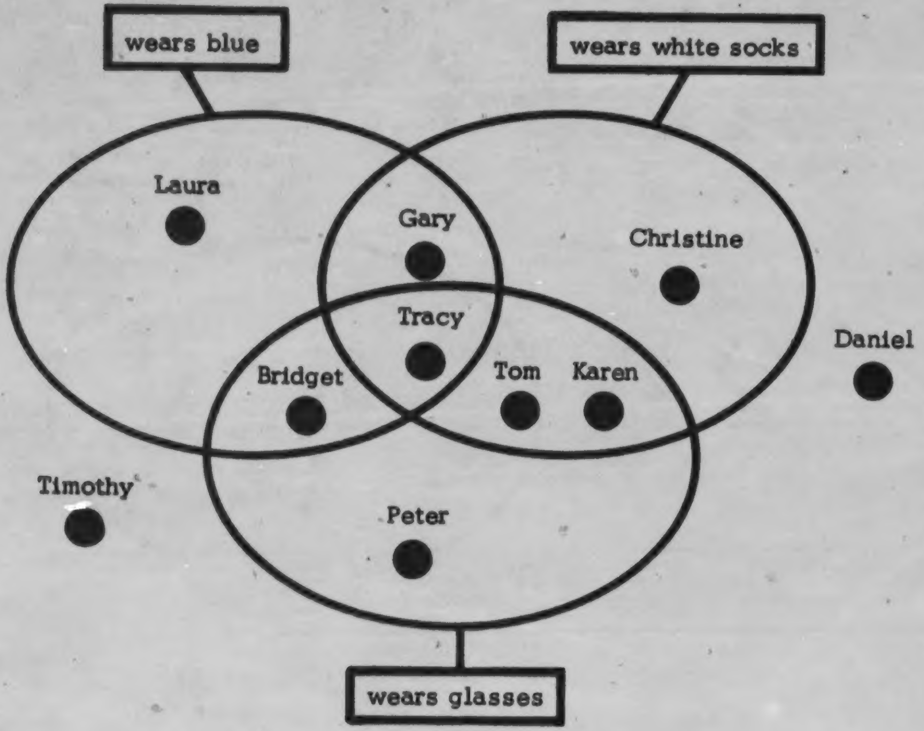
S: It can't be for Kenneth.

T: Does someone else have an idea?

S: Karen.

Ask the students to check with you that Karen is not wearing blue, is wearing white socks, and is wearing glasses.

Repeat this exercise drawing a dot inside all three strings (Tracy's dot in the picture below) and then again with a dot outside all three strings (Daniel's dot).


wears blue
wears white socks
Laura
Gary
Christine
Tracy
Daniel
Bridget
Tom
Karen
Timothy
Peter
wears glasses

Students may suggest that a dot outside all three strings could be for other things than people or students in your classroom.

S: My nose.

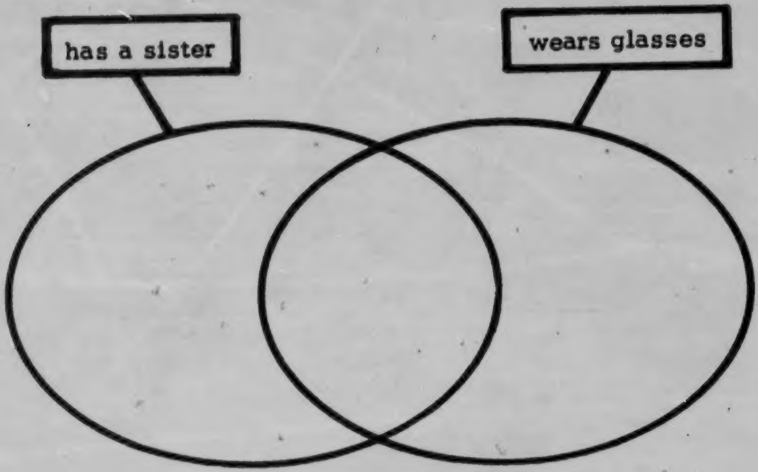
S: An elephant.

T: Yes, those things are not people and do not wear clothes or glasses.

L 3

Exercise 3

Erase the board and then draw two new strings. As you do, explain that the blue string is for people who have a sister and the red string is for people who wear glasses.



When you ask the next question, the students cannot correctly respond without first asking you some questions. Do not allow the students to just guess where a dot should be drawn. Discourage guessing by asking if they are sure that is where the dot belongs.

T: I am thinking about my friend Nick. Who can draw a dot for Nick in this picture?

S: Does Nick have a sister?

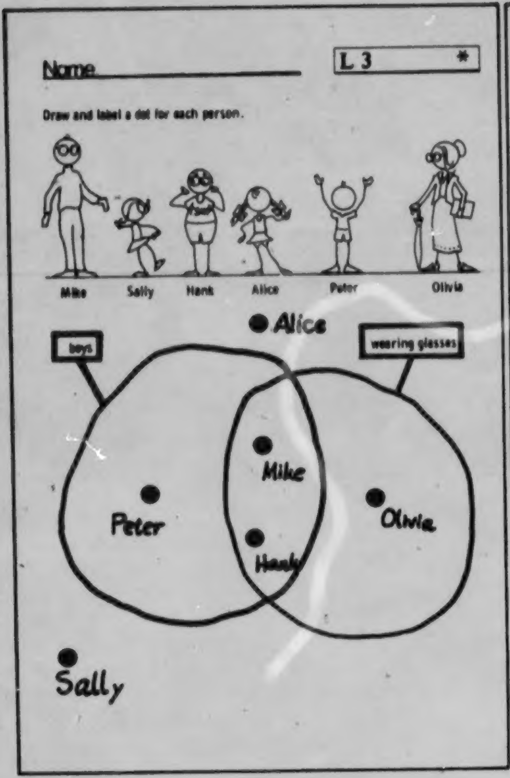
T: That's a very good question. Yes, he does have a sister.

S: Does he wear glasses?

T: No, he does not.

Invite a student to draw and label a dot for Nick. These questions might not be asked immediately by your students. If they are confused or begin to draw dots randomly, suggest that they ask you some questions about Nick. Continue this exercise with other children unknown to the students in your class so that you get dots in different regions.

Worksheets L3* and ** are available for independent work. Be sure your students understand that they are to draw and label a dot for each person pictured.



Name

L 3 *

Draw and label a dot for each person.

Mike Sally Hank Alice Peter Olivia

boys

wearing glasses

Alice

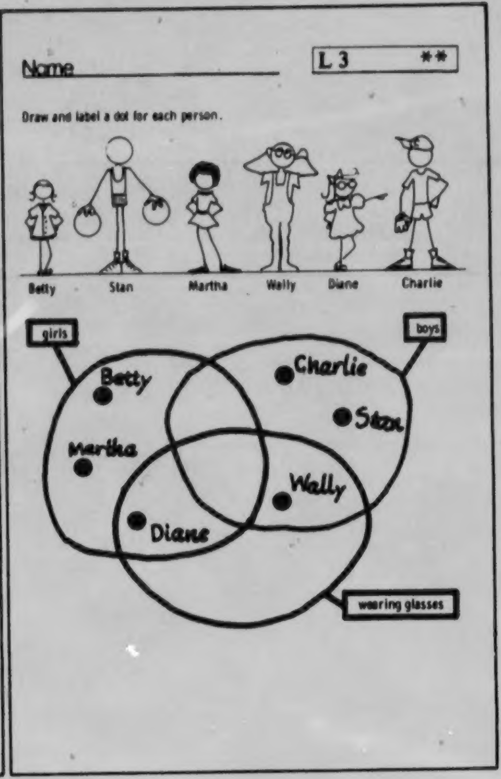
Peter

Mike

Hank

Olivia

Sally



Name

L 3 **

Draw and label a dot for each person.

Betty Stan Martha Wally Diane Charlie

girls

boys

wearing glasses

Betty

Charlie

Stan

Martha

Wally

Diane

L4 Introduction to A-Blocks

CAPSULE LESSON SUMMARY

Introduce the A-Blocks and describe each piece with three attributes: color, shape, and size. Find pieces which differ in one way, two ways, and three ways. Place pieces in a string picture and find pieces which could be in a certain region of the string picture.

MATERIALS

Teacher: A-Block String Game Kit; colored chalk; box

Student: None

NOTE: Before this lesson begins, you will need to prepare some of the materials from the A-Block String Game Kit. These materials will be used again in all future lessons on A-Blocks and the String Game, so keep them together in the envelope with the kit.

Punch out one set of shapes (twenty-four pieces). If you have a magnetic chalkboard or gridboard, magnetize each piece by sticking a small piece of the magnetic material to the back. If you do not have a magnetic board available, stick a loop of masking tape to the back of each piece. Masking tape loses its stickiness quickly, so be prepared to reinforce the pieces with new loops of tape during the lesson.

DESCRIPTION OF LESSON

Put the twenty-four A-Block pieces (shapes) in a box about the size of a shoe box. Sort them so that you can locate any given one quickly.

Exercise 1

T: In this box I have some cardboard pieces in different shapes, colors and sizes. They are called "A-Blocks". What do you think are the different shapes of the pieces?

S: Squares.

S: Triangles.

S: Circles.

Many students have seen the A-Blocks before, perhaps last year, and are familiar with them. In any case, these responses usually come immediately. If not, tell them the three shapes involve. As each shape is mentioned, write the word on the chalkboard.

T: What are the different colors of the pieces?

S: Red.

S: Blue.

S: Green.

S: Yellow.

As each color is mentioned, write the word on the chalkboard.

T: There are two different sizes; what are they?

S: Big and little.

Write these words on the chalkboard.

Tell the class it will be their job to get the pieces out of the box. Call on students one at a time to describe to you which piece they want to see. As the pieces are described and taken out of the box, put them on the chalkboard for all to see.

T: You tell me which piece you want, and I will take it out of the box for you.

S: A square.

T: I have many squares. Which one do you want?

S: Any of them.

Hold up a square, say a big red square.

T: Here's a square. I will put this piece back in the box. Now who can tell me which piece it was?

S: A square.

T (holding up a green square): This one?

S: No, a red square.

T (holding up the little red square): This one?

S: No. It was a big red square.

T (holding up the big red square): Was it this piece?

S: Yes.

Put the piece on the chalkboard.

T: Someone else ask me for a piece.

S: A big triangle.

T: Yes, I have several big triangles. Which one do you want?

S: A big blue triangle.

Take out the big blue triangle and put it on the chalkboard.

T: Very good. Sam told me the size, the color, and the shape of the piece he wanted to see.
Can someone describe another piece with the same shape?

S: A big yellow triangle.

Put the big yellow triangle on the board and continue asking for pieces. If someone describes a piece already on the board, point to it and ask for one we have not yet seen. If the activity begins to drag, ask a student to describe all the rest of the triangles and put them up as they are described. In this way, get all the triangular pieces on the chalkboard.

T: How many triangles do we have?

S: 8.

T: How many circles do you think there are?

S: 8.

Take out all (the rest of) the circles and put them on the board.

T: What else could be in the box?

S: Just some squares.

T: Which ones?

Take them out as they are descirbed and put them on the board.

T: How many pieces are there altogether?

S: 24.

T: Can you explain?

S: I counted them.

S: 8 triangles, 8 circles, and 8 squares make 24 pieces in all.

Write this calculation on the board.

$$8 + 8 + 8 = 24$$

T: Can anyone tell us another number story for this.

Accept any number story for 24, but write only "3 x 8 = 24" on the board. If necessary, suggest this multiplication story yourself.

$$8 + 8 + 8 = 24$$
$$3 \times 8 = 24$$

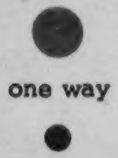
Exercise 2

Take a large yellow circle and put it on a separate section of the board.

T: Who can show me a piece that is different from this piece in just one way?

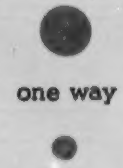
L 4

Suppose a student, Jason, chooses a small green circle.



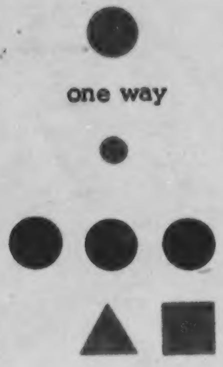
T: Well, Jason, your piece is a different color but it is also a different size, so it is different from my piece in two ways.

Remove the small green circle and ask the student to make another choice. Suppose Jason selects a small yellow circle.



T: Is Jason's piece a different color? (No)
Is it a different shape? (No)
Is it a different size? (Yes)
So the pieces are different in just one way. Who can show another piece different from mine in just one way?

Invite several students, one at a time, to select a piece. Each time let the rest of the class check that a correct piece was chosen. It is not necessary that all the pieces which differ in just one way be found; but, for your information, all the possibilities are shown in the next illustration.



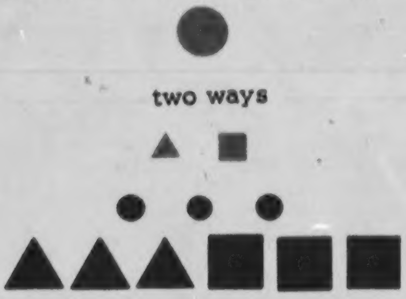
one way

(different only in size)

(different only in color)

(different only in shape)

If the lesson has been moving along at a brisk pace, the class might enjoy finding some of the pieces which differ from yours in two ways and perhaps even in three ways. All the possibilities are shown below.



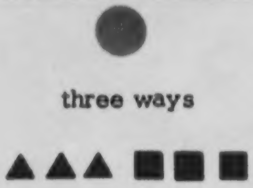
two ways

(different only in:

size and shape)

size and color)

shape and color)



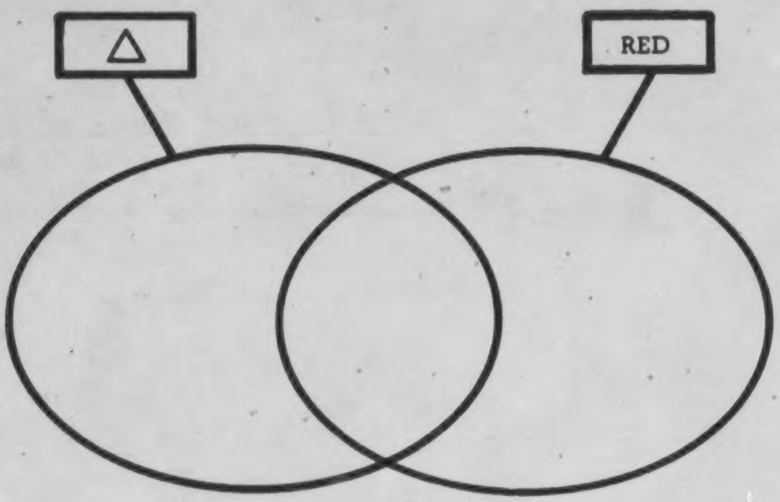
three ways

(different in:

size, shape, and color)

Exercise 3

Put all the pieces back in the box. Draw two large overlapping strings in two colors and label them as in the next illustration.



T: The blue string is for all the triangles. The red string is for all the red pieces.

Hold up the large red square.

T: Who can put this piece in the picture?

Call on a student to put the piece in the picture and ask the class for agreement or disagreement. If the piece is incorrectly placed, ask for an explanation and call on another student to place the piece. If the piece is correctly placed, ask:

T: Why do you think Kristy put it inside the red string?

S: Because the red string is for red pieces.

T: Why did Kristy put it outside the blue string?

S: Because the blue string is for triangles, not squares.

T: Yes, you must think about both strings.

T: Let's put another piece in our picture. Where should we put this little yellow circle?

Hold up the little yellow circle, and call on a student to put it in the picture. Again, ask the class for agreement or disagreement.

T: Does it go in the red string?

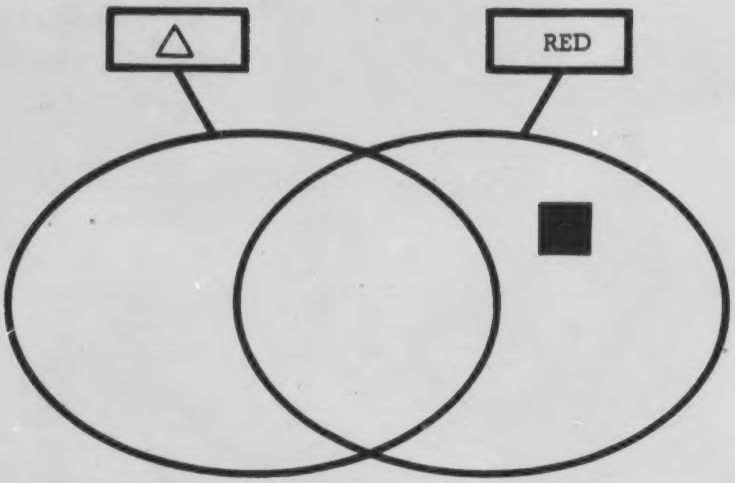
S: No, it is not red.

T: Does it go in the blue string?

S: No, it is not a triangle.

T: Then it is outside both strings.

Let a student put the piece correctly in the picture, if it has not already been done.



Continue in this way, asking students to place pieces in the picture. Not every piece needs to be discussed, but try to get at least one piece in every region. Be sure the students understand the two questions that help one decide where a piece goes: Is it red? Is it a triangle?

Sometimes vary the activity by asking:

T: Can you find a piece that belongs here?
(or)
I'll put this piece here. Is it correct?

Leave the pieces presently on the board and continue the exercise with some new types of activities.

Ask a student to describe a piece still in the box, and then invite the student to put it in the correct place in the picture. Repeat this activity several times.

Put a dot in the string picture. Ask a student to find a piece in the box that the dot could be for. Let the class discuss whether the choice is correct. If it is correct, put that piece on the board next to the dot. Ask another student to find a piece in the box that the same dot could be for. If there are no more possibilities or a student thinks there are none, ask the student to explain. Repeat this activity with a dot in another region.

L5 Exchanging Names

CAPSULE LESSON SUMMARY

Draw a picture of dots for the students in the class, exchange names among the class, and draw arrows for "I have your name". Individually, draw a similar arrow picture for each row (table) in the class.

MATERIALS

Teacher: Colored chalk; small pieces of paper (about 10 cm by 10 cm)

Student: Unlined paper; colored pencils

DESCRIPTION OF LESSON

Exercise 1

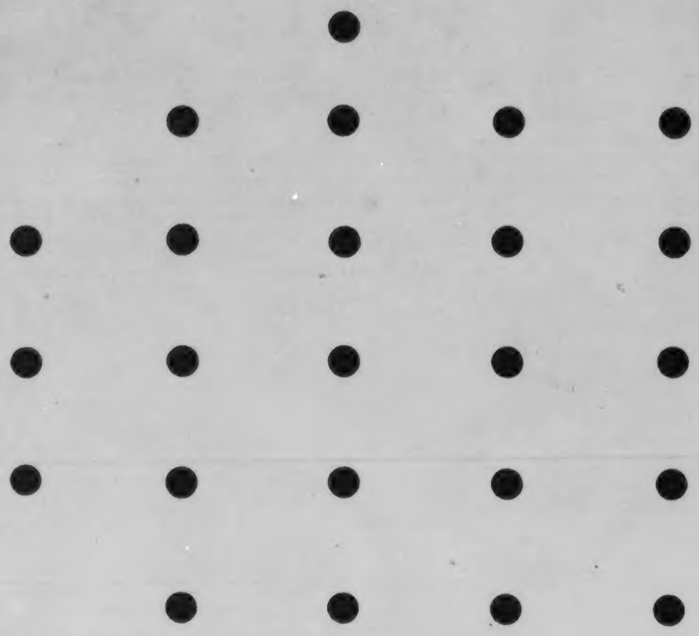
T: I am going to draw some dots on the board. Watch me and silently count them as I draw.

Use the largest available chalkboard and draw a dot for each student in the room and one for yourself. Use as much space on the board as possible; spread out the dots as much as you can. Arrange the dots in the same relative positions as the students are sitting. (The top of the board corresponds to the front of the room where you are standing.) Make a special point of examining the class several times while you draw. When you have drawn about half of the dots, say:

T: Think about what these dots could be for.

You will likely begin getting reactions from the class, but keep drawing. Depending on the arrangement of your classroom, your finished picture of dots may be organized in rows as in the next illustration.

L 5

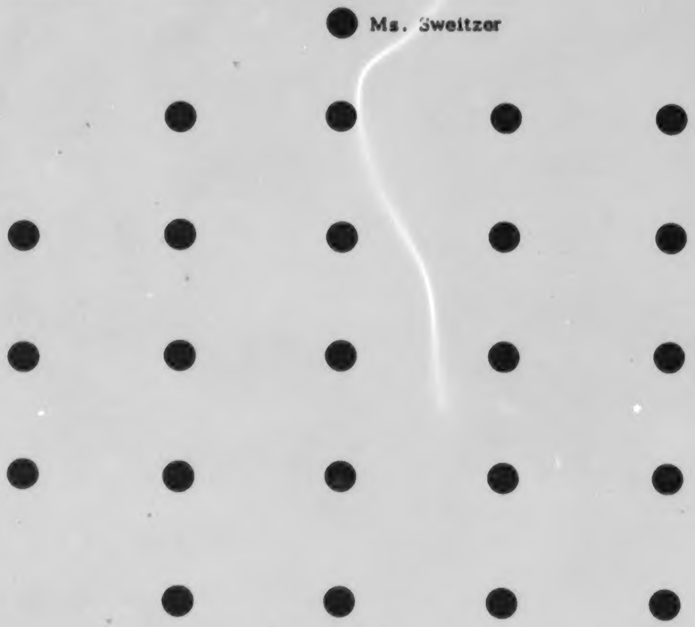


T: How many dots?

S: 24.

T (pointing to the dot at the top of the picture): I am here in this picture.

Write your name next to this dot.



T: Can you find your dot on the board?

Let several students come to the board and point to their dots. If a student points to the correct dot, label it with his or her name. Try to get dots labeled from various parts of the room. After five or six dots are labeled, point to an unlabeled dot and ask:

T: Will the person whose dot is here please stand up?

When the correct student stands, label the dot. Do this several times. If some students have difficulty locating themselves, you could ask the class to assist them. Point to an unlabeled dot and say:

T: Who is here?

S: Lynn.

Ask Lynn to stand up and check with the rest of the class that the dot is for her. Then label it.

Continue this activity, sometimes calling on a student to come to the board and sometimes pointing to a dot and asking the student to stand at his or her desk, until all the dots have been labeled.

Exercise 2

Leave your picture on the board. Pass out small pieces of paper (about 10 cm by 10 cm) and ask the students to carefully write their names on the papers, then fold them in half. Write your name on a similiar piece of paper and fold it. Next, collect all the names in a box.

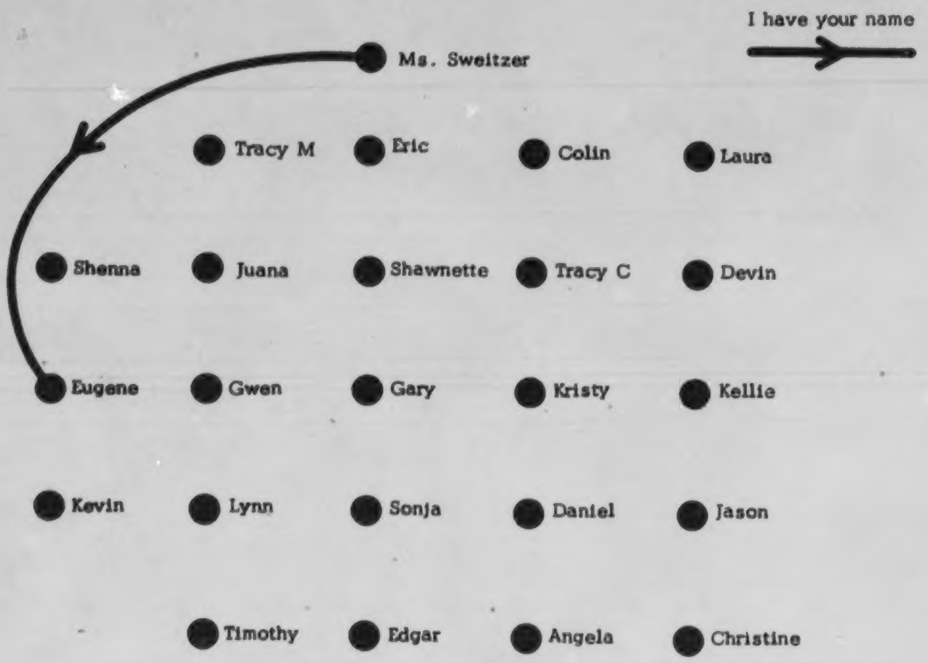
T: I am going to mix up the names and then pass them out. When you get a paper, open it and read the name on it but do not tell anyone whose name you have.

L 5

When you do this, secretly try to arrange that some student gets his or her own name. Take the last paper in the box yourself.

T: I have Eugene's name, so I draw an arrow from my dot to Eugene's dot.

After drawing the arrow, explain that the red arrow is for "I have your name". Near the picture, draw a red key arrow labeled "I have your name".


I have your name
Ms. Sweitzer
Tracy M
Eric
Colin
Laura
Shenna
Juana
Shawnette
Tracy C
Devin
Eugene
Gwen
Gary
Kristy
Kellie
Kevin
Lynn
Sonja
Daniel
Jason
Timothy
Edgar
Angela
Christine

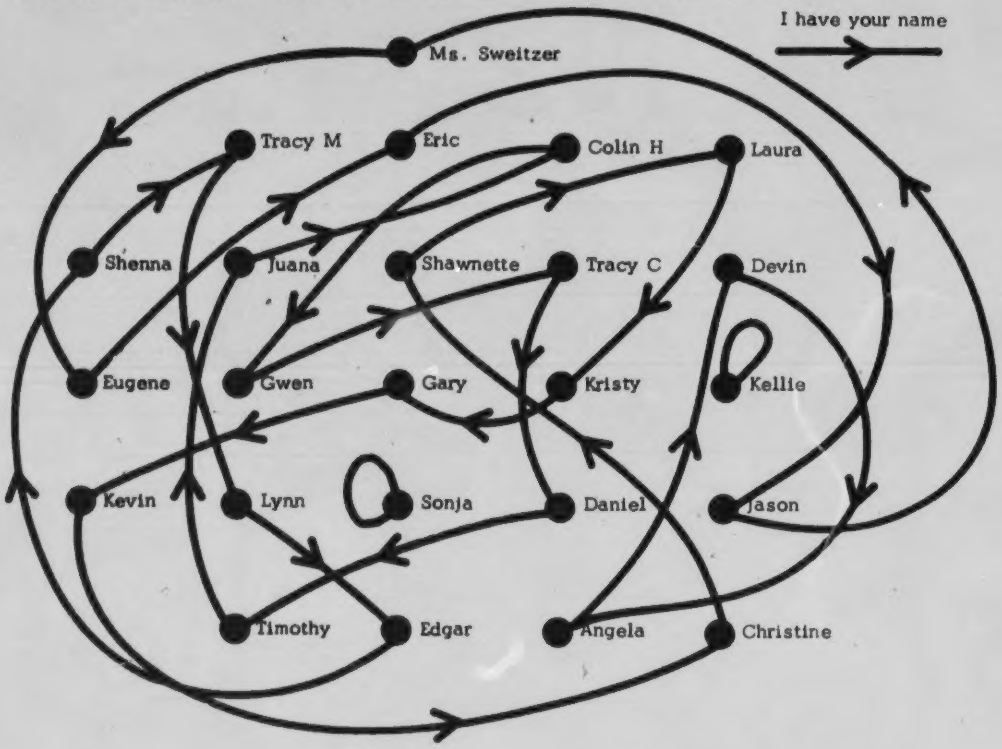
T: Who would like to trace an arrow to show us whose name you have?

Invite students, one at a time, to trace arrows. When a correct arrow is traced, draw it on the board. Continue this until most of the arrows have been drawn. The picture will eventually become very crowded (and the class most likely will comment on how complicated it has become), but try to draw arrows which are easy to follow.

If an error is made, it will certainly be noticed either when it is made or very shortly thereafter. An error will result in more than one arrow ending at a particular dot. In such a case, tell the class (they may observe themselves) something is wrong and ask them to explain the problem.

When there are only a few missing arrows, ask the class to predict which name a student has before it is revealed.

A completed picture from our development class is shown below.


I have your name
Ms. Sweitzer
Tracy M
Eric
Colin H
Laura
Shenna
Juana
Shawnette
Tracy C
Devin
Eugene
Gwen
Gary
Kristy
Kellie
Kevin
Lynn
Sonja
Daniel
Jason
Timothy
Edgar
Angela
Christine

The following are some questions you could ask about the completed picture.

T: How many arrows end at each dot? (1)
How many arrows start at each dot? (1)
What does this loop mean? (A child got his or her own name.)
How many arrows are there in our picture? (24, or the same as the number of dots.)

Exercise 3

Erase the board.
Distribute unlined paper and colored pencils.

T: Imagine that you played this game with just the people in your row (or at your table, or whatever is appropriate so long as there are between five and ten students in each group). Draw a picture of what could happen.

Be sure the students understand that they need only draw dots for the students in their row (at their table) and that their pictures will show names are exchanged among the people in their row (at their table). Walk around and observe the drawings. It might help some of the students to think about the situation if you ask them to explain their drawings to you.

L6 Detective Story with Coins #1

CAPSULE LESSON SUMMARY

Present a detective story about a secret amount of money in a piggy bank. Use clues which give information about the particular coins in the bank. Do a mental arithmetic activity counting by twos and by tens. Draw a +10 spiral to illustrate counting by tens in the class.

MATERIALS

Teacher:	Coins (one dime, three nickels, one penny); small container; colored chalk
Student:	Paper; colored pencils

DESCRIPTION OF LESSON

Exercise 1

Before the lesson begins, put one dime, three nickels, and one penny into a small container. Choose a container so that the coins inside are not visible and you can hear the coins clank against the sides. A small opague plastic vial is such a container.

T: How many of you have piggy banks? What do you do with a piggy bank?

S: I save pennies in my bank.

S: My father gave me some foreign coins to put in my bank.

S: I put pennies and nickels and dimes and quarters in my bank.

Choose one of your students to be the star of your story.

T: Tracy got a new piggy bank and it came with something already in it. She shook it and listened to the clanking sound the coins inside made. "I wish I knew how much money is in my bank," Tracy said to herself.

L 6

Shake the container and let the class listen.

T: Clank! Clank! Clank! " said the coins. Suddenly, Tracy realized the coins were talking to her. "Would you like to know how much money is in your new piggy bank?" they asked. "Yes! Yes, I would. Can you tell me?" cried Tracy. The coins answered: "We cannot tell you the secret, but we will give you some clues. If you are a good detective, you can discover our secret."

First clue

"The first clue is: we are five coins and we are only dimes, nickels, and pennies."

Write this information on the board.

DIMES

NICKELS

PENNIES

Ask the students to suggest some possibilities for what coins are in the bank. You should get suggestions such as: three dimes and two nickels; two dimes, two nickels and a penny; five dimes; four nickels and a penny; and so on. After a short while, you should have received many different suggestions.

T: Tracy was not very happy. "There are too many possibilities," she said to the coins.

Second clue

"We will give you another clue. Pay close attention to this clue: at least one of us is a dime; at least one of us is a nickel; and at least one of us is a penny." Now, what do you think could be in the bank?

Draw this chart on the board.

10¢ DIMES	5¢ NICKELS	1¢ PENNIES

Let the students give you suggestions and write correct combinations in the chart. Continue this until you get the list below.

10¢ DIMES	5¢ NICKELS	1¢ PENNIES	
○ ○ ○	○	○	36¢
○ ○	○ ○	○	31¢
○ ○	○	○ ○	27¢
○	○ ○ ○	○	26¢
○	○ ○	○ ○	22¢
○	○	○ ○ ○	18¢

As you get each new suggestion, check with the class that it is correct, ask how many cents it makes, and write the amount of money next to that line in your chart.

L 6

T: What is the largest amount of money that could be in the bank?

S: 36¢.

T: What is the smallest amount of money that could be in the bank?

S: 18¢.

T: Now you know there is one of these amounts of money in Tracy's new piggy bank (point to the six possibilities). "Are you ready for another clue?" the coins asked.
"Yes, yes I am," said Tracy and she listened carefully.

Third clue

"If you exchanged each dime for two nickels, then we would be six coins," said the coins.

Go over the list line-by-line with your class. Cross out those lines which do not fit this clue. You may find it helpful to display next to a line in your chart the result of the exchange. If so, erase this before going on to the next clue.

10¢ DIMES	5¢ NICKELS	1¢ PENNIES		AFTER EXCHANGE	
				NICKELS	PENNIES
~~○○○~~	~~○~~	~~○~~	~~36¢~~	○○○○○○○	○
~~○○~~	~~○○~~	~~○~~	~~31¢~~	○○○○○○	○
~~○○~~	~~○~~	~~○○~~	~~27¢~~	○○○○○	○○
○	○○○	○	26¢	○○○○○	○
○	○○	○○	22¢	○○○○	○○
○	○	○○○	18¢	○○○	○○○

T: Now we know that there is either 26¢ or 22¢ or 18¢ in the bank. Tracy shook her bank and listened. She heard the coins say: "You may have one more clue. Pay close attention and think about this clue."

Fourth clue

"If you put one more nickel in your bank, then there would be more than 30¢." Tell me with whispers how much money is in Tracy's piggy bank now. (Answer: 26¢)

You may have to remind the class that there is either 26¢ or 22¢ or 18¢ in the bank now. After a while, choose a volunteer who did not get to whisper an answer to you to say aloud how much money is in the bank. Check the three possibilities with the class so that they can eliminate 22¢ and 18¢. Find that there is 26¢ in the bank. Let a student open the container and count the money.

Exercise 2

T: We are going to go around the class counting by twos, but before we start let's see how many students are here today.

S: 26 (or the number of students in your class).

T: We will start our counting by twos with Eric and end with Christine. Eric will say "2". What number do you think Christine will say?

Let the students make some guesses. You are likely to get answers such as 50, 26, 42, and 52. You should not insist on getting the correct answer at this time.

T: Let's count and see.

Afterwards, point out that those who guessed 52 (in this case) were correct.

T: Now, let's pretend that each of you has one dime and we want to know how much money you have altogether. How can we find out?

S: Count by tens.

T: That's a good idea; go around the class counting by tens. Suppose we start with Kevin and end with Tracy. Kevin will say "10" for his dime. What number do you think Tracy will say?

Some students might look around the room and count in their heads. Others might make wild guesses. You are likely to get answers such as about two dollars, 100 cents, more than two dollars, and 260 cents. Again, don't insist on getting the correct answer at this time.

T: Let's count and see.

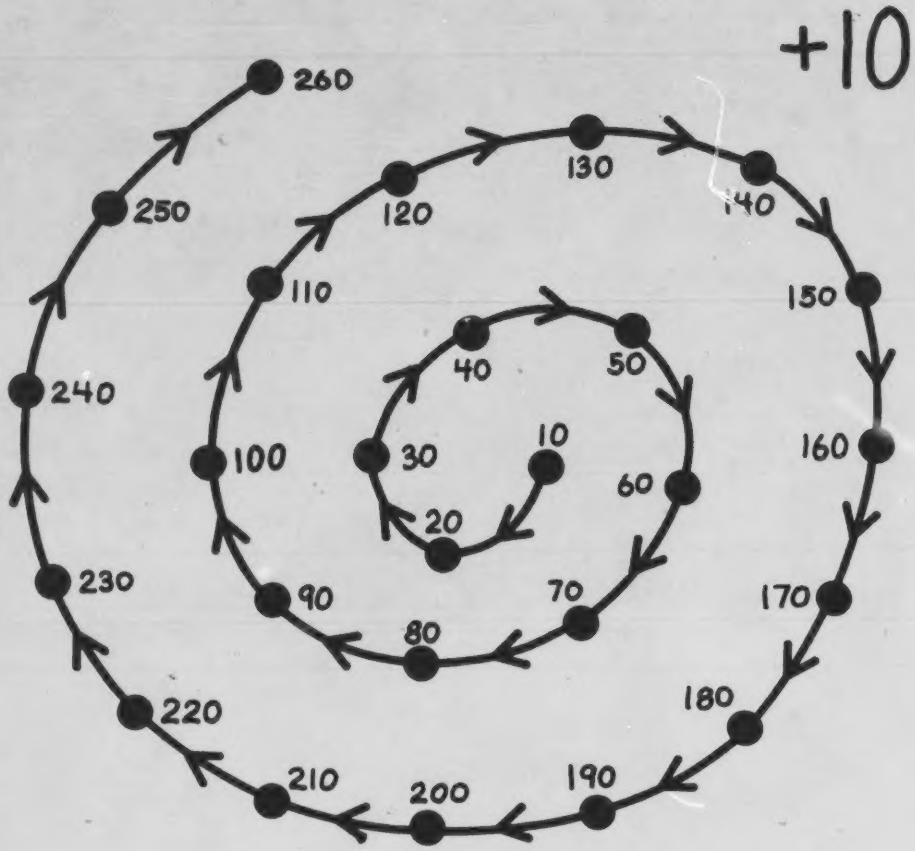
.

.

.

T: Tracy said "260", so we have 260 cents. That is the same as 2 dollars and 60 cents. Let's draw a "counting by tens" picture to show what we did.

Draw a + 10 spiral on the board. Your spiral will have the same number of dots as students in the class.


+10
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250
260

As you draw and label each dot, mention who said that number. If there is still time and the students are enjoying this "counting" activity, repeat the above dialogue pretending this time that each student has a nickel. This time, however, ask the students to draw a + 5 spiral on their own.

L 7

L7 String Pictures with A-Blocks

CAPSULE LESSON SUMMARY

Review the different A-Block pieces as to shape, color, and size. Place them all correctly into both a two-string picture and a three-string picture.

MATERIALS

Teacher: A-Blocks String Game Kit; box; colored chalk; magnetic board (optional)

Student: None

NOTE: Before this lesson begins, prepare a team board as pictured below either directly on your chalkboard, or on the side of a file cabinet, or on another accessible board as is convenient for you. Be sure that this board is metallic if you have magnetized the A-Block pieces.

TEAM A	TEAM B

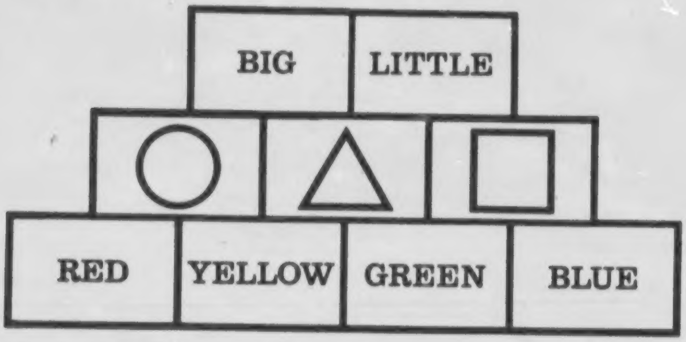
DESCRIPTION OF LESSON

Put the twenty-four A-Block pieces (shapes) in a box about the size of a shoe box. Sort them so that you can locate any given one quickly.

Divide the class into two teams. Try to arrange that the teams are roughly equal in ability.

Exercise 1

Briefly discuss the different kinds of A-Block pieces there are; the size, shape, and color. Post a list of attributes above the team board. This list can be prepared using one of the posters in the A-Blocks String Game Kit.



T: Your first task will be to get all the pieces out of this box and onto this board (point to the team board). We will take turns; someone on Team A will tell me which piece to put on Team A's side of the board, and then someone from Team B will tell me which piece to put on Team B's side. Each time you must describe completely a piece that is still in the box. Team A goes first.

S: Big red circle.

Put the big red circle on Team A's side of the board.

T: Very good. Kevin told me the size, color, and shape of the piece he wanted. Now it is Team B's turn.

S: Little red circle.

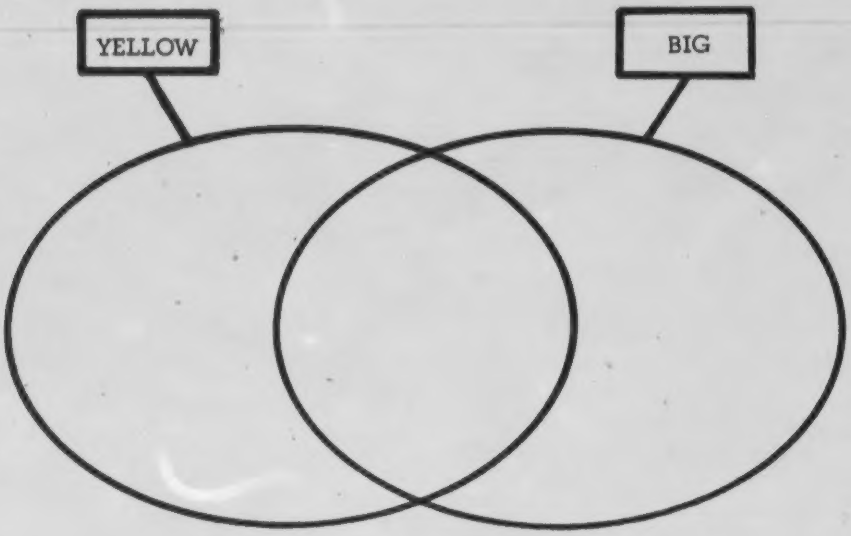
Put the little red circle on Team B's side of the board.

T: Very good, Angela described a piece that is still in the box and I can put that piece on her Team's side of the board.

Continue in this way keeping a brisk pace until all the A-Block pieces are on the board. If a student describes a piece that is on the team board, point out that that piece has been chosen already and he or she will have to choose a different piece, one that is still in the box. Try to call on slower students early in the game so that there are many choices open to them.

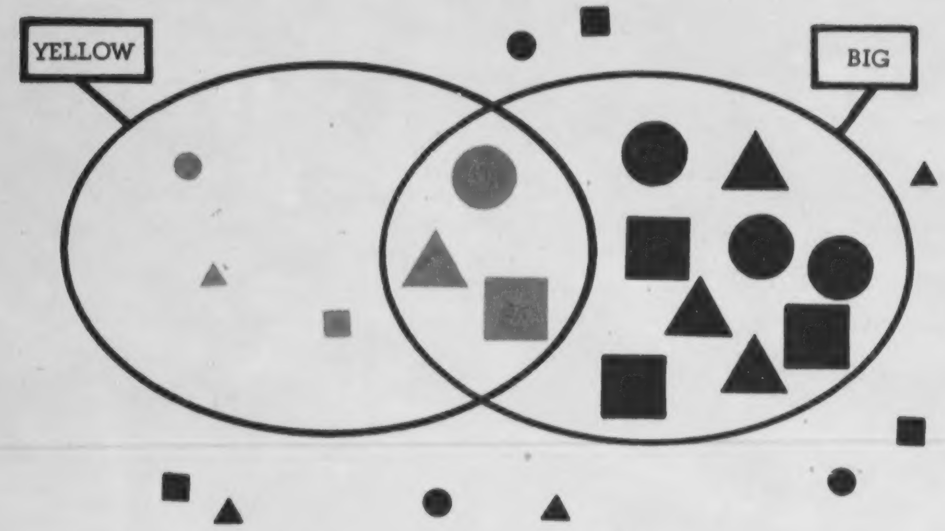
Exercise 2

Draw two different colored, overlapping strings on the board and label them as in the next illustration.



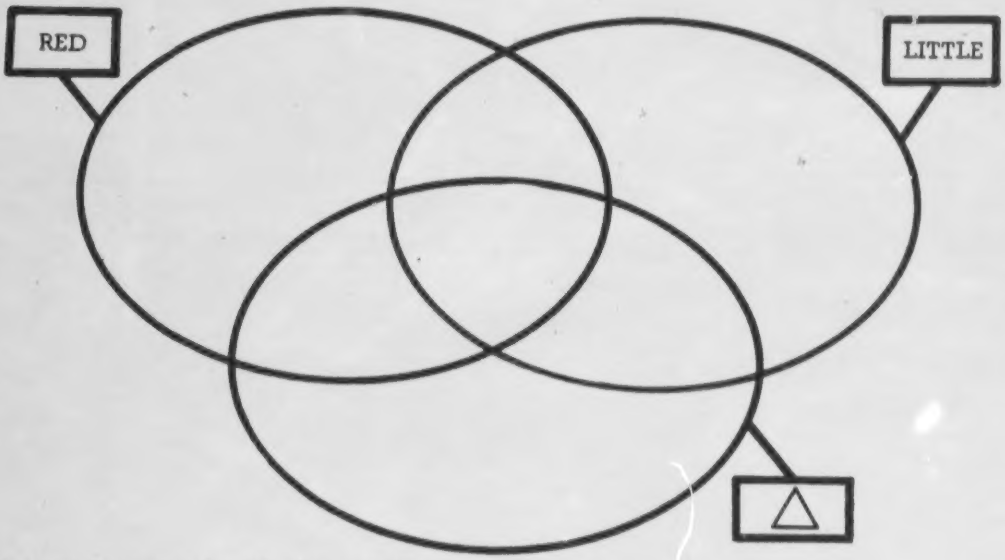
T: Now the teams will take turns trying to place their pieces in this string picture. Each of you will try to place a piece from your team's side of the board in this picture. If you put it in its correct place, I will say "Yes" and it stays there. If you do not, I will say "No" and you must return it to your team's side of the board. The first team to get all of its pieces in the picture is the winner.

Play the game, alternating teams and alternating turns among the members of each team. When all the pieces are in the picture, your board will look like this.



Exercise 3

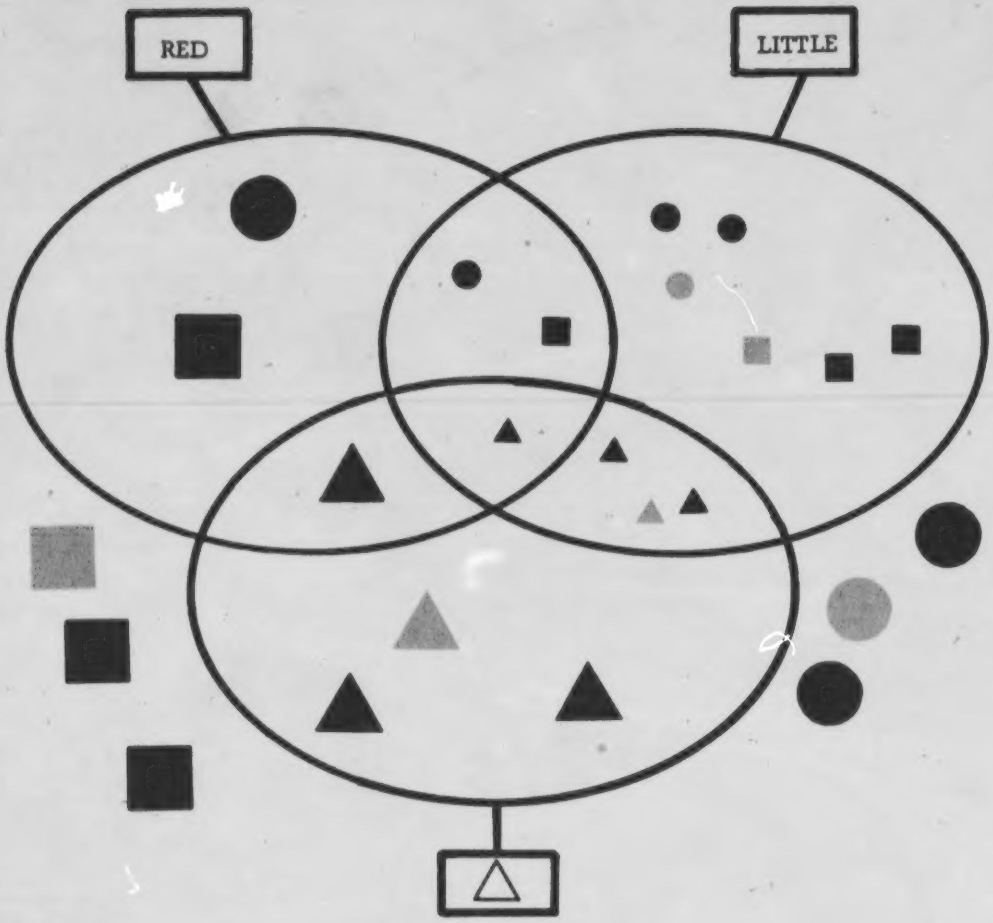
Redistribute the A-Block pieces on the team board. Then draw three different colored, overlapping strings on the board. Label the strings as in the next illustration.



Repeat Exercise 2 with this situation.

L 7

When all the pieces are in the picture, the board will look like this.



L8 Messages

CAPSULE LESSON SUMMARY

Decode messages which are given in arrow pictures where the arrows indicate which letter comes first, second, third, etc. in the message.

MATERIALS

Teacher: Colored chalk

Student: Paper; pencil; Worksheets L8* and **

DESCRIPTION OF LESSON

NOTE: Before this lesson begins, you should draw the arrow picture for Exercise 1 on the board. Make the picture large and well spaced so that the students can clearly follow each arrow from starting point to ending point. You may find it more convenient to draw this picture and the picture for Exercise 2 on large sheets of unlined paper (easel pad size). This would allow you to check the accuracy of your drawings before the lesson and save time during the lesson. You would then tape each picture to the board as you begin to talk about it.

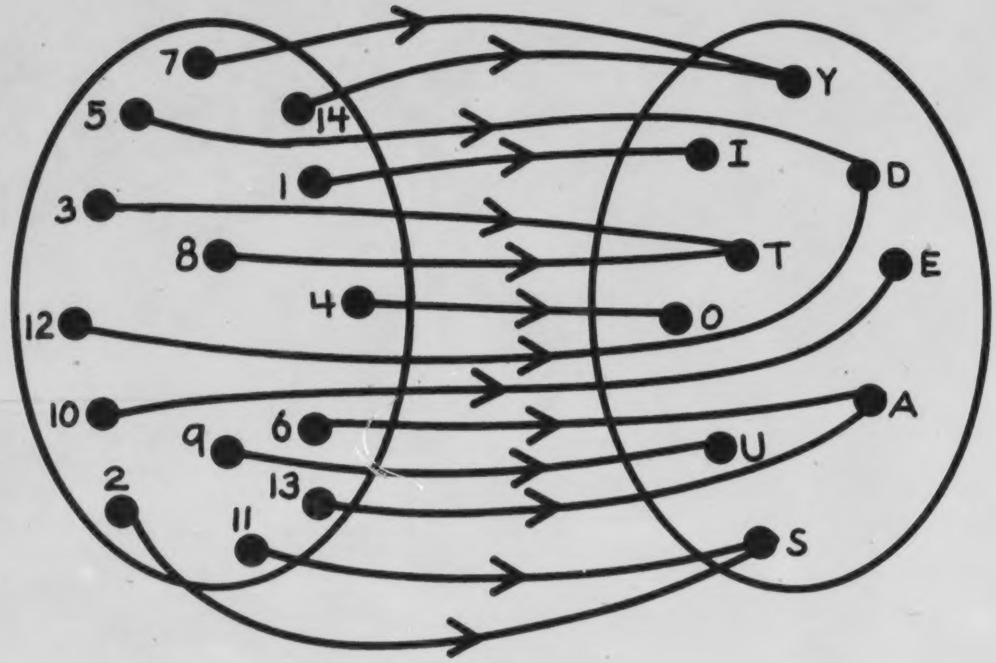
Exercise 1

Put the arrow picture from the next illustration on the board.

T: I have put a message to you on the board. Look closely at my picture and try to find out what is the message.

Give the class a couple minutes to study the picture. While the students are thinking, draw 14 blanks below the picture and space them as illustrated.

L 8



__ __   __ __ __ __ __   __ __ __ __ __ __ __?

Some students may suggest ways to decipher the message. Others may like to comment on the picture (for example, all the arrows go in the same direction or each arrow starts at a number and ends at a letter). Let them freely comment or make suggestions for a few minutes. Perhaps someone will suggest that the first letter of the message is "I". If this happens, write "1" under the first blank and write "I" in the blank. Otherwise, point to "1" and ask:

T: What is the first letter of my message?

S: I.

Write "1" under the first blank and write "I" in the blank.

I _   _ _ _ _ _   _ _ _ _ _ _ _ ?
1

T (tracing the arrow from 1 to I): That's right. Let's go on. (Point to the second blank.) What is the second letter of my message?

S: S.

Ask the student to trace the arrow from 2 to S. Then write "2" under the second blank and write "S" in the blank.

I S   _ _ _ _ _   _ _ _ _ _ _ _ ?
1 2

Continue in this way until the message is complete. For each letter in the message, call on a student to trace the appropriate arrow before writing the letter in the blank. This activity should go very quickly and give many students an opportunity to participate.

I S   T O D A Y   T U E S D A Y ?
1 2   3 4 5 6 7   8 9 10 11 12 13 14

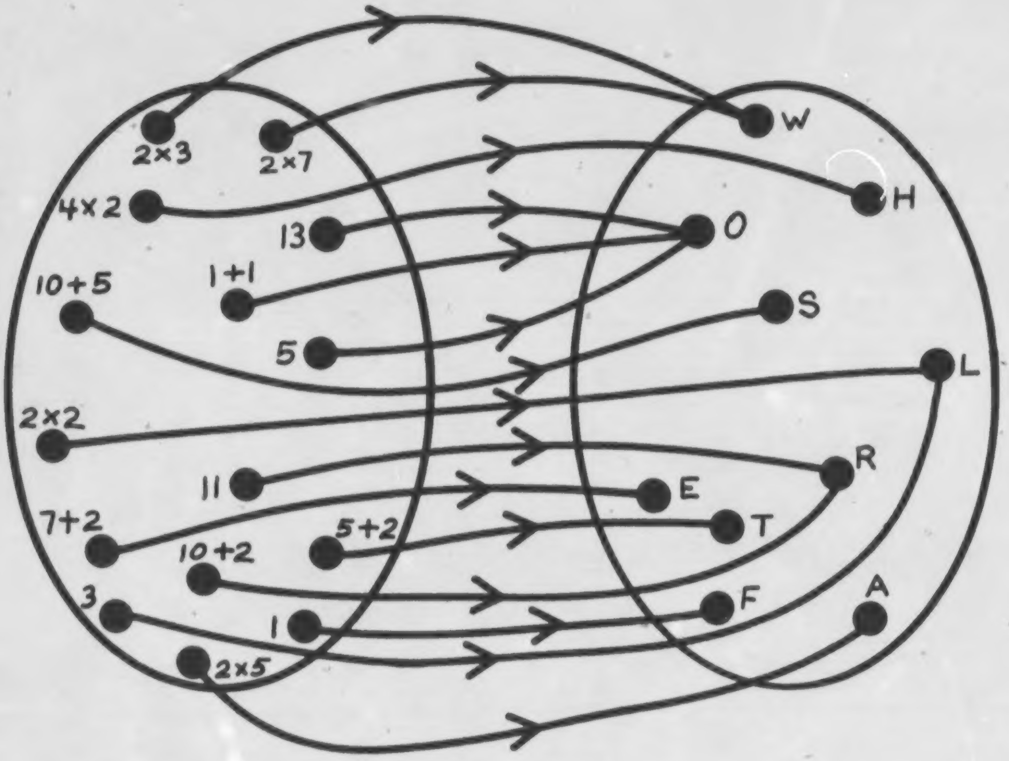
T: You certainly discovered my message quickly. I have another message, but this time I will make it more difficult.

L8

Exercise 2

Distribute or ask the students to take out paper and pencils.

Put this arrow picture on the board and under it draw 15 blanks for the message.



— — — — — —   — — —   — — — — — —.

If you have drawn the picture ahead of time and can now tape the paper to the board, give the class a minute or so to study the picture. If the students are watching you draw the picture, ask them to try to figure out your message as you draw. Suggest they keep secret anything they find out about the message.

T: On your paper, draw only these blanks. You do not have to copy the picture. Use my picture and try to find out what is my message. Write it on your paper.

Let the students work independently on this problem for five to ten minutes, giving individual help to students who have difficulty getting started. When several students have finished or if there is general difficulty, return to a collective activity of completing the message.

T: What is the first letter in this message? (F)

Ask a student to trace the arrow from 1 to F. Then write "1 " under the first blank and put "F" in the blank.

T: Who can find the second letter in this message?

Ask a student to trace the arrow from 1 + 1 to O. When a student points to "1 + 1", emphasize that 1 + 1 = 2 and then erase "1 + 1" replacing it with "2". Write "2" under the second blank and put "O" in the blank.

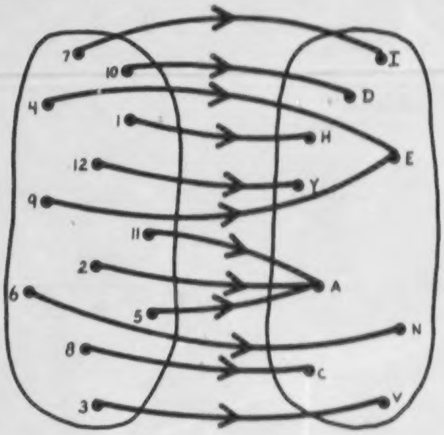
Continue in this way until the message is complete. Ask students to check the calculations as the numbers from 2 to 15 are located. The completed message is:

F	O	L	L	O	W	T	H	E	A	R	R	O	W	S.
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Worksheets L8* and ** are available for the rest of the lesson. If some students complete these worksheets quickly, ask them to write their names or some other message in this coded form. Give them unlined paper on which to do this.

Name ____________ L8 *

Decode this message.

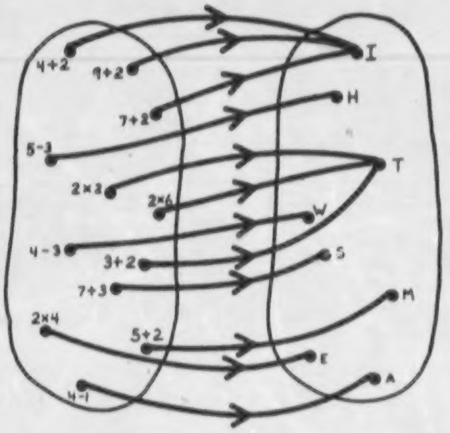


7
10
4
1
12
9
11
2
6
5
8
3
I
D
H
E
Y
A
N
C
V

HAVE A NICE DAY.

Name ____________ L8 **

Decode this message.



4+2
9+2
7+2
5-3
2×3
2×6
4-3
3+2
7+3
2×4
5+2
4-1
I
H
T
W
S
M
E
A

WHAT TIME IS IT?

L9 String Game with A-Blocks #1

CAPSULE LESSON SUMMARY

Review the A-Block pieces as to size, shape, and color by taking them one at a time out of a box as they are described. Play the A-Block String Game.

MATERIALS

Teacher: A-Blocks String Game Kit; colored chalk; magnetic board (optional)

Student: None

NOTE: Before this lesson begins, you need to prepare some materials from the A-Blocks String Game Kit. If you use a magnetic board (chalkboard or grid board), punch out and magnetize the attribute cards from the kit. Do this in the same way as you magnetized the shapes for Lessons L4 and L7, except in this case the magnetic material is attached to the front face of the cards so that they can be placed face down (hidden) as labels for strings. If you do not have any magnetic board available, prepare the attribute cards with loops of masking tape in a way similar to the way the shapes were prepared. Be careful not to obscure what is written on the card and be prepared to make repairs during the lesson as masking tape loses its stickiness quickly.

Prepare a team board as was done for Lesson L7.

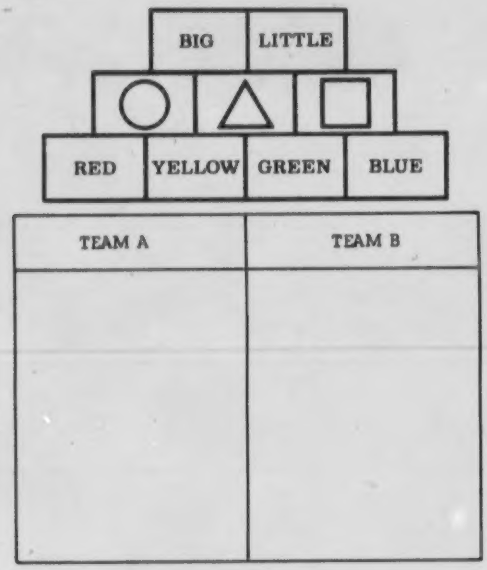
DESCRIPTION OF LESSON

Put the A-Block pieces (shapes) in a box about the size of a shoe box. Sort them so that each one is easy to locate. Divide the class into two teams and try to arrange that the teams are roughly equal in ability.

L 9

Exercise 1

Briefly review the different kinds of A-Block pieces and post a list of attributes above the team board.

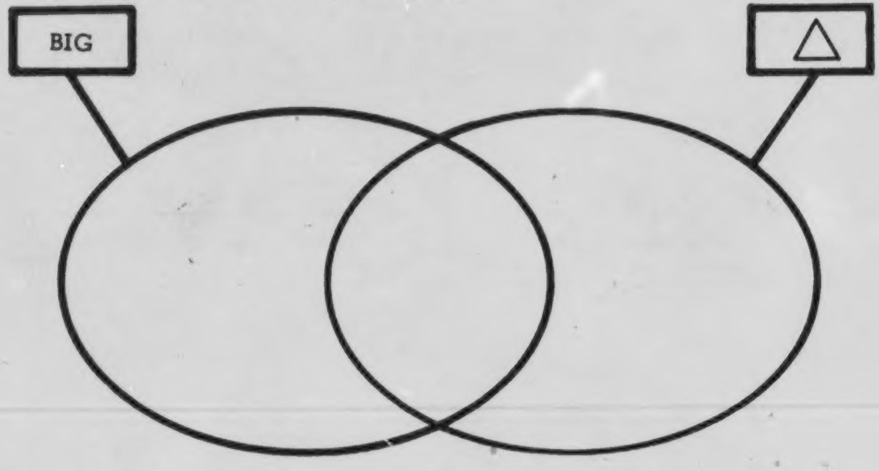


T: We are going to play a String Game with these shapes. First you have to help me take the A-Block pieces out of this box. We will take turns; someone on Team A will tell me which piece to put on Team A's side of the board, and then someone from Team B will tell me which piece to put on Team B's side of the board. Remember, you have to describe a piece that is still in the box and so I know exactly which piece you want me to put on your side of the board.

Alternating between the teams, let students describe pieces to put on the team board. Insist that the description be complete. If the piece described is already on the board, point it out and ask for another. Try to call on weak students early so that there are many choices open to them. Encourage all the students to be thinking about which pieces they will ask for when you call on them.

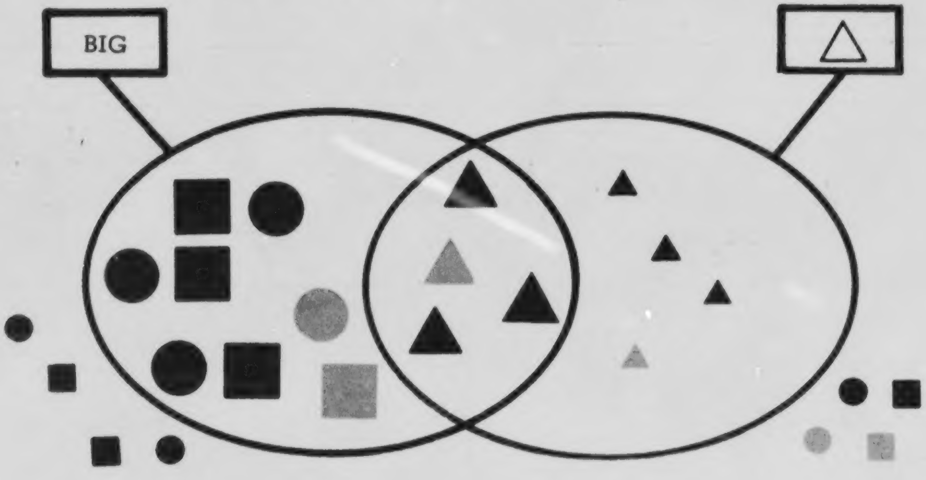
Exercise 2

Draw two large overlapping strings on the playing board using different colors. Label the strings as in the next illustration.



T: Now the teams will take turns trying to place their pieces in the string picture. Each of you will try to place just one piece from your team's side of the board. If you place it correctly, I will say "Yes." If you put it in the wrong place, I will say "No" and you must return the piece to your team's side of the board. The first team to get all of its pieces in the string picture is the winner.

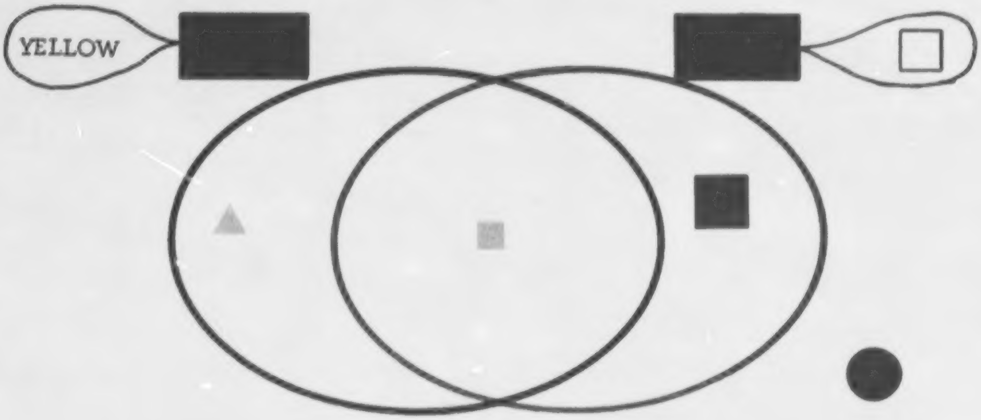
Play this game alternating teams and alternating turns among the members of a team. The picture below shows all the A-Block pieces placed correctly.



L 9

Exercise 3

Prepare to play the String game in the usual way with face down string cards.[*] Redistribute game pieces on the two sides (TEAM A and TEAM B) of the team board. Set up the playing board as shown below. Bubbles indicate what is hidden on the string cards. Four correctly placed pieces are the starting clues.



Remember that the rules of the game call for you to be the judge. If a piece is correctly placed, say "Yes" and immediately instruct the player to try to place a second piece. No player should have more than two consecutive plays. If a piece is incorrectly placed, say "No" and ask the player to return the piece to his or her team's side of the team board.

For this game, the player who correctly places the last piece from his or her team's side of the team board may then attempt to identify each of the string cards. A team wins by being first to get all of its pieces in the string picture and to identify the strings.

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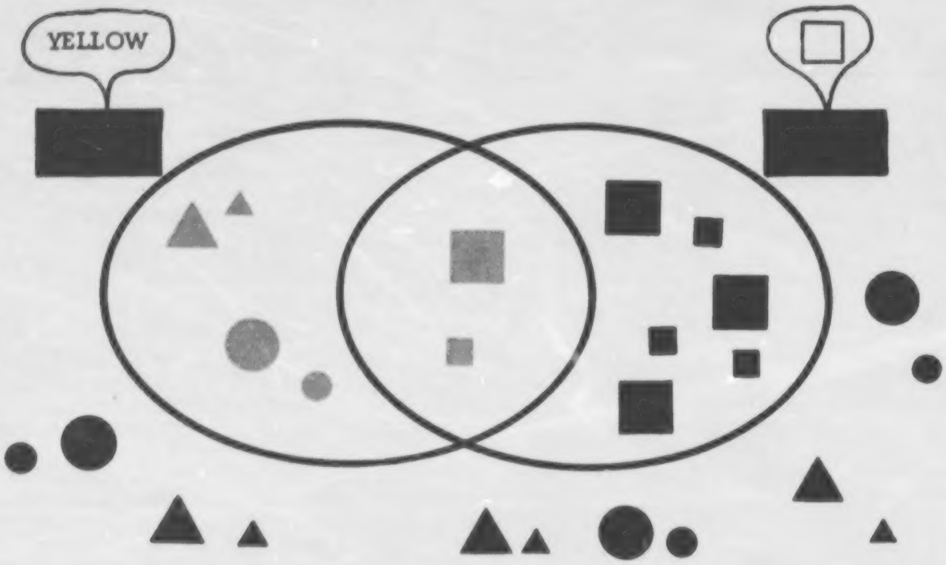
[*] See the appendix on the String Game for a description of equipment and preparation for the game played in the usual way.

69

The following picture shows correct placement for all of the game pieces. You could use this picture as a "crib-sheet" while the game is being played.



7Q

L 10

L10 Composition #1

CAPSULE LESSON SUMMARY

Play a composition game with colored arrows where the only rule for the game is: yellow followed by blue is red.

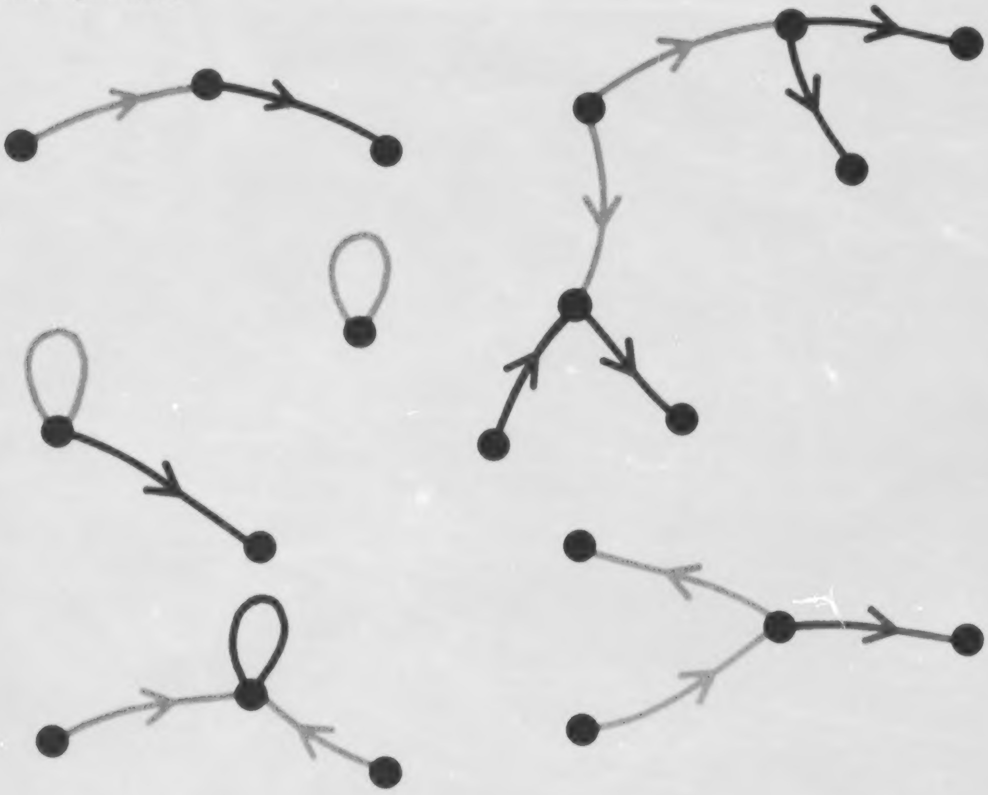
MATERIALS

Teacher: Colored chalk
Student: None

DESCRIPTION OF LESSON

Exercise 1

Before the lesson begins, draw this arrow picture on the board. Spread the picture out over the entire board, and draw arrows whose colors can be easily distinguished.

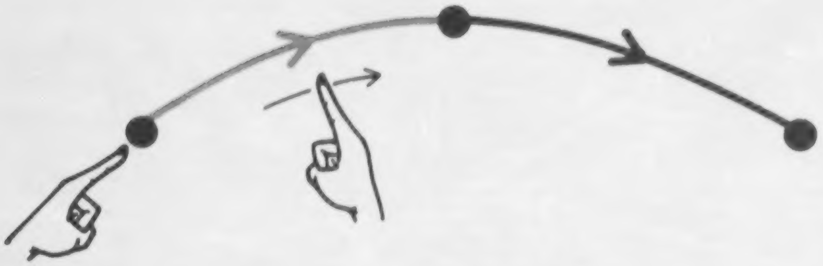


L 10

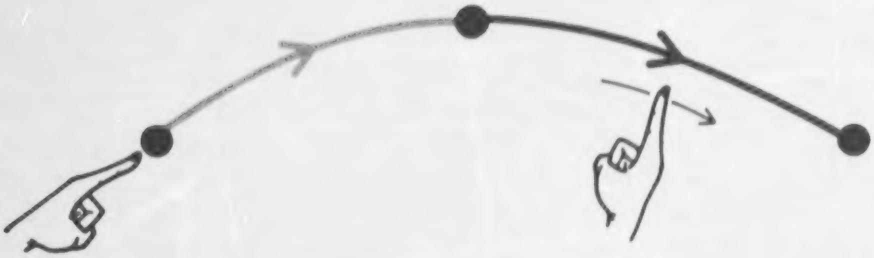
T: We are going to play a game with these yellow and blue arrows. The object of the game is to draw red arrows; but there is a rule for drawing red arrows.

Move to the upper left portion of your picture. Explain the rule for drawing red arrows (as stated in the left-hand column) while making the motions described in the right-hand column.

T:	This is the rule of the game . . .	Place your left forefinger on a dot at which a yellow arrow starts.
	Each time there is a yellow arrow . . .	With your left forefinger in place at the dot, follow the yellow arrow with your right forefinger in the direction of the arrowhead.



T:	. . . followed by a blue arrow . . .	Stop the motion of your right forefinger at the middle dot, tap the dot, and then follow the blue arrow.



L 10

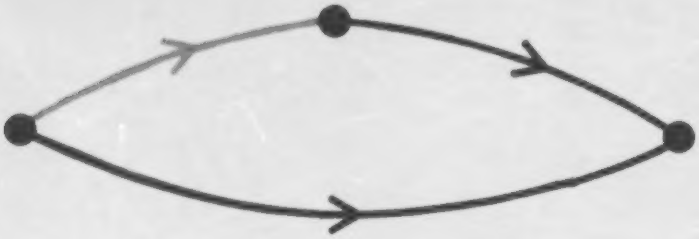
T: . . . then we can draw a red arrow from the dot where the yellow arrow starts . . .

Tap this dot several times with your left forefinger.

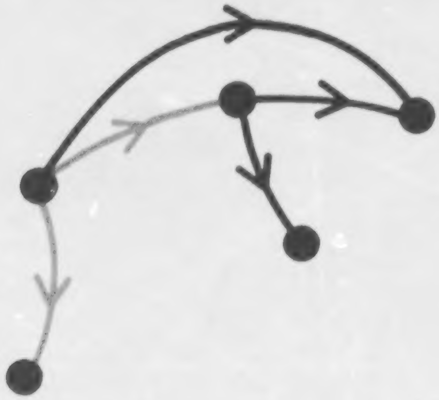
. . . to the dot where the blue arrow ends.

Tap this dot several times with your right forefinger.

First trace and then draw the red arrow.



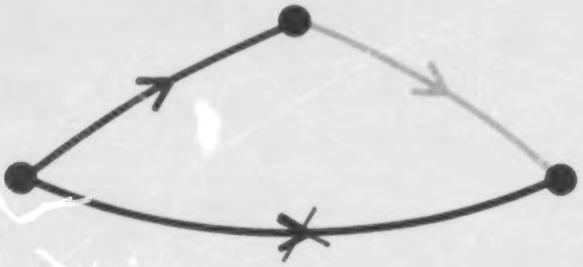
Move to the upper right portion of your picture. Repeat the statement of the rule using the same words and motions as above. Trace and then draw this red arrow.



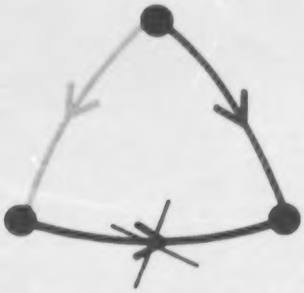
T: Who sees where we can draw another red arrow?

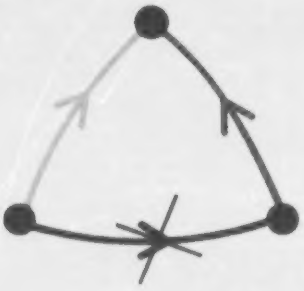
Invite students one at a time to the board to show where another red arrow can be drawn. Ask each student to first trace a yellow arrow followed by a blue arrow (emphasize that the direction of an arrow must be followed and stop a student who traces against the direction of an arrow), and then to trace the red arrow. If correct, give the student a piece of red chalk with which to draw a red arrow. The rest of the class should help you catch mistakes.

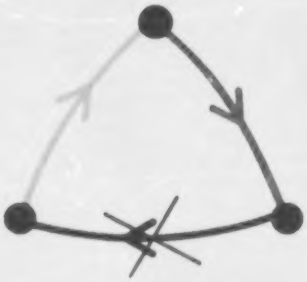
Note on common mistakes and difficulties: The most common mistake at first will occur when students confuse the rule "yellow followed by blue" with "blue followed by yellow".



Another mistake might occur when the directions of the arrows are not correctly followed.



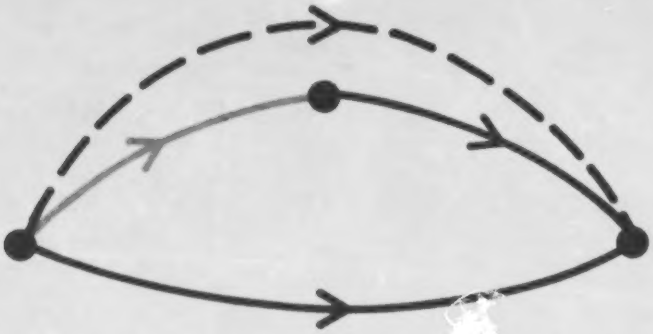




Remind students making such mistakes that they must follow the direction of the arrows. The arrows are like one-way streets and they are not allowed to go the wrong way.

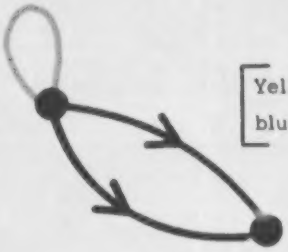
L 10

If it happens that a student looks at a portion of the arrow picture and proposes to draw a red arrow as indicated by this dotted arrow,



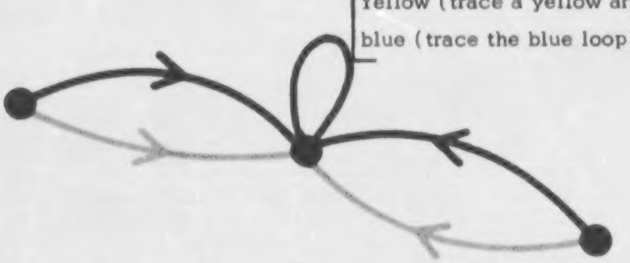
another student will probably remark that the red arrow has already been drawn. There is nothing wrong with the dotted arrow; it is merely redundant. That portion of the picture is complete without it.

It is likely that the loops on the left side of the picture will cause difficulty. If no student suggests one of these red arrows, you should suggest one of them.



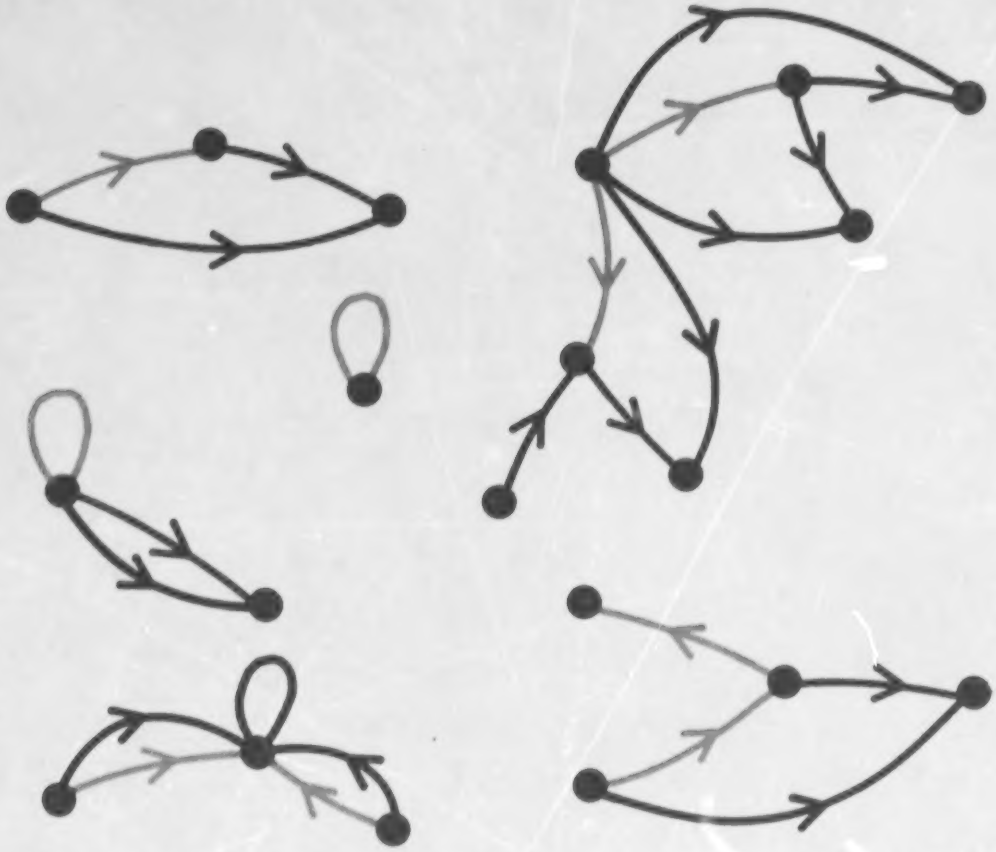
[Yellow (trace the yellow loop) followed by blue (trace the blue arrow), then red]

[Yellow (trace a yellow arrow) followed by blue (trace the blue loop), then red]



It is important here to keep your left forefinger on the dot where a yellow arrow (loop) starts and then trace with your right forefinger ending at the dot where the blue arrow (loop) ends. The red arrow then goes from the left forefinger (start) to the right forefinger (end).

When all the red arrows that can be drawn have been found, your picture should look like this.



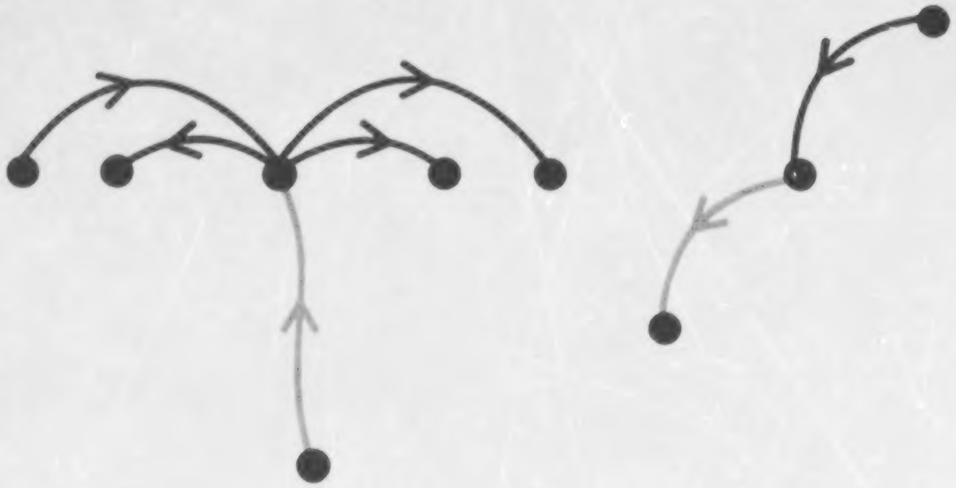
Exercise 2

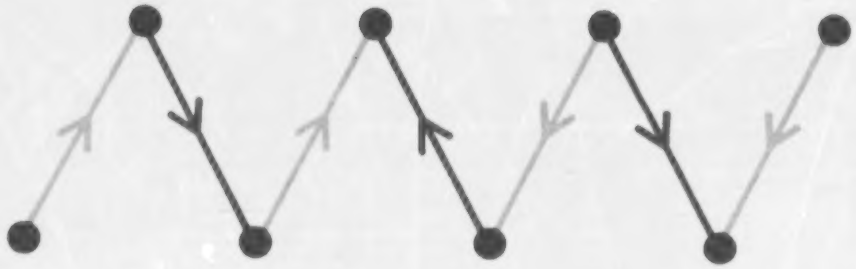
Erase the board.

T: Now you know the rule for drawing red arrows: a yellow arrow followed by a blue arrow, then a red arrow. Watch as I draw a new picture and see how many places you can find to draw red arrows.

L 10

Draw this arrow picture spread out on the board.

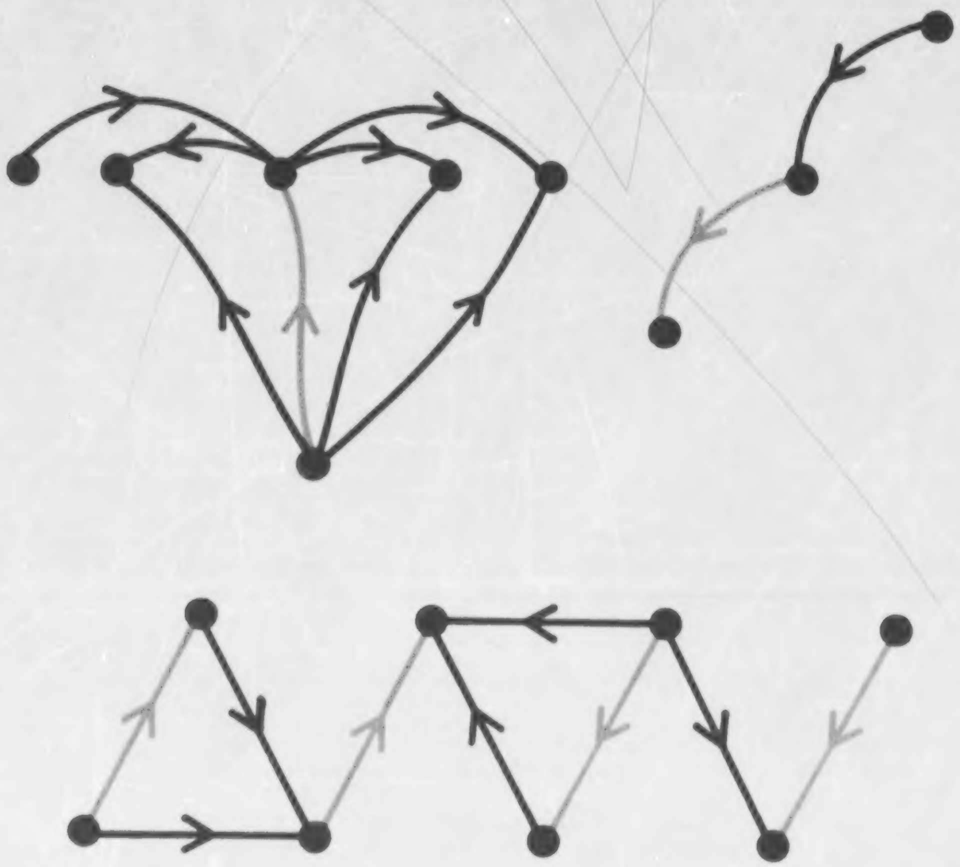




One at a time, invite students to come to the board and show the class where red arrows can be drawn. The class should judge whether students have correctly followed the rule: yellow followed by blue is red.

Encourage students to first trace a yellow arrow followed by a blue arrow and then to trace where a red arrow can be drawn. After class agreement (and indeed correct), give students red chalk with which to draw the red arrows. Try to persuade the entire class to participate in this activity.

When all the red arrows have been drawn, your picture should look like this.



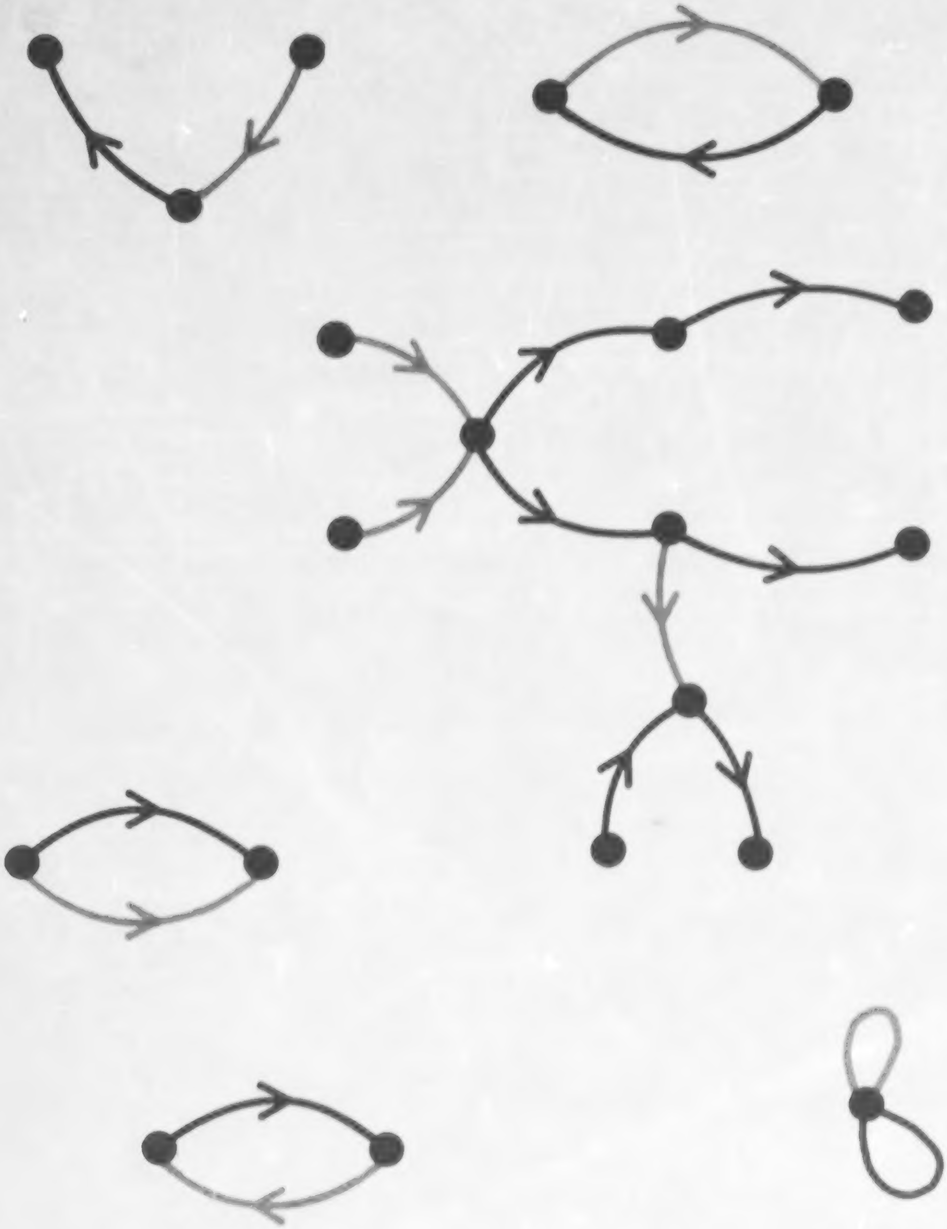
Exercise 3

Erase the board.

T: You are getting very good at the game. I will try to make it more difficult; this time there will be some tricky parts in the picture. Watch as I draw a new picture and see how many places you can find to draw red arrows. When I finish drawing, we will play the game again.

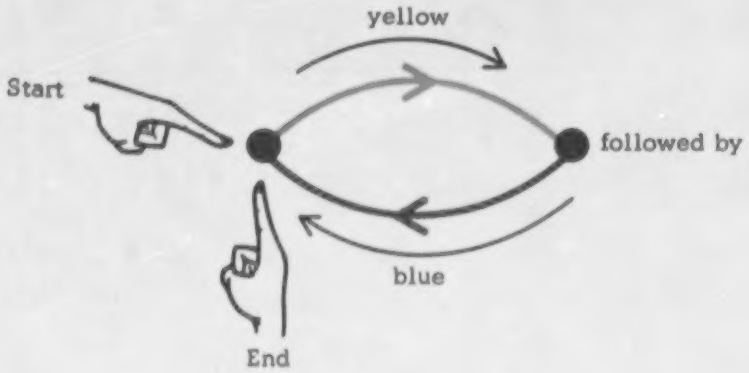
L 10

Draw this arrow picture spread out on the board.



Repeat Exercise 2 with this situation. Remind the class to look carefully because you have tried to make it more difficult and there are some tricky parts.

After a while, if no student finds one of the red loops, move to the upper right portion of the picture and trace the yellow arrow following by the blue arrow using the same words and motions suggested earlier.



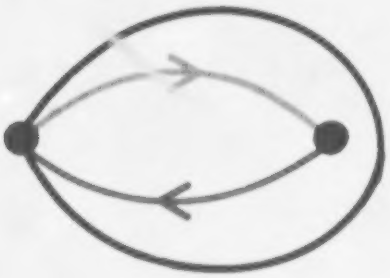
T: Where did I start (tap the dot with your left forefinger)?

Where did I end (tap the dot with your right forefinger)?

I start and end at the same point. How should we draw a red arrow starting and ending at this point?

S: It's a loop.

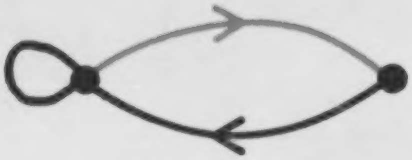
It is likely a student will suggest a loop which encircles the entire configuration.



80

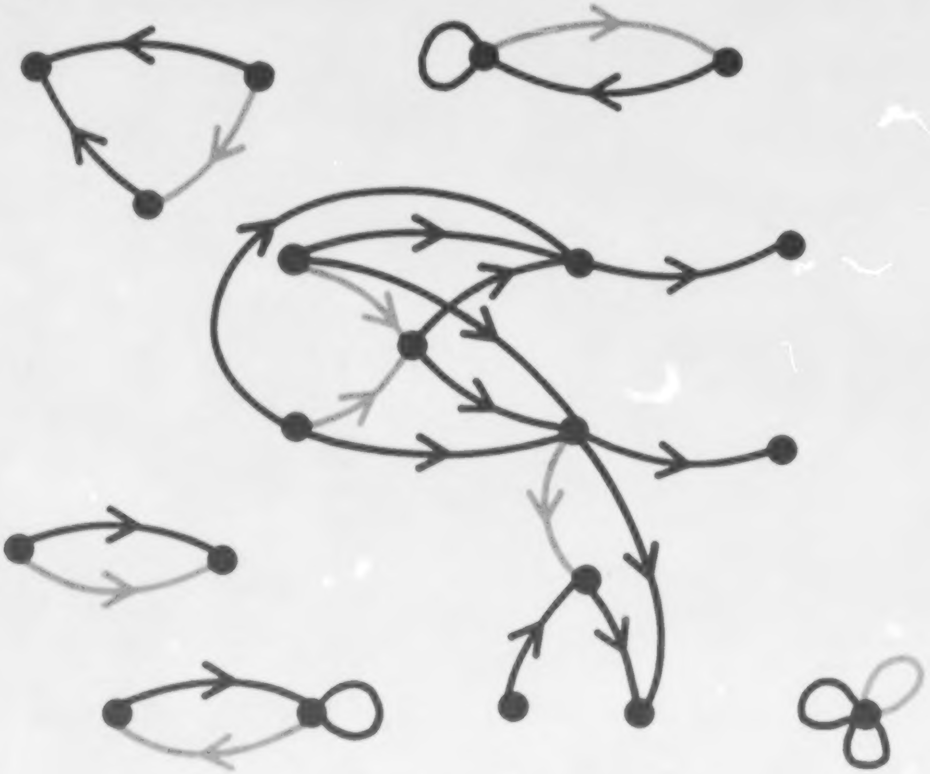
L 10

In this case, accept it and let the student draw it. Later, or if you draw the loop yourself, you might suggest that the loop be drawn as shown below.



Students should be able to find other red loops in the picture, but do not be concerned if they do not find them all, or if they do not find all the red arrows themselves. Your comments should direct their attention at missing red arrows and loops.

When all the red arrows and loops have been drawn, your picture should look like this.



L11 Detective Story with Coins #2

CAPSULE LESSON SUMMARY

Solve a detective story about a secret amount of money with clues about the coins, a clue which says the amount of money can be put on the Minicomputer with exactly two checkers, and an arrow picture clue. Find how much money the students would have all together if each student had exactly one dime, and draw a +10 spiral to help solve the problem.

MATERIALS

Teacher: Coins (one quarter, one nickel); Minicomputer set; colored chalk

Student: Paper; colored pencils

DESCRIPTION OF LESSON

Exercise 1

Before the lesson begins, put just one quarter and one nickel in your pocket or a small container such as a change purse, ring box, match box, etc.

T: I have some money in my pocket (or this small container) and you have to discover how much money. I will give you some clues, so if you listen carefully, you can discover how much money I have without guessing.

First clue

My first clue is that I have two coins. They could be quarters, dimes, nickels, or pennies.

L 11

Begin to draw this chart on the board.

25¢ QUARTERS	10¢ DIMES	5¢ NICKELS	1¢ PENNIES	AMOUNT

T: How much money could be in my pocket?

S: 50¢. You could have two quarters.

S: 15¢. You could have one dime and one nickel.

As the students make suggestions, write them in the chart. Encourage the class to find all the possibilities; each time an amount of money is suggested, be sure the student says which coins make that amount of money. Your completed chart should have ten possibilities, although most likely they will not have been suggested in this orderly way.

25¢ QUARTERS	10¢ DIMES	5¢ NICKELS	1¢ PENNIES	AMOUNT
○ ○				50¢
○	○			35¢
○		○		30¢
○			○	26¢
	○ ○			20¢
	○	○		15¢
	○		○	11¢
		○ ○		10¢
		○	○	6¢
			○ ○	2¢

Leave this chart on the board.

T: Second clue

My next clue is that you could have the amount of money in my pocket with just nickels.

For this clue you might have to illustrate with an example what is meant. Be sure the students understand that this clue does not mean you have two nickels in your pocket.

T: For example, you could have 15¢ with three nickels. So I could have 15¢ in my pocket. What other amounts of money could be in my pocket?

L 11

S: 50¢. You can have 50¢ with ten nickels.

Go through the possibilities in the chart, crossing out or erasing those amounts that cannot be made with just nickels. This clue leaves six possibilities.

25¢ QUARTERS	10¢ DIMES	5¢ NICKELS	1¢ PENNIES	AMOUNT	
○ ○				50¢	(ten nickels)
○	○			35¢	(seven nickels)
○		○		30¢	(six nickels)
~~○~~			~~○~~	~~26¢~~	
	○ ○			20¢	(four nickels)
	○	○		15¢	(three nickels)
	~~○~~		~~○~~	~~11¢~~	
		○ ○		10¢	(two nickels)
		~~○~~	~~○~~	~~6¢~~	
			~~○ ○~~	~~2¢~~	

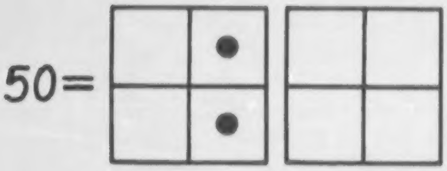
T: Now you know that I have one of these six amounts of money in my pocket.

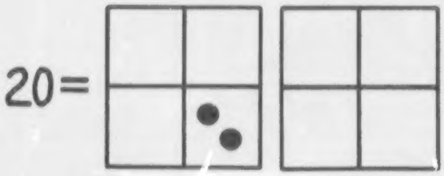
Third clue

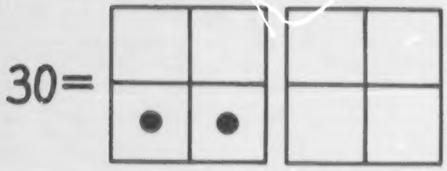
My next clue is: You can put this amount of money on the Minicomputer with exactly two checkers.

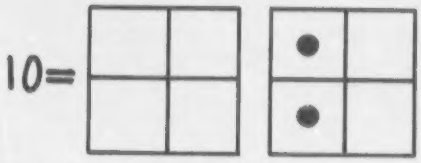
T: Which of these numbers (point to the remaining six possibilities) can you put on the Minicomputer with exactly two checkers?

Display two Minicomputer boards and let the students try each of the six numbers. They should find that four possibilities remain.









Cross out or erase the other two possibilities (35¢ and 15¢) after it has been agreed that they cannot be put on the Minicomputer with exactly two checkers.

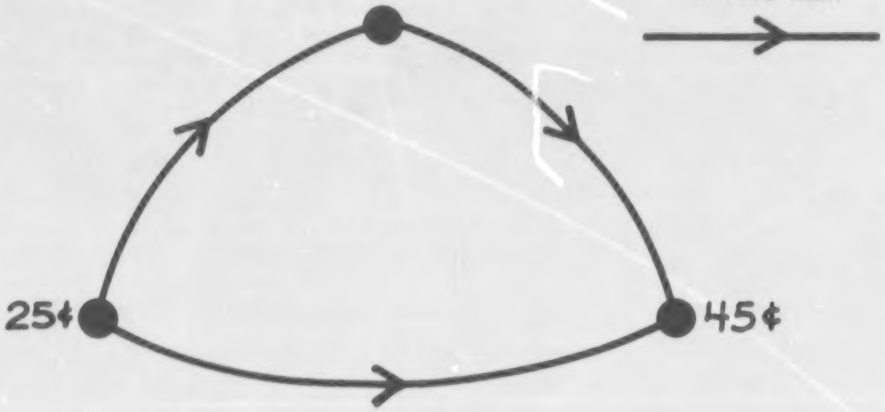
T: Now you know that the amount of money in my pocket is 50¢ or 30¢ or 20¢ or 10¢. I'll give you just one more clue. When you know for sure how much money is in my pocket, write it on your paper.

Fourth clue

The amount of money in my pocket is in this picture.

L 11

Draw this arrow picture on the board.



Check some of the students papers. After most students have the correct amount, let one student announce that you have 30¢ in your pocket and show them the quarter and nickel.

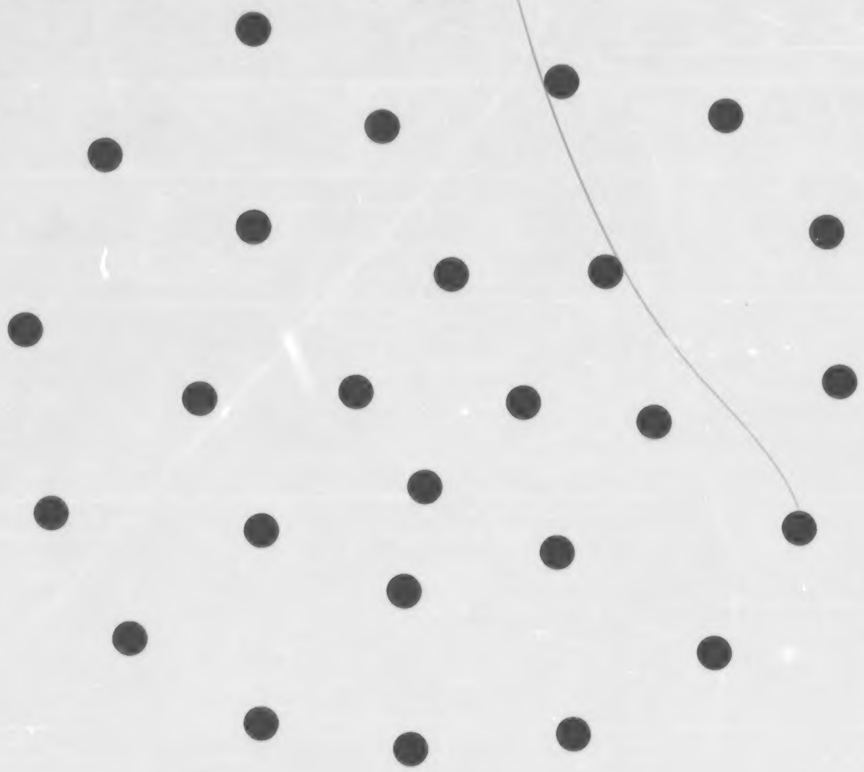
Exercise 2

Distribute paper and colored pencils.

T: Let's pretend that every student in this class has one dime and we want to find how much money you have all together. Think about the problem for a minute while I draw a dot for each of you on the board. How many dots should I draw?

S: 26 (or the number of students in your class).

Draw the dots on the board in a spiral pattern.



T: We are going to draw a picture here to help us find out how much money you would have all together. Before we do, does anyone have an idea how much?

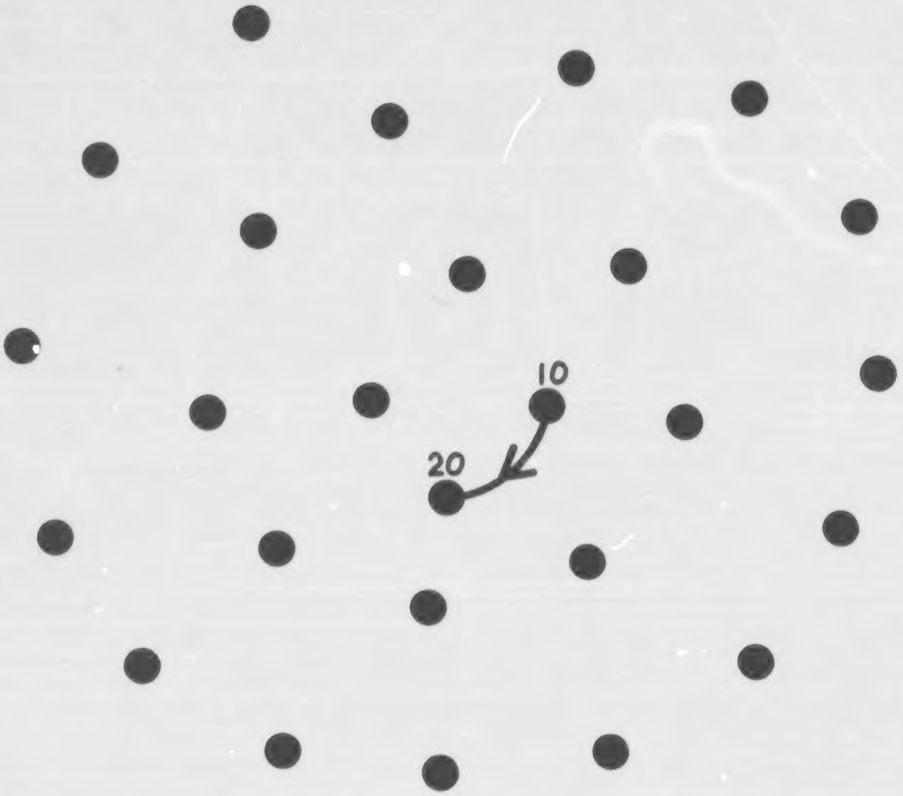
Let several students give responses and write their "guesses" on the board. Then start with a student in one corner of the room.

T: This dot (point to the center dot) will be for Robin. She has one dime, so I label it "10". The next person (behind or in front of Robin) also has one dime. Together they have how much money?

S: 20¢.

T: So I draw a red arrow and label the next dot "20".

L 11

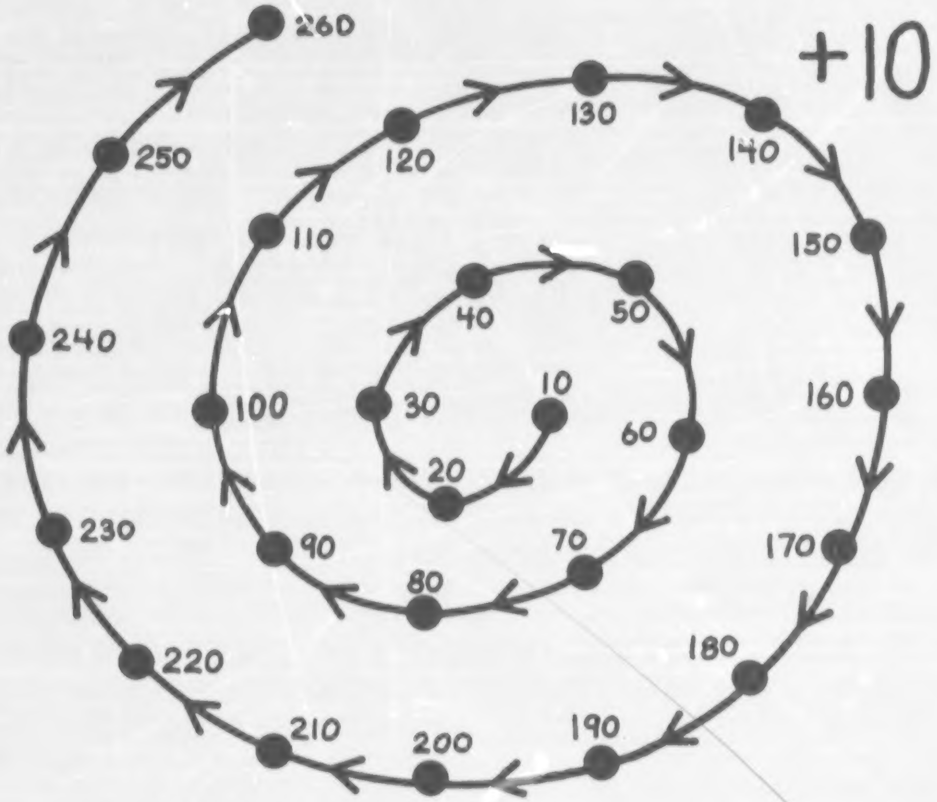


T: What do you think my red arrow is for?

S: + 10.

T: Good; now let's continue going around the room adding each person's dime into the amount of money.

Ask the student whose dot you are labeling to tell you how to label it. Try to get the students to respond quickly so that it is obvious you are just counting by tens.


+10
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250
260

T: How much money all together?

S: 260¢ or $2.60.

Check this result with the initial guesses. Ask a student to write the amount of money on the board as "$2.60".

T: Think about this problem. Suppose each person has a nickel instead of a dime. Then, how much money would you have all together? Write what you think on your paper. Then draw a picture to show adding together all the nickels.

You may have to help some students get started on a +5 picture. Students who finish this problem quickly could be encouraged to think about a similar problem for a quarter and to draw a +25 picture.

L 12

L12 String Game with A-Blocks #2

CAPSULE LESSON SUMMARY

Prepare to play the game by taking the A-Block pieces out of a box one at a time as they are described by team members. Play a game placing the pieces in a three string picture with visible labels. Play the String Game in the usual way with hidden labels.

MATERIALS

Teacher: A-Blocks String Game Kit; colored chalk; magnetic board (optional)

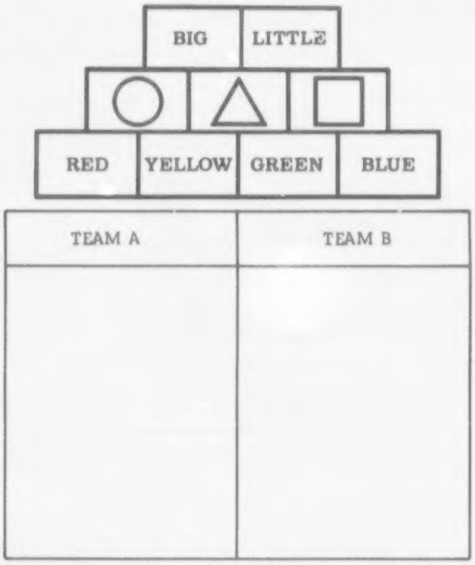
Student: None

DESCRIPTION OF LESSON

Put the A-Block pieces (shapes) in a box about the size of a shoe box. Sort them so that each one is easy to locate. Divide the class into two teams and try to arrange that the teams are roughly equal in ability.

Exercise 1

Briefly review the different kinds of A-Block pieces and post a list of attributes above the team board.



T: We are going to play a String Game with these shapes. First you have to help me take the A-Block pieces out of this box. We will take turns; someone on Team A will tell me which piece to put on Team A's side of the board, and then someone from Team B will tell me which piece to put on Team B's side of the board. Remember, you have to describe a piece that is still in the box and so I know exactly which piece you want me to put on your side of the board.

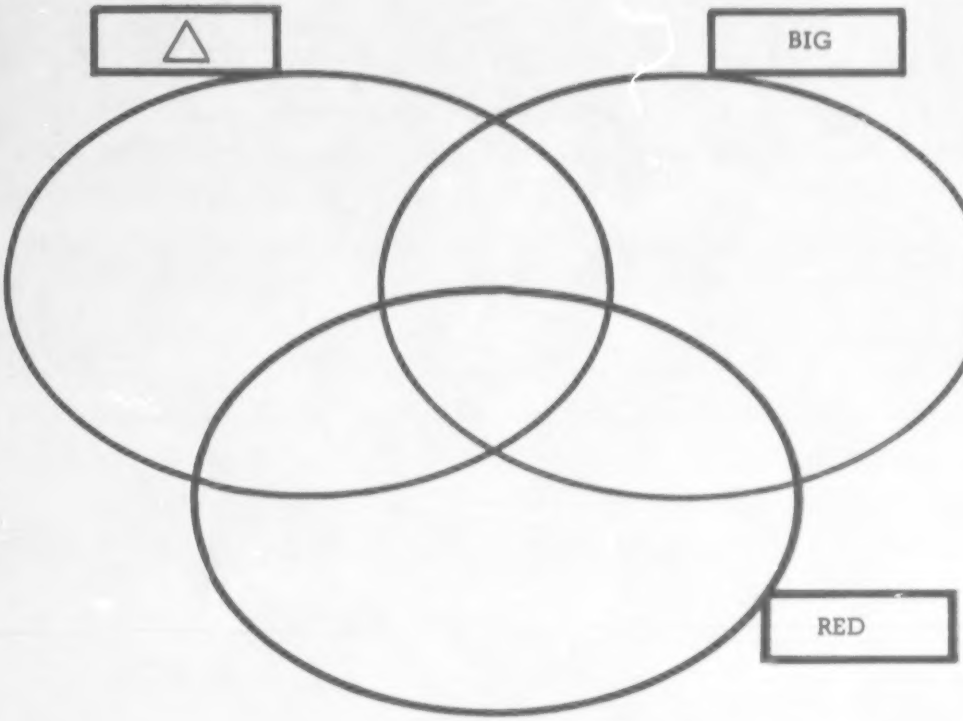
Alternating between the teams, let students describe pieces to put on the team board. This exercise serves to remind the students of what are all the A-Block pieces. Encourage the students to be decisive about which piece they want you to put on their team's side of the team board. If the piece described is already on the board, point it out and ask for another. Try to call on weak students early so that there are many choices open to them. Encourage all the students to be thinking about which pieces they will ask for when you call on them.

Exercise 2

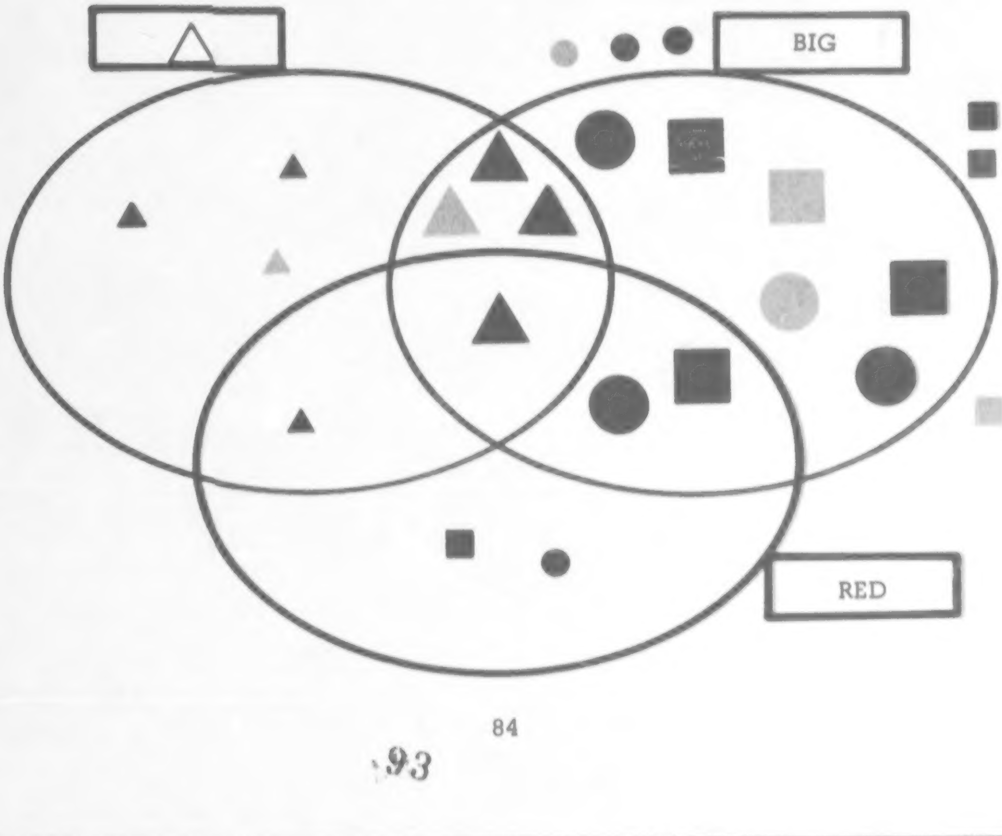
Draw three large overlapping strings on the playing board using different colors. Label the strings as shown in the next illustration.

With this situation, play a game alternating turns between the two teams and among the members of a team. Play the game just like the usual String Game except that the labels are visible so no starting clues are necessary. The bottom picture of the next illustration shows the correct placement of all the pieces. Use this picture as a "crib-sheet" while the game is being played.

L 12

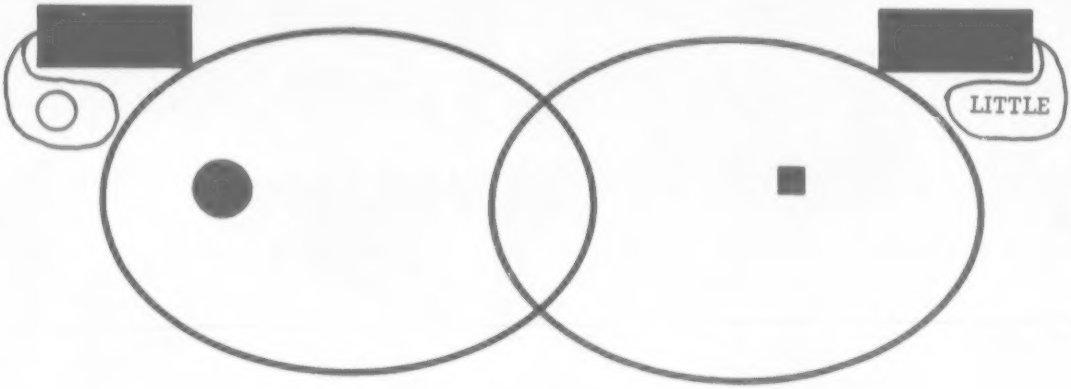


Correct placement of pieces

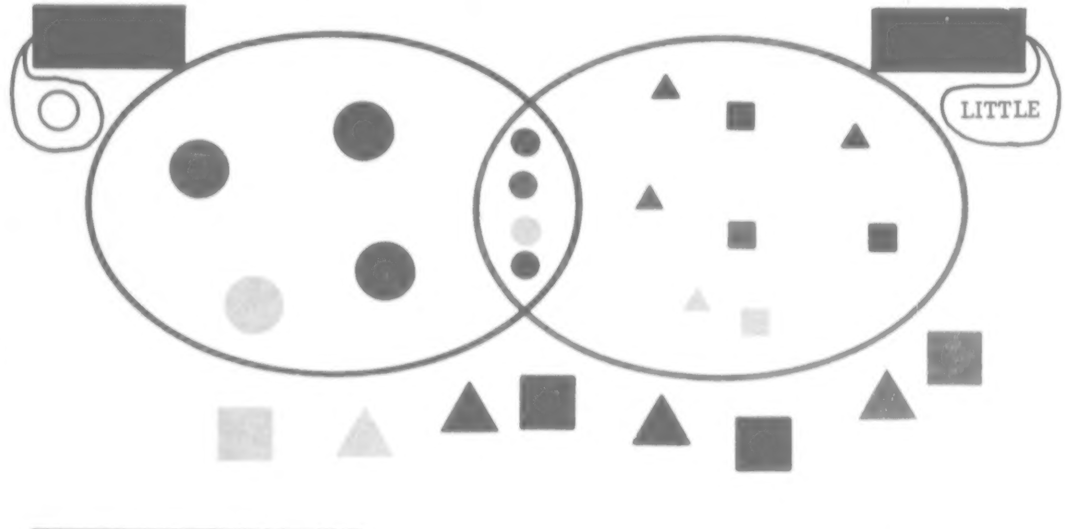


Exercise 3

Prepare to play a two string version of the String Game in the usual way with face down string cards[#]. Redistribute game pieces on the two sides (TEAM A and TEAM B) of the Game Board. Set up the playing board as shown below. The bubbles indicate what is hidden on the string cards, and there are two starting clues.



The next picture shows correct placement for all of the game pieces. Use this picture as a "crib-sheet" while the game is being played.



[#] See the Appendix: The String Game for a complete description of the game.

L 13

L13 String Pictures About Our School

CAPSULE LESSON SUMMARY

Locate children and other objects in string pictures about your school. Progressively, make the picture more and more detailed with strings for several different groups of children at your school. Examine some different ways of picturing the same situation.

MATERIALS

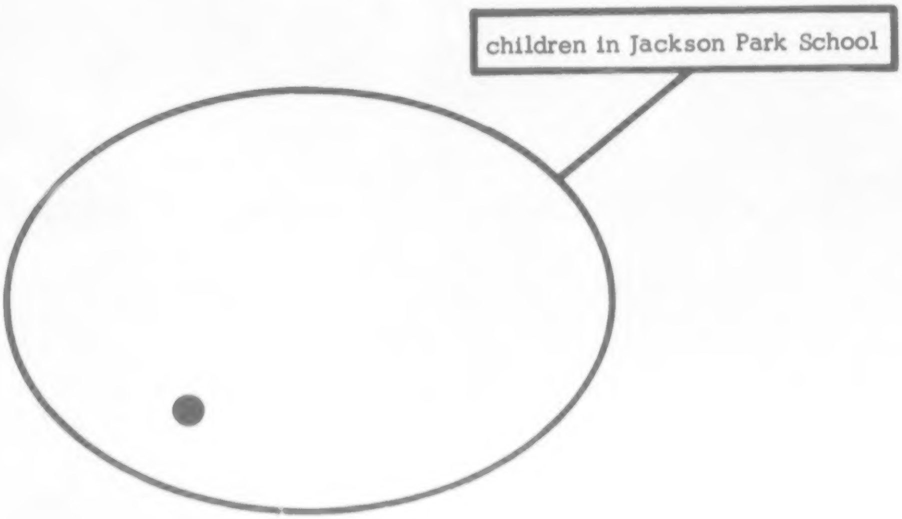
Teacher: Colored chalk

Student: Unlined paper; colored pencils

DESCRIPTION OF LESSON

Distribute unlined paper and colored pencils or crayons.

Draw a large string on the board and label it "children in your school name".



T: This string is for all the children in our school.

Draw a dot inside the string and ask:

T: What could this dot be for?

S: It could be for me.

S: It could be for my friend Michael.

S: It could be for Angela.

Let the students make many suggestions both of students in your classroom and students in other classrooms in the school. For students in other classrooms, be sure they have been well identified so the class is sure they are children who attend your school.

Draw a dot outside of the string and ask:

T: What could this dot be for?

S: It could be for my cousin, Joshua. He goes to Lincoln School.

S: It could be for my little sister. She doesn't go to school.

S: It could be for my grandmother.

S: It could be for you, teacher.

S: It could be for my dog.

Let the students make many suggestions. Be sure the class agrees on the correctness of each answer. Do not impose upon their creativity in giving answers unless there is some doubt.

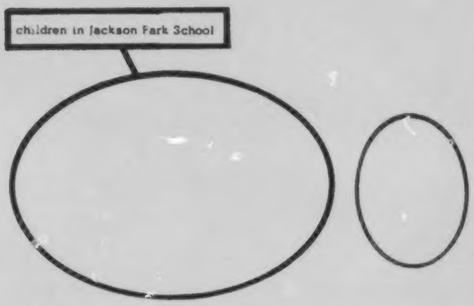
L 13

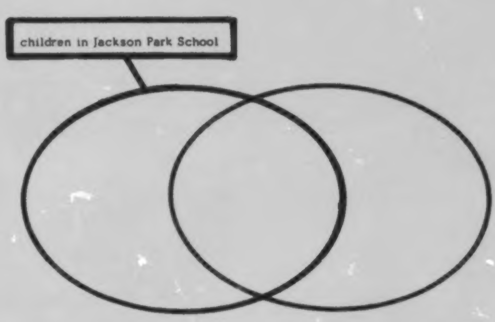
T: Copy this string on your piece of paper. (Encourage large drawings.) Listen carefully. On the same piece of paper, draw another string (say blue) for all the children in our classroom.

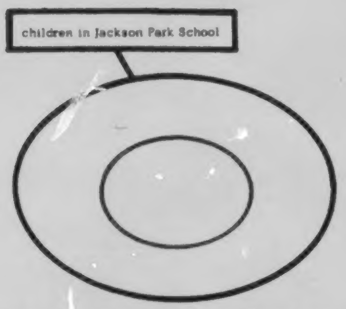
Observe the students' drawings; you are likely to see several kinds. Do not discourage anyone but do ask some suggestive questions of students who have difficulty drawing any picture.

T: I see several very interesting drawings. Some of your drawings look similiar to these.

Draw three pictures (separated) on the board.


children in Jackson Park School


children in Jackson Park School


children in Jackson Park School

For each picture, ask the class what they think about the picture. Discuss the pictures one at a time. If it is not suggested by a student, suggest yourself that you think there is something wrong with the first picture.

T: Do you remember that Angela (in our class) was inside the red string. Is she inside or outside of the blue string?

S: Inside.

T: Can it be?

S: No; she has to be inside both strings.

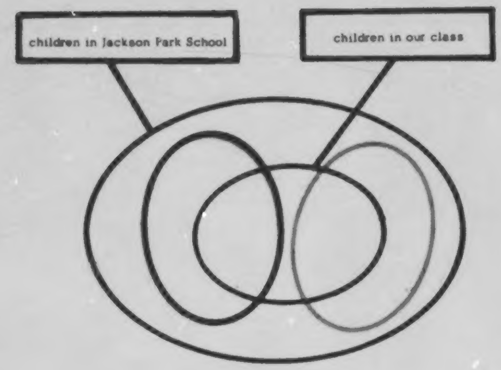
The students should conclude that the first picture is not possible. Erase it and go on to discuss the other two pictures. Although some students might suggest that the third picture is the correct one (being the most descriptive for the time being), your students should conclude that both pictures are okay. Do not force them to make observations about which is better or about empty sets unless it happens naturally. The acceptance of a picture should be strictly on the basis that every thing or person has its place in the picture.

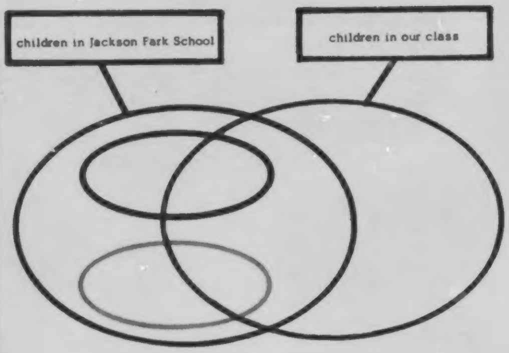
Ask students who have incorrect pictures to copy one of these two pictures (use the reverse side of the paper). The others will have to be patient for a moment.

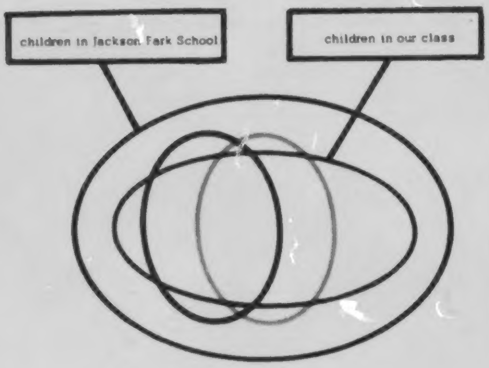
T: Now, choose two new colors (say, green and yellow). In one color (green), draw a string for all the boys in the school. In the other color (yellow), draw a string for all the girls in the school.

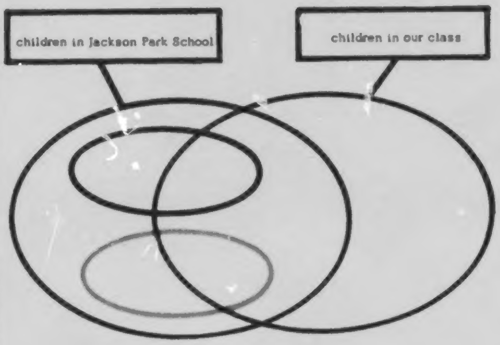
Observe students' drawings and help those who are confused by asking some leading questions. After a short while, put some of the drawings you have observed on the board. Display the drawings as you put them on the board to give students recognition. Let the class discuss each drawing. Some possible correct pictures are shown in the next illustration; do not expect your students to find all possibilities.

L 13


children in Jackson Park School
children in our class

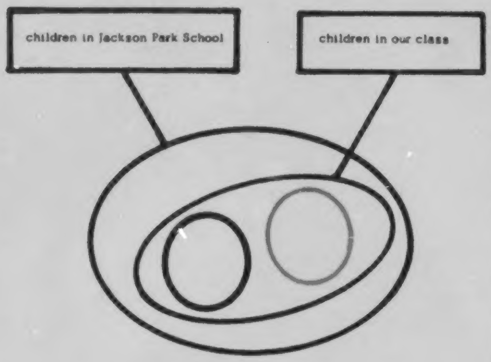

children in Jackson Park School
children in our class

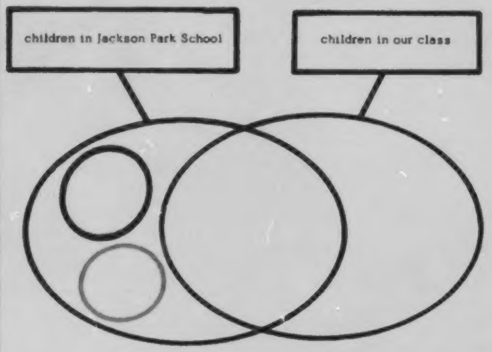

children in Jackson Park School
children in our class

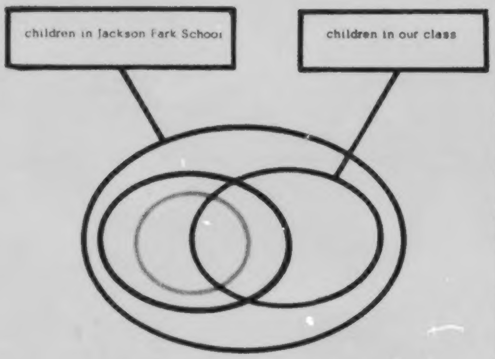

children in Jackson Park School
children in our class

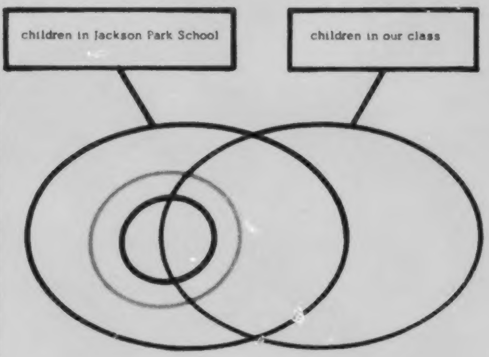
While discussing a drawing, you should lead the discussion to consider whether or not everything (person) has its place in the picture. Ask about specific children in the school but outside of your class, both boys and girls; and other children, people, or things. This discussion is very open, and you must decide how far it can proceed based on the interests and abilities of your students.

Some incorrect drawings you might observe are shown below. If one of these is discussed, the class should note that there is somebody they cannot place in the picture.


children in Jackson Park School
children in our class


children in Jackson Park School
children in our class


children in Jackson Park School
children in our class


children in Jackson Park School
children in our class

If your class is handling this activity very well and seems to still be interested, ask them to draw (in a new color) a string for all the girls in the class who are wearing ribbons in their hair. For better students, you could challenge them to draw a string for all the children in the school who wear glasses.

L 14

L14 Changing Seats

CAPSULE LESSON SUMMARY

Draw a picture of dots for all students in the class. Locate all the students in the picture. Include arrows in the picture and change seats by following the arrows. Play the changing seat game several times observing who are in their own seats after each play.

MATERIALS

Teacher: Colored chalk
Student: None

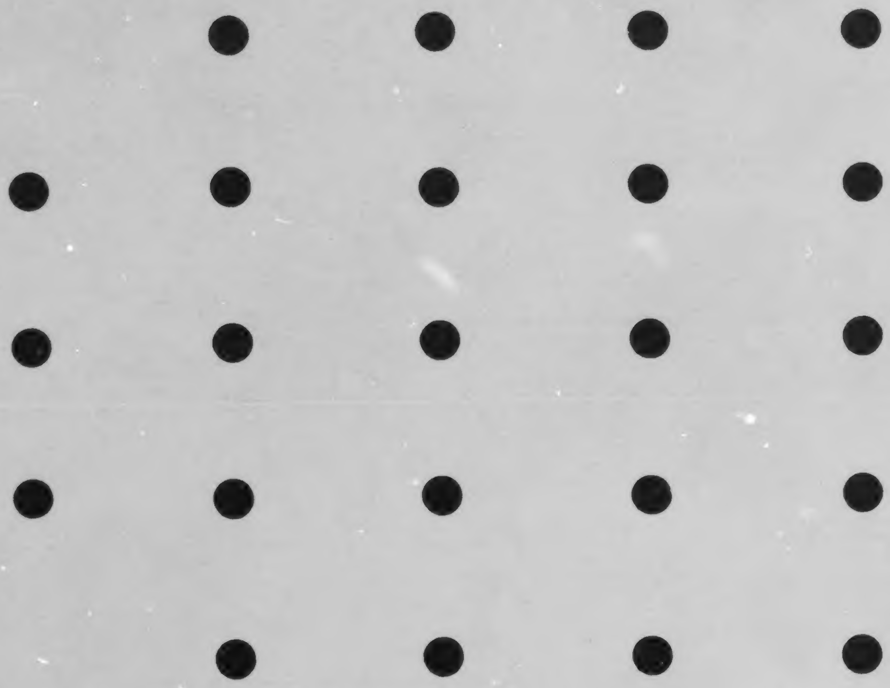
DESCRIPTION OF LESSON

Before this lesson begins, you may wish to plan an arrow picture which will be easy for your class to follow. The one used in our development class is at the end of this lesson and is the basis for some of the dialogue here.

T: I am going to draw some dots on the board. Watch me and silently count them as I draw.

Use the largest available chalkboard and draw a dot for each student in the room. Spread out the dots as much as possible but arrange them in the same relative positions as the students are sitting. (The top of the board corresponds to the front of the room or where you are standing.) Make a point of looking at the class a few times while you are drawing.

Depending on the size and arrangement of your classroom, your board might be organized in rows and columns as in the next illustration.



T: How many dots did I draw?

S: 23.

T: What do you think these dots could be for?

S: They are for us.

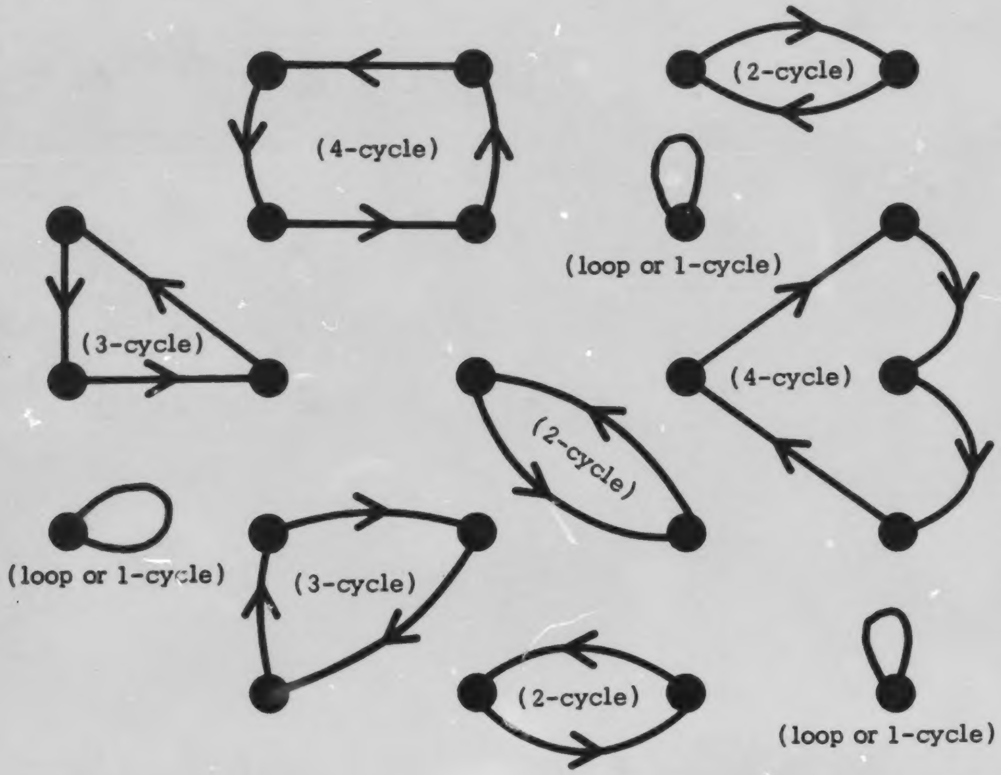
T: That's close to correct. They are for your seats. Do you know where the dot for your seat is in this picture?

Let several students come to the board and point to the dots for their seats. Point to a dot and ask the student whose seat is there to raise his or her hand. Continue this activity until you are sure all the students know where the dots for their seats are in the picture, but do not label the dots.

L 14

T: We are going to play a game; you will have to follow an arrow. So, first let me put some arrows in this picture. Pay special attention to the arrow that starts at the dot for your seat.

As quickly and carefully as possible draw arrows in your picture. Be sure exactly one arrow starts at each dot and exactly one arrow ends at each dot. Include some loops, 2-cycles, 3-cycles, and 4-cycles as indicated in the following picture.


(4-cycle)
(2-cycle)
(loop or 1-cycle)
(3-cycle)
(4-cycle)
(2-cycle)
(loop or 1-cycle)
(3-cycle)
(2-cycle)
(loop or 1-cycle)

T: How many arrows did I draw?

S: 23. One arrow starts at each dot.

S: Also one arrow ends at each dot.

Encourage such observations, but do not force the class to make them.

T: The game we are going to play is to change seats. First you must look carefully at our picture, follow the arrow that starts at the dot for your seat, and find whose seat you are going to take.

Before you let the students play the game, be sure everyone understands where they are going to go. Point to a dot and ask (for example):

T: Eugene, your seat is here. When we change seats, where will you go?

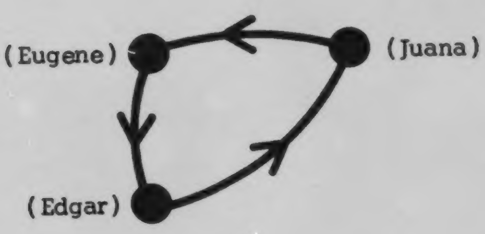
S: To Edgar's seat.

T: And Edgar, where will you go?

S: To Juana's seat.

T: And Juana, where will you go?

S: To Eugene's seat.



NOTE: Names in parentheses indicate whose seat is where at the start. They are not written on the board.

You need not ask all the students where they go, but ask the class as a whole if they are ready; if they have checked the picture carefully to know whose seat they are going to take.

T: Everyone quiet. Get ready. Change.

After the students have made their moves and are quieted, point to a dot and ask:

T: Now, who is in this seat?

Do this several times. The class should discover whether anyone made a mistake when the move was made. It is likely that a student would have been corrected during the play of the game by another student.

T: Are some of you still in your own chairs? Raise your hands. Why are you still in your own chair?

S: There was a loop at my dot, so I had to go to my seat.

Make a list on the board of all the students who are in their own chairs. For example,

ROUND 1

Tracy

Daniel

Kellie

T: Let's play the game again. Look at the picture and find where you are now. Follow the arrow that starts at the dot for the seat you are in and find whose seat you are going to take this time.

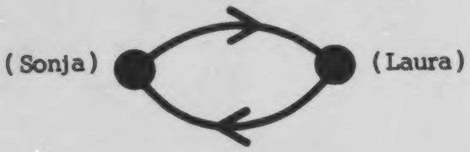
Before the students play the game again, be sure everyone knows where they are going to go.

T: Everyone quiet. Get ready. Change.

After the students move and are quieted, check with several students where they are in the picture now.

T: Who are in their own chairs? Raise your hands. Sonja, you are back in your own chair. How did that happen?

S: First I went to Laura's seat. Then I had to come back to my own seat.



Make another list on the board of all the students who are in their own chairs. For example,

ROUND 1	ROUND 2
Tracy	Tracy
Daniel	Daniel
Kellie	Kellie
	Sonja
	Laura
	Tammy
	Kevin
	Timothy
	Angela

T: Let's play the game again. But before we do, let's try to make a list of all the students who will be in their own chairs after we play again. We will check our list after we play.

L 14

Write the list of students suggested by the class on the board headed by "ROUND 3".

T: Look at the picture and find where you are now. Follow the arrow and find whose seat you are going to take this time. Everyone quiet. Get ready. Change.

After the students move and are quieted, check the list against those who are actually back in their own chairs. Make corrections as needed. For example,

ROUND 1	ROUND 2	ROUND 3
Tracy	Tracy	Tracy
Daniel	Daniel	Daniel
Kellie	Kellie	Kellie
	Sonja	Edgar
	Laura	Eugene
	Tammy	Juana
	Kevin	Lynn
	Timothy	Gary
	Angela	Chuck

T: Tracy, Daniel, and Kellie are always on our list. Why?

S: Because they always have to go to their own seats - they have loops at their dots.

T: Sonja, you are not in your own chair now. Where did you go?

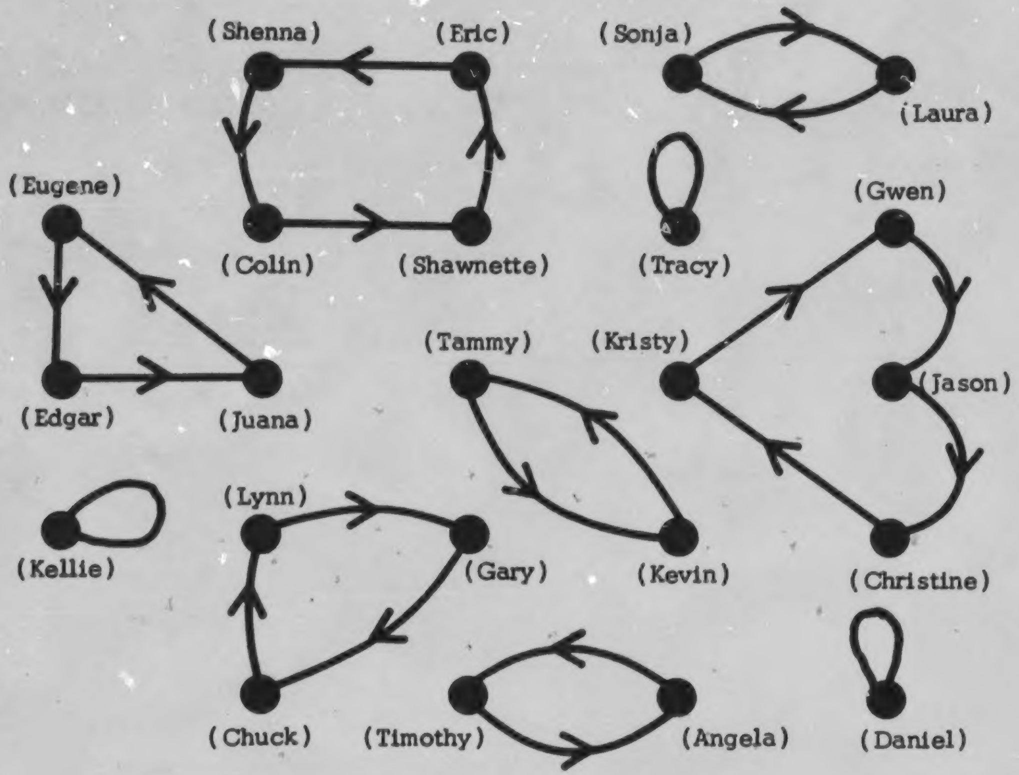
S: To Laura's seat.

Ask several such questions about the lists. Then repeat this activity for a fourth round of the game. Ask the class to first predict who will be on the list after the fourth round, make a list as they suggest, and check the list after the fourth round.

ROUND 1	ROUND 2	ROUND 3	ROUND 4	
Tracy	Tracy	Tracy	Tracy	Shenna
Daniel	Daniel	Daniel	Daniel	Eric
Kellie	Kellie	Kellie	Kellie	Shawnette
	Sonja	Edgar	Sonja	Colin
	Laura	Eugene	Laura	Kristy
	Tammy	Juana	Tammy	Gwen
	Kevin	Lynn	Kevin	Jason
	Timothy	Gary	Timothy	Christine
	Angela	Chuck	Angela	

If the class has enjoyed commenting on this game, ask some questions about these four lists (similar to the previous ones) and ask them to predict how many rounds of the game would have to be played before everyone would be back in his or her own chair. This is a difficult problem, and you should not expect to get an exact answer at this time. Some students may observe some patterns which enable them to predict 12 (the correct answer in our class).

L 14


(Shenna)
(Eric)
(Sonja)
(Laura)
(Eugene)
(Colin)
(Shawnette)
(Tracy)
(Gwen)
(Tammy)
(Kristy)
(Jason)
(Edgar)
(Juana)
(Lynn)
(Kellie)
(Gary)
(Kevin)
(Christine)
(Chuck)
(Timothy)
(Angela)
(Daniel)

Comments:

1. The names are in parentheses because they were not written on the board. They only indicate the starting situation; i.e., whose seat is where.
2. No cycle larger than a 4-cycle was drawn so that the change would not become too complicated.
3. Better students were matched in a cycle with slower students so that they could help them make the correct moves.
4. The loops were drawn at dots for children who are confident individuals and would not be embarassed by having to always stay in their own seats.

L 15

L15 Project: Making a Temperature Bar Graph

CAPSULE LESSON SUMMARY

Introduce a long term project of keeping a record of the morning temperature in degrees celsius. Play the String Game or Number Line game.

MATERIALS

Teacher: Thermometer; grid paper
Student: None

For this lesson you will need to obtain an outdoor celsius thermometer which can be mounted outside of a classroom window and read from inside the room. This thermometer should be calibrated each degree and labeled at least every 10 degrees.

You will also need to prepare a chart on which to record daily temperatures. (See page 102 for a reduced copy of such a chart.) To do this we suggest taping together three or four sheets of the largest size grid paper available to you; grid paper with 2 cm by 2 cm squares is ideal. Post this chart on a bulletin board or some other convenient place in your classroom.

DESCRIPTION OF LESSON

On the morning that this lesson is taught (preferably the first class day in January), make note of the temperature in degrees celsius.

Begin the lesson by showing the class the thermometer and allowing the students a few minutes to examine it close up. Discuss how one reads the temperature on a thermometer, perhaps reading and calling on individual students to read the (indoor) classroom temperature on this thermometer. Make special note to tell them that the scale here is for degrees celsius (write this word on the board) and note the classroom temperature as (for example) 22° C. Discuss the fact that sometimes the temperature is above zero and sometimes below zero. For a temperature below zero, for example five below zero, we write "$\hat{5}$° C".

L 15

T: We are going to put this thermometer outside our classroom window, and every morning[*] we will record the temperature on this chart.

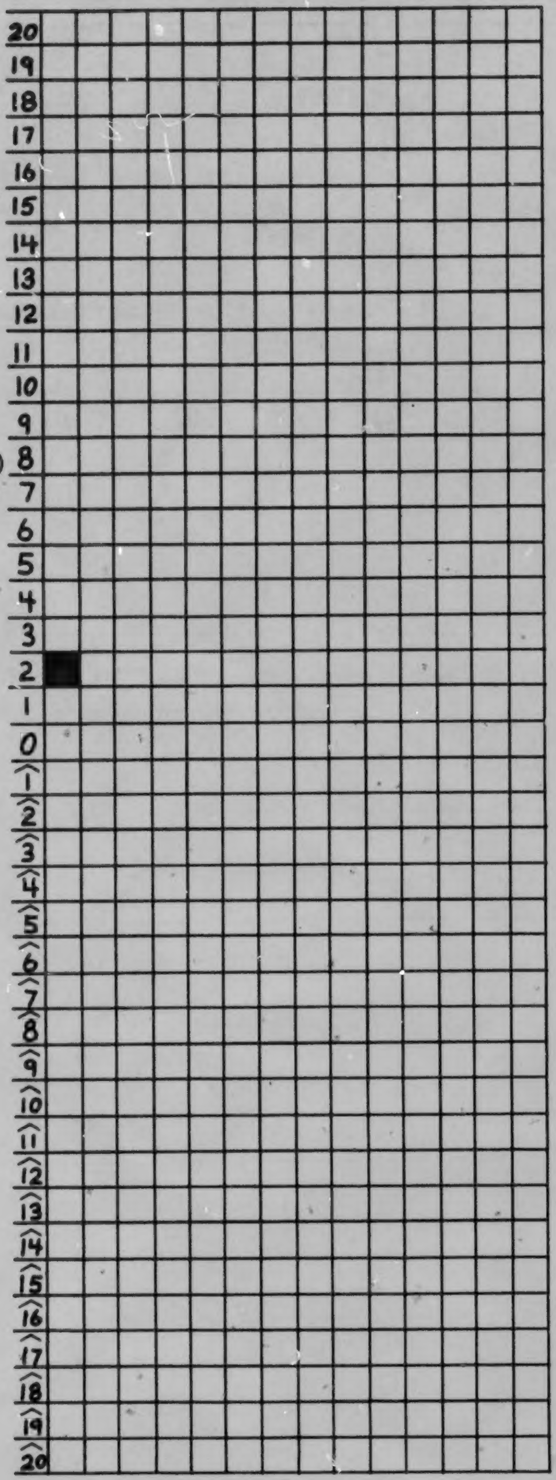
This morning I read my outdoor thermometer at home so we could record today's morning temperature. It was (for example) 2°C. Where do you think we should record this on our chart?

Invite a student to locate (in this case) 2 (above zero) on the chart.

T: To record the temperature, we shade the square next to "2".

Ask a student to do this as indicated in the chart here.

TEMPERATURE CHART



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[*] It would be best to read and record the temperature at the same time every day; we suggest first thing each morning.

L 15

NOTES:

1. Your class will be keeping a record of the morning temperature in degrees celsius for the next two months (school days in January and February). Let different students read the thermometer and record the temperature each day. Every student should have at least one or two chances to do this.

2. The first time a student reads the thermometer, check it with him or her. If the mercury is between two marks, read the temperature to the nearest degree.

3. Each day you might ask one or two questions. For example,
   - a) What do you think the temperature will be this morning? After taking the temperature, ask who had the closest guess.
   - b) Yesterday the temperature was 2° C and today it is 3° C. Is it warmer or colder today? How many degrees colder?
   - c) What is the coldest termperature we have recorded so far? I heard that this morning was the coldest morning of the year. What do you think the temperature will be this morning?

For the remainder of the class time, play either the String Game or the Number Line Game.

L16 Composition #2

CAPSULE LESSON SUMMARY

Play the composition game with the rule: yellow followed by blue is red. Reverse the order in which the colors of the arrows are considered to introduce a new rule: blue followed by yellow is green. Play a game using both rules.

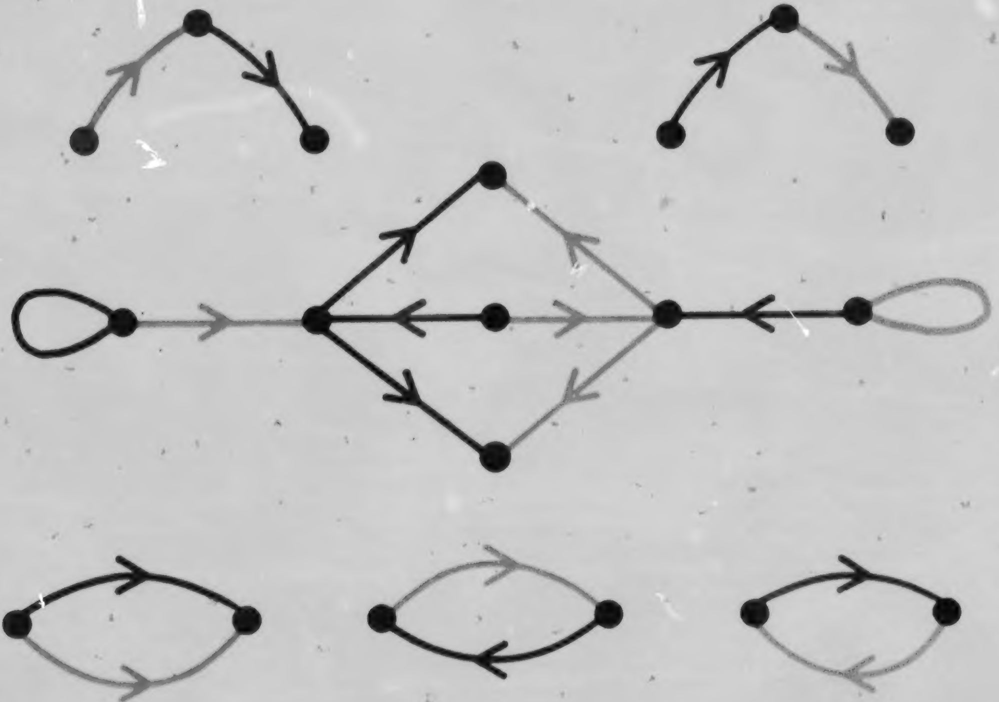
MATERIALS

Teacher: Colored chalk
Student: None

DESCRIPTION OF LESSON

Exercise 1

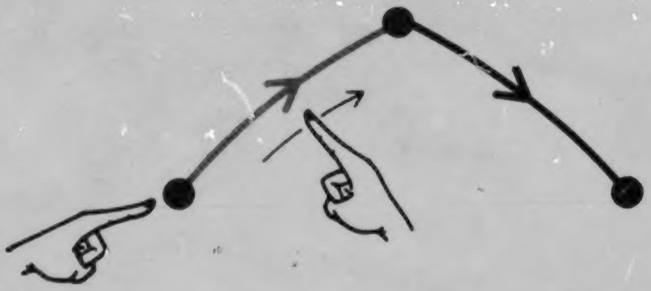
Before the lesson begins, draw this arrow picture on the board. Spread the picture out over the entire board and draw arrows whose colors can be easily distinquished.



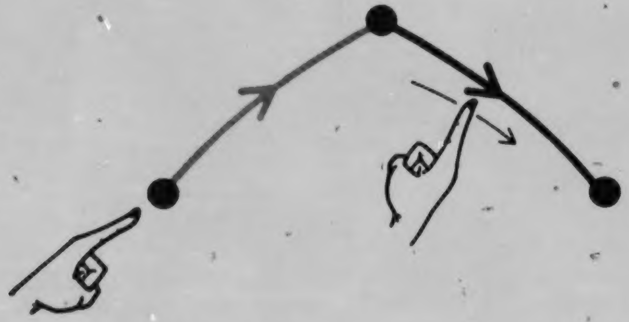
T: Do you remember the game where whenever a yellow arrow is followed by a blue arrow, then you can draw a red arrow?

Use the upper left corner of the arrow picture to review the rule of the game. Use motions as described in Lesson L10 Composition #1.

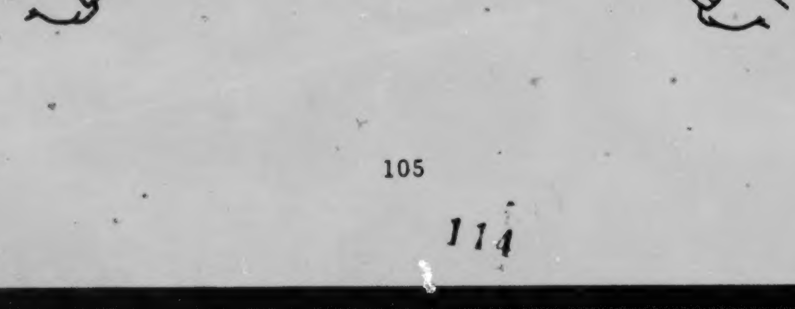
T: A yellow arrow (trace the yellow arrow) . . .



. . . followed by a blue arrow (trace the blue arrow) . . .



. . . is a red arrow (trace and then draw a red arrow).



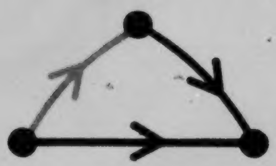
L 16

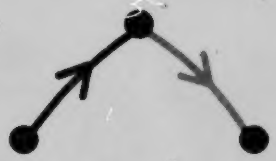
Remind the class that your left forefinger marks the start of the red arrow and that your right forefinger moves to the end of the red arrow.

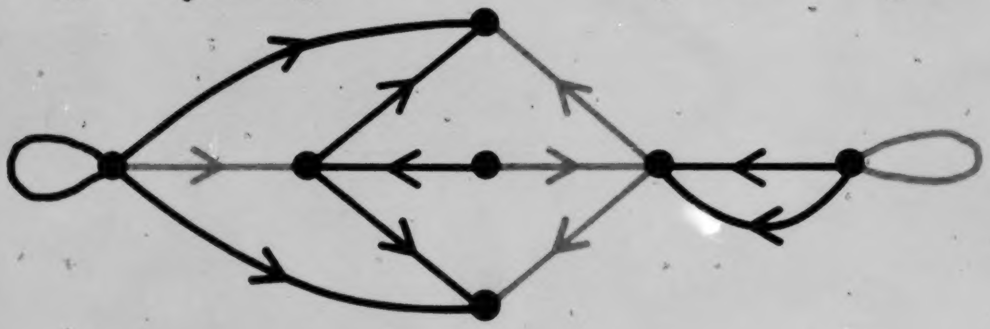
T: Who can show us where we can draw other red arrows?

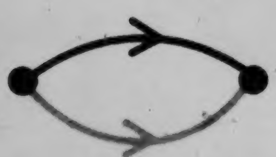
Invite students one at a time to the board to show where red arrows can be drawn. Ask each student to trace a yellow arrow followed by a blue arrow and then to trace the anticipated red arrow. Ask the class to help you watch and catch mistakes such as tracing against the direction of an arrow.[*] Draw red arrows yourself after they have been correctly traced. Help a student who has difficulty by holding the student's left forefinger at a dot where a yellow arrow starts. Then guide the student's right forefinger to trace a yellow arrow followed by a blue arrow.

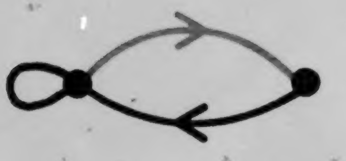
The following pictures shows all the red arrows that can be drawn.

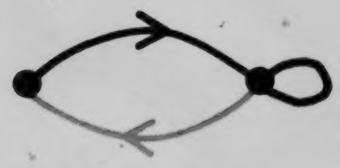












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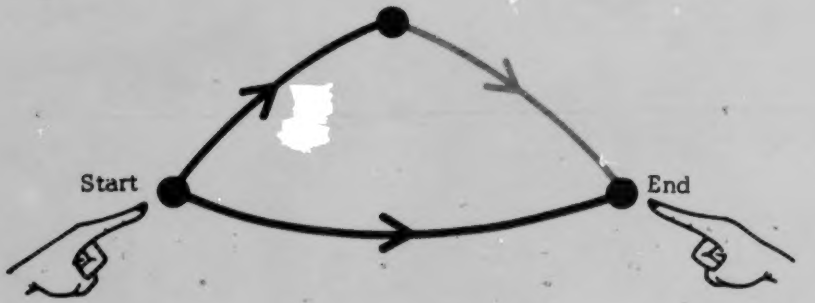
[*] For an illustration of some common mistakes see Lesson L10 Composition #1.

T: Let's introduce a new rule to our game.

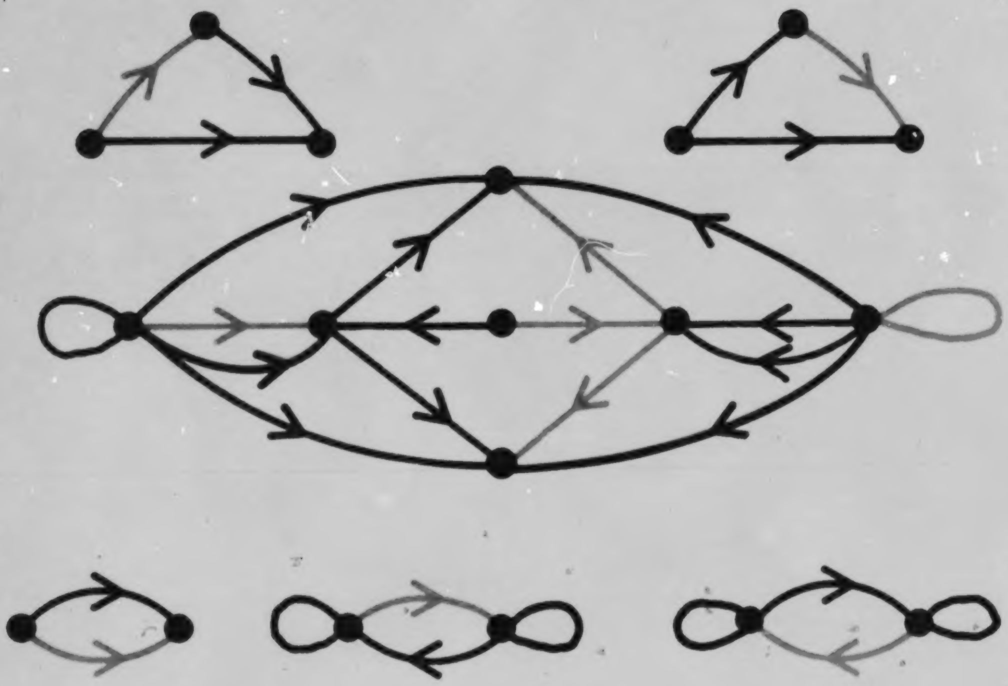
Use the upper right corner of the arrow picture to introduce the new rule. Again use motions as described in Lesson L10.

T: A blue arrow (trace the blue arrow) followed by a yellow arrow (trace the yellow arrow) is a green arrow (trace and then draw a green arrow).



Invite students one at a time to trace arrows on the picture to show where other green arrows can be drawn. Draw green arrows yourself after they have been correctly traced.

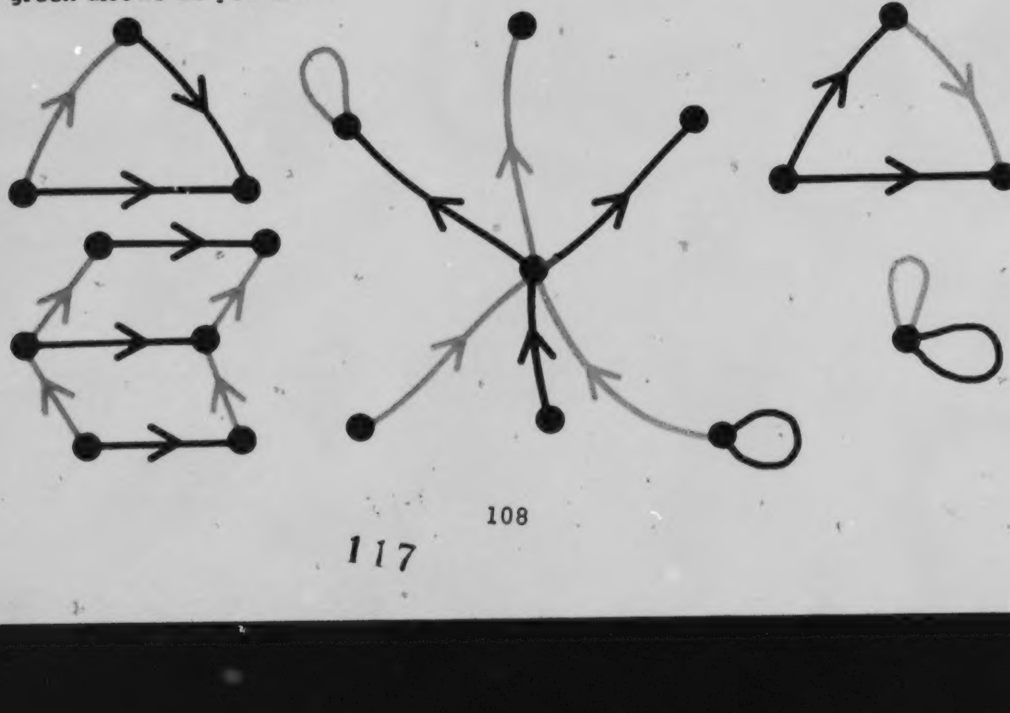
The next illustration shows all the green arrows that can be drawn.



Erase the board except for the upper left and right corners; these parts of the picture serve as reminders of the rules.

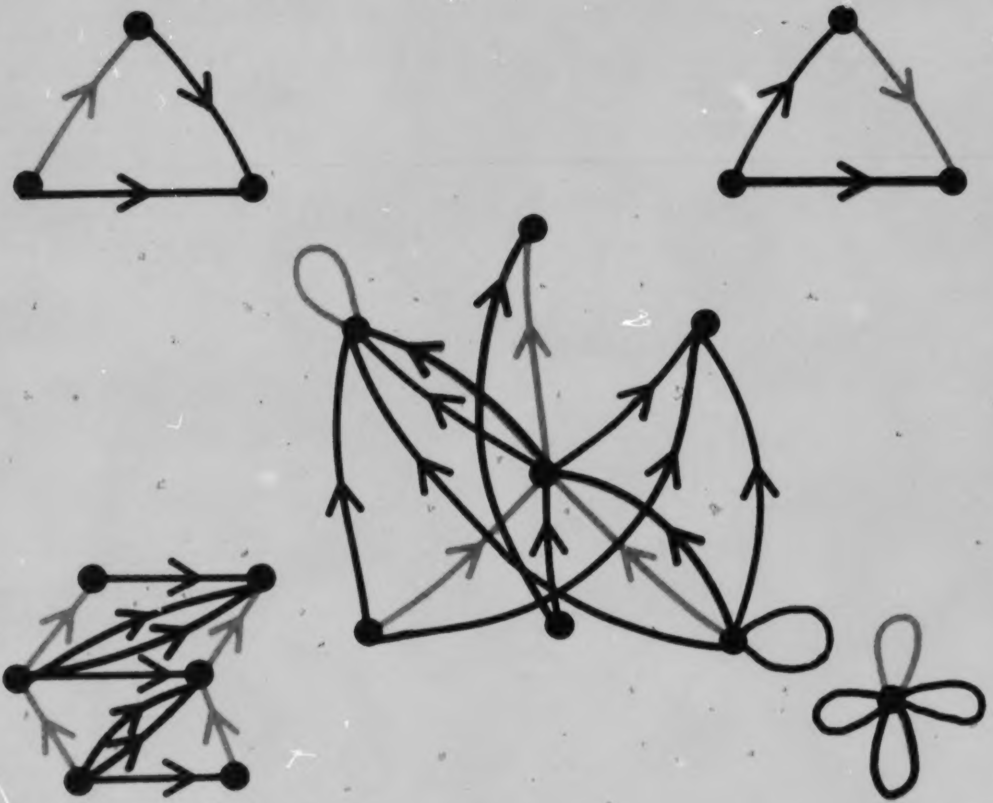
Exercise 2

Draw this picture on the board. Ask the class to begin looking for red and green arrows as you draw.



Invite students, one at a time, to trace and then to draw red and green arrows. Here the students are free to find either a red or a green arrow which can be drawn. Encourage all the students to participate, especially those who did not have a turn in Exercise 1.

A completed picture is shown below.



L 17

L17 The Happy Puppet

CAPSULE LESSON SUMMARY

Read the storybook THE HAPPY PUPPET and invent new stories about the various string pictures and arrow pictures in the book.

MATERIALS

Teacher:	Colored chalk; "The Happy Puppet" storybook
Student:	"The Happy Puppet" storybook

DESCRIPTION OF LESSON

Sometime before this lesson begins, you should read the storybook THE HAPPY PUPPET on your own. Read also the comment by Edward Martin on the last page of the booklet. While doing this, think about the various situations and how your students might react to them. You should feel free to bring your imagination and your experience and your general familiarity with the interests and concerns of your students to bear on the lesson. We shall simply make some suggestions on how children might react and how you might prompt creative thinking.

In order to create the right climate for the fanciful adventures of the dynamic "Happy Puppet", the story should not be read too slowly. Many second graders are not yet very good readers, so unless you have some unusually good readers, you should do most of the reading yourself. The students may follow the reading in their own books. They should have time to look at the pictures and to answer the questions by pointing to dots or to parts of the realistic drawings, or by telling their own stories. Allow spontaneous reaction to the story and expression of feelings about the "Happy Puppet".

If your students get very involved in telling their own stories during this lesson, do not be concerned about finishing the storybook. There will be another telling stories lesson later in the year during which you could finish this storybook.

Pages 1 and 2

Read both pages to the class. Encourage discussion about what a hand-puppet is and how one might use hand-puppets to tell make-believe stories. On page 2 there are many interesting pictures about which children may wish to comment. Ask simply what they see and you will get a variety of responses.

S: I see lots of arrows and some loops.

S: I see "+", "x", and "÷".

S: I see "44 + 44" and "88" and they are the same.

S: Also, 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 = 88.

S: 88 is on the Minicomputer flags.

Pages 3 and 4

Read both pages to the class. Let the students comment on the changing interpretation of the eight dots and, perhaps, give some new interpretations of their own. Be sure to note how there are always eight dots; the number of dots does not change even though the interpretation may.

Students might not be familiar with the word "delights". Discuss its meaning and how it fits the "Happy Puppet" game.

Pages 5 and 6

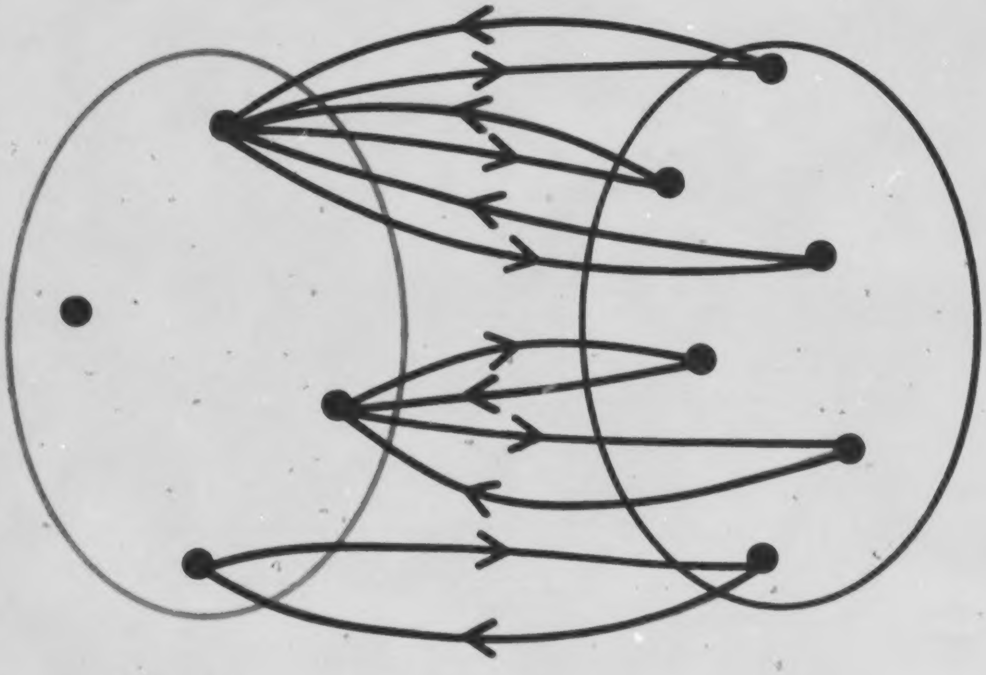
Read both pages collectively pausing to allow students to answer the questions. Ask students to locate the different cars in the string picture. After some discussion of this situation, encourage students to invent their own stories about the string picture. Let the class discuss whether a student's story fits the picture. You may find it helpful to draw the picture of page 5 on the board during this discussion.

L 17

Pages 7 and 8

Read both pages collectively pausing to allow the students to point to appropriate dots and to answer the questions.

In each situation, discuss what the red arrows are for and what the return arrows could be for. You may find it helpful to draw the picture on the board. Let students help you draw return arrows.



Pages 9 and 10

Read and discuss pages 9 and 10 collectively. Encourage students to tell their own stories about the arrow picture. Some may recall the exchanging names activity the class played several weeks ago or the game of changing seats. Do not expect clearly stated stories that exactly fit the picture; but do encourage the class to check whether a story does fit the picture.

Pages 11 and 12

Read these pages collectively pausing to ask students to point to appropriate dots and answer the questions. After reading page 12, ask the students to show where the various toys are in the string picture.

Pages 13 and 14

Allow the students to read these pages on their own or read them collectively, whichever you think is most appropriate for your class. There are no pictures to be investigated nor to be discussed on these pages.

APPENDIX: The String Game

The String Game is used in many versions throughout the CSMP Elementary School Program. It gives students an opportunity, in a game-like atmosphere, to become familiar with the language of strings, while at the same time it involves them in the kind of reasoning that will be developed and reinforced in various contexts throughout the program.

The String Game is first played with A-Blocks (shapes), but later on in the Intermediate Grades variations which make use of numbers make their appearance. In this appendix we provide you with the necessary information and examples to enable you to play the game using A-Blocks, and we suggest some of the various possibilities open to you when you play the game with shapes. Although only a few lessons in UPG Part I are scheduled for the String Game, we hope you will not feel restricted to playing the game only at these times. It has been found that it is most beneficial and enjoyable for students if you make a regular practice of playing the game about once every two weeks, or whenever you have an extra 10-15 minutes during the course of the day.

Equipment

PLAYING BOARD

The most easily manageable equipment for this game depends upon your having a magnetic* chalkboard available. Many of the permanently mounted chalkboards in classrooms are magnetic, and you should test yours using one of the demonstration Minicomputer checkers. If your permanent chalkboards are not magnetic, try any portable chalkboard (grid board, etc.) that the school has available. If you do not have a magnetic chalkboard available, you can use your regular chalkboard.

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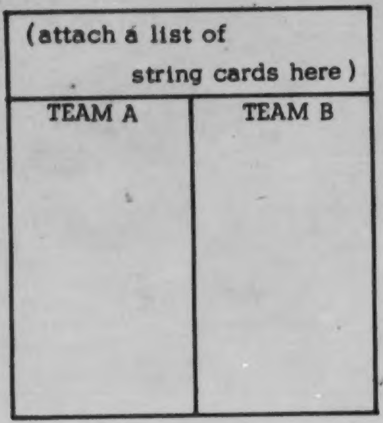
* Strictly speaking the chalkboard itself is not magnetic, but rather magnet-sensitive; i.e., magnets adhere to it. However, for simplicity we call such boards "magnetic".

TEAM BOARD

The Team Board is divided into three regions as illustrated below.



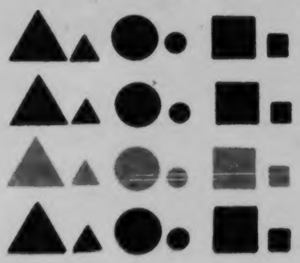
a) Magnetic: If you do locate a magnetic chalkboard, you will also need a magnetic or metallic Team Board. With a large magnetic chalkboard, say at least three chalkboard squares, the Team Board could be drawn on the chalkboard. However, if you have a relatively small (portable) magnetic chalkboard, then you will need to obtain a sheet of metal (minimum size: 60 cm by 80 cm) or locate a convenient metallic surface in the classroom such as the side of a file cabinet on which to put the Team Board. In this case draw the markings for the three regions of the Team Board on a large sheet of (easel pad) paper and tape this paper to your metallic surface.

b) Non-magnetic: If you do not have a magnetic chalkboard available for the playing board, then your Team Board could be a large piece of poster board (minimum size: 60 cm by 80 cm). Make the markings for the three regions of the Team Board with a heavy felt-tip pen.

GAME PIECES and STRING CARDS

One set of game pieces and string cards are needed for each version of the game. A list of the string cards (A-Blocks String Game Poster) is to be attached to the upper section of the Team Board - it is a constant reminder during the game of the possible labels for the strings.

Game Pieces


String Game
with
A-Blocks

String Cards

RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE

Game pieces, string cards, and the poster list of the string cards can be found in the A-Blocks String Game Kit.

a) Magnetic: The game pieces (A-Blocks) can be magnetized by sticking to the back of each one a small piece of magnetic material. (Magnetic material is included in the A-Blocks String Game Kit, or it is available in many hobby and notions departments of stores.) Similarly, string cards are magnetized by sticking a small piece of magnetic material to the front of the card, taking care not to obscure what is written thereon.

b) Non-magnetic: Game pieces can be attached to the Team Board by means of loops of masking tape stuck to the backs. A string card should have a loop of masking tape stuck to the front so that what is written on the card is not obscured. With this type of equipment, you should prepare yourself for running repairs by having sufficient masking tape available so that if a loop of tape loses its stickiness, it can be replaced on the spot. An alternative is to use a small wad of a plastic caulking compound (Rope Caulk or Mortite, for example) in place of the loop of masking tape. This material will maintain its stickiness over a much longer period of time.

Preparation for the Game

Draw two (or three, depending on which variation you are using) large, overlapping strings on the playing board using two (or three) different colors. Next to each of these strings attach one string card face-down. Locate the Team Board near and convenient to the playing board. Share the game pieces between the two sections of the Team Board labeled "TEAM A" and "TEAM B" by randomly placing one-half of the pieces in one section and the remainder in the other section. Divide the class into two teams using whatever method is acceptable to your class; e.g., boys vs. girls; rows 1, 2, 3 vs. rows 4, 5, 6; etc. Call one team "Team A" and the other "Team B". Before any student takes a turn, correctly place an equal number (at least one) of each team's game pieces in the string picture. This removes from the first players the necessity of operating on the basis of pure guesswork. You can influence how long the game will take by the number of pieces you place in the string picture before the game begins.

Object of the Game

Each team tries to place all of its game pieces correctly (according to the face-down string cards) in the string picture. The winning team is the one which first places all of its game pieces correctly and identifies the face-down cards correctly, after playing according to the rules.

Rules of the Game

1) The game is played in silence. This is to avoid one student telling another where to play a certain piece, or what the strings are for. Each student should have the opportunity to analyze the game alone. Infringement of this rule by anyone is penalized by the talker's team losing its next turn.

2) The teams alternate, and the members take turns within each team. A player comes to the board and selects a piece from his or her team's collection to place in one of the regions of the string picture.

3) You are the judge. If the piece is correctly placed, say "yes". The piece remains in the string picture and the player immediately has a second turn (no player may have more than two consecutive turns). If the piece is incorrectly placed, say "no". The player returns the piece to the team's unplayed collection and play passes to the other team.

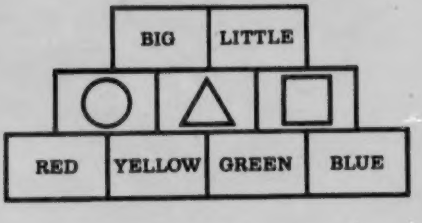
As an aid in judging, prepare a "crib-sheet" showing the correct position of each game piece or at least reminding you of what is on the face-down cards. If at any time you discover that you have made an error, say so immediately and rectify the mistake. Either move an incorrectly placed piece whose position you had approved to its correct region or replace in the string picture a correctly placed piece that had been rejected because you had disapproved its position.

4) When a team has correctly placed all of its pieces, the player who placed the last piece may thereupon attempt to identify each of the string cards. If these are all correct, the team has won. If a mistake is made (even if it is only in the case of one of the string cards), simply indicate that the identification is incorrect and let the game continue.

5) If a team has exhausted its stock of game pieces and the strings have not been identified, that team continues on its turn to attempt to identify the strings, while the other team works to place its game pieces.

Version A: The String Game with A-Blocks

This simplest version of the game uses 24 A-Blocks as game pieces and only nine string cards.

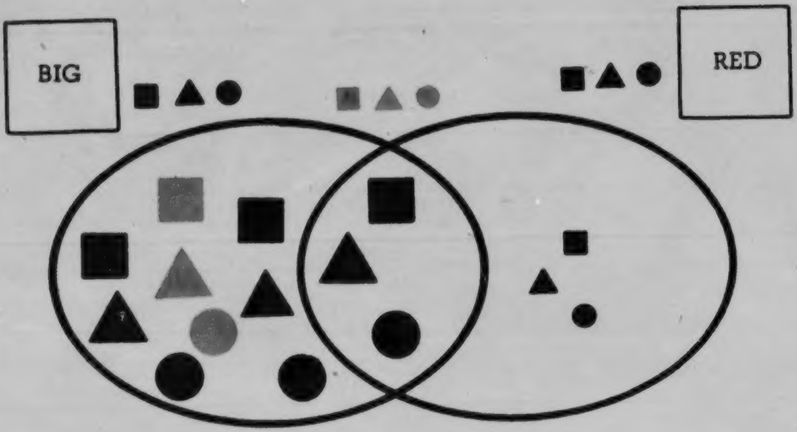


This list of string cards should be posted on the board or attached to the game board.

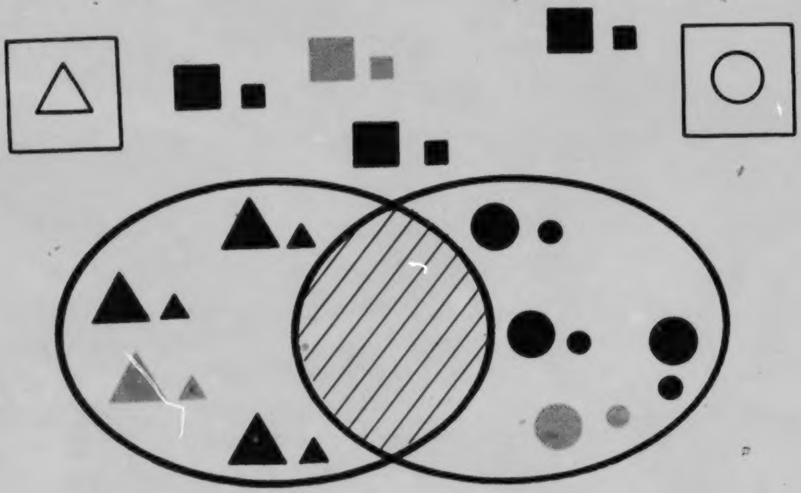
We provide here several example "crib-sheets" for variations of the game with two and three strings.

Variation #1: Two Strings

Example 1: No Empty Regions

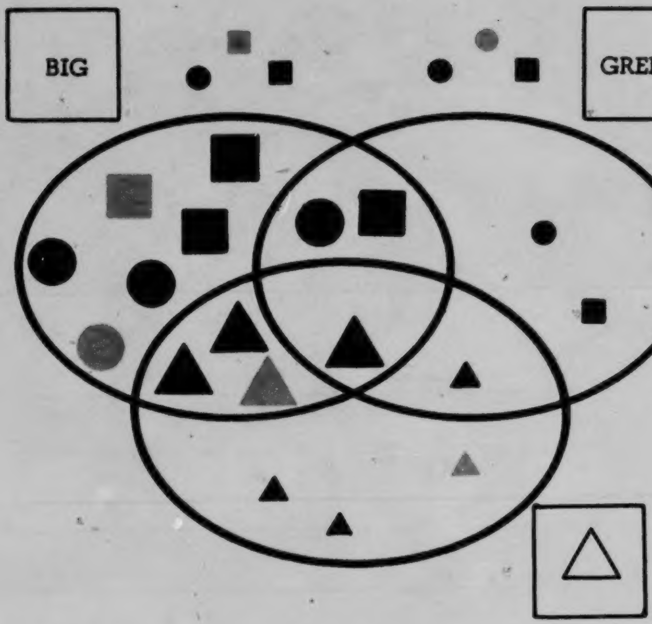


Example 2: One Empty Region

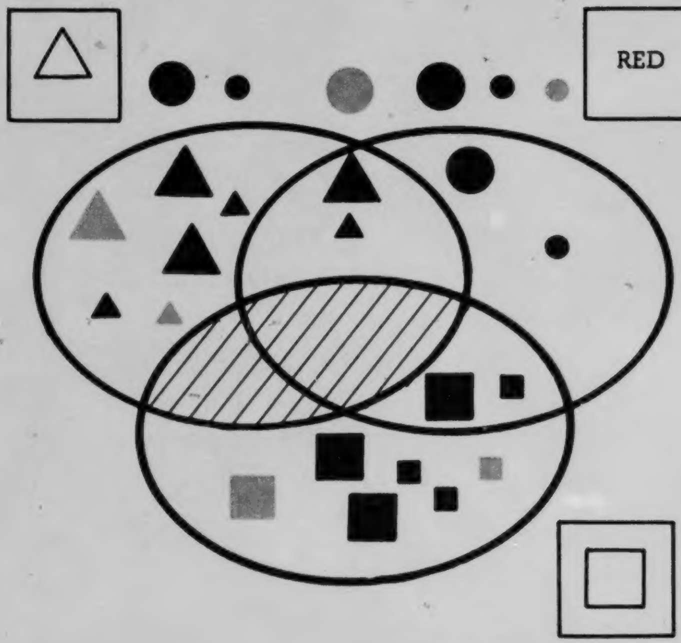


Variation #2 : Three Strings

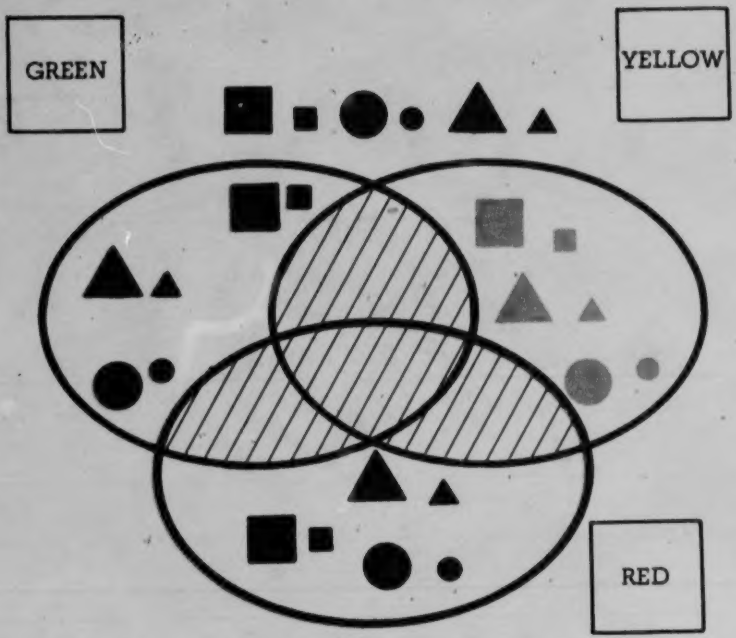
Example 1 : No Empty Regions



Example 2 : Two Empty Regions



Example 3: Four Empty Regions



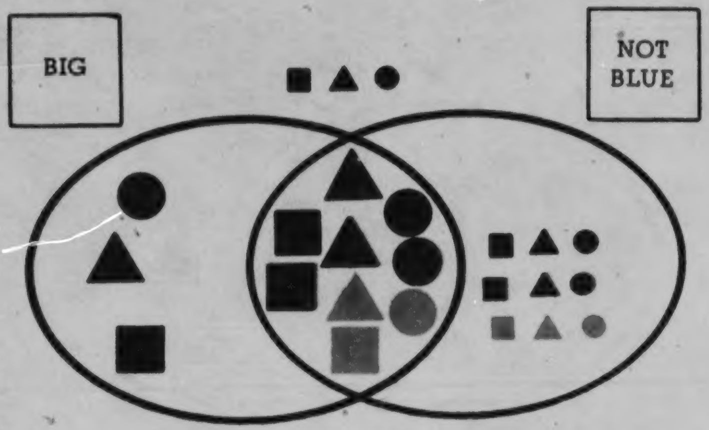
Version B: The String Game with A-Blocks, using "Not" Cards

A more complicated version of the game with A-Blocks uses all 16 of the string cards. A list of all 16 string cards should be posted or attached to the Team Board.

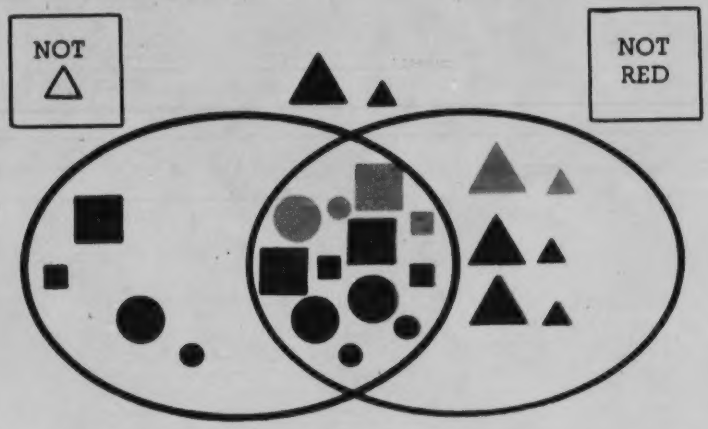
NOTE: Since "NOT BIG" and "LITTLE" describe exactly the same A-Blocks (and similarly for "NOT LITTLE" and "BIG"), such a "not" card is redundant. If a student questions why we do not include these possibilities, let the class discuss that it is unnecessary.

We provide here several example "crib-sheets" for this version of the String Game played with two strings.

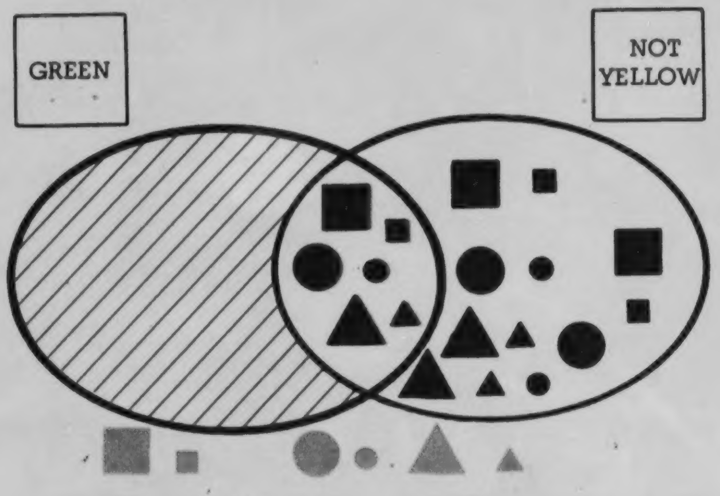
Example 1 : No Empty Regions


BIG
NOT BLUE

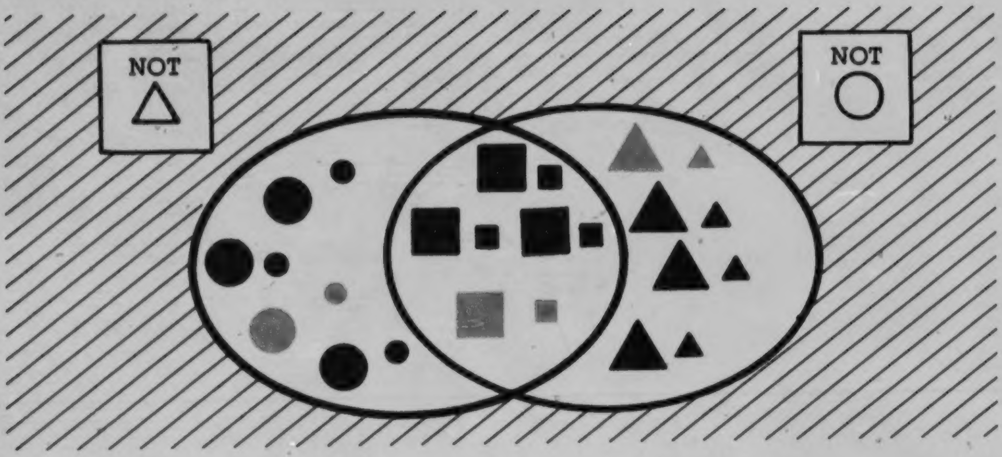
Example 2 : No Empty Regions



Example 3 : One Empty Region


GREEN
NOT YELLOW

Example 4: One Empty Region



Of course, by hatching the "outside" region of this diagram we mean to indicate that no game pieces can be placed there correctly. Strictly speaking, that region is not empty because, for example, the number 50 is in the outside region.

You should be warned that the "NOT" version of the game played with three strings is almost impossible to judge without a "crib-sheet" and is correspondingly difficult to play. Hence you would be well-advised to avoid it until such time as you think the two-string version is no longer challenging enough for the majority of your students.

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GEOMETRY
AND
MEASUREMENT

Comprehensive School Mathematics Program

CSMP Mathematics for the Upper Primary Grades

Part I

Geometry and Measurement

CEMREL, Inc.

13-1

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GEOMETRY AND MEASUREMENT

INTRODUCTION

Geometry is useful and interesting, an integral part of mathematics; geometric thinking permeates mathematics and relates to most of the applications of mathematics. Geometric concepts provide a vehicle for exploring the rich connections between arithmetic concepts (such as numbers and calculations) and physical concepts (such as length, area, symmetry, and shape). Even as children experience arithmetic notions in their daily lives, they encounter geometric notions. Common opinion recognizes that each child should begin developing a number sense early. There is no sound argument to postpone the development of geometric sense. These are compelling reasons why the systematic development of geometric thinking should be a substantial component of an elementary school curriculum.

A benefit of early development of geometric thinking is that it provides alternative opportunities for "slow learners". Children develop number concepts at individual rates. Relative lateness of a child's numerical development often signifies trouble remembering number facts. A common response to this problem is assignment of arithmetic drill, which serves only to confirm the diagnosis of slowness. However, numerical development may not be an accurate indication of other development, in particular, of geometric development. Often the labels "slow" and "fast" are incorrect when alternative activities are presented.

We are careful to allow students a great deal of time to explore quantitative relationships among physical objects before studying arithmetic. In the same spirit, students should be allowed to explore geometrical relationships through free play and guided experiences. Before studying geometrical properties of lines and shapes, they should have free play with models and with drawing instruments in order to create new figures and patterns. Before considering parallelism and perpendicularity as relations on geometric lines, they should have free play in creating patterns with parallel and perpendicular lines. Before learning length and area formulas, they should have informal experiences to establish and sharpen intuitive notions of area and length. These early experiences are aesthetically rewarding and just as stimulating and suggestive of underlying concepts as early experiences leading to arithmetic. Such experiences are a crucial first step in the development of a geometric sense.

As with all educational development occurring through informal experiences, the payoffs come unexpectedly and spasmodically. In the development of geometric thinking, the suggested sequence of activities is so varied, reaches into so many other facets of the program, and continues for so long a span that it would be nonsense to set a priori milestones of accomplishments.

Thus, we must not rush the students through the free play activities that precede the more substantive lessons. These preliminary activities have implicit significance for geometric thinking and also provide aesthetic opportunities and explicit practice. We must restrain our inclination to view these activities as mere play. On the other hand, some activities appear to have important consequences or clearly relate geometry to other parts of the program. Here the inclination might be to spend more time on these activities than is suggested. Again this should be resisted, remembering that similar activities will continue to be experienced later in a slightly different context.

The key word is "experience" not "mastery".

But what sorts of experiences will lead students to think geometrically? The standard curriculum usually consists of measurement activities, recognition of some special geometric figures, and association of these figures with their technical names; in short, a good deal of vocabulary but little opportunity for geometric thinking.

A glance at the lessons of this strand indicates that the CSMP Upper Primary Grades curriculum includes measurement activities (leading to the use of metric units as well as topics not usually in traditional programs at this level).

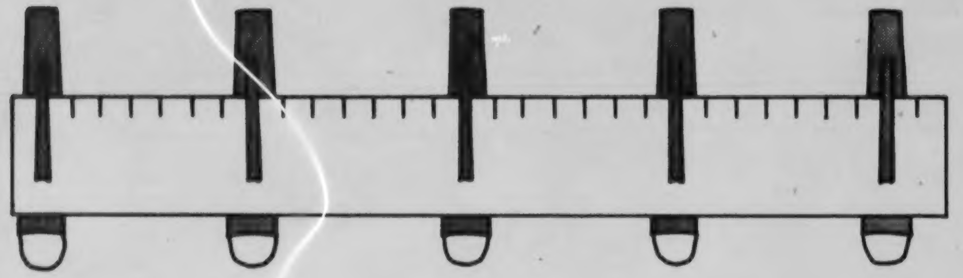
Measurement activities eventually lead to notions of distance. Traditionally one waits for a good deal of maturity before studying properties of figures, such as circles, that are defined in terms of distance. The reason is that Euclidean distance (distance "as the crow flies") requires sophisticated notions of "exact" measurement in terms of the real number system in order to be able to measure every line segment. But in the CSMP curriculum very young children (even in kindergarten) begin thinking about distance in the simplified setting of "taxi-geometry". Taxi-distance (as a taxicab travels) between points is measured in terms of whole numbers by easy counting processes. Experiences in taxi geometry lead to some creative situations, yet do not require special technical skills or vocabulary.

Putting the emphasis on experience rather than on mastery frees us to engage in imaginative activities with children that we ordinarily would not attempt. For example, even when considering such commonplace topics as area and length, we should feel free to devise activities that help students recognize patterns, develop cognitive strategies, and relate the topic to others in the program, while suppressing the urgency to get the "basic" formulas and to master "basic" skills. What is basic here, at this level, is that children are thinking rather than doing tasks on command. The skills and mastery will come eventually, in their own time and place in the curriculum.

NOTE ON GRIDS

A demonstration grid board is needed for all of the taxi-geometry lessons. This should be a large square grid board at least 12 squares by 12 squares (and grid lines spaced 6 to 8 centimeters apart) on which you can draw with colored chalk or with colored pens. A portable, commercially made, permanent grid chalkboard is ideal. However, if such a grid board is not available to you, try one of these alternatives.

- Printed grids on large sheets of paper* are provided with full classroom sets of materials for CSMP Mathematics for the Upper Primary Grades, Part I. Tape one of these sheets to the board, use thick felt-tipped pens for drawings, and discard the sheet after use. Or mount one of the sheets on cardboard and cover it with clear contact paper to provide a protective surface. Use erasable wax crayons for drawings.
- For each lesson, carefully draw a grid on the chalkboard. One can make a tool for drawing such grids efficiently. Use a music staff drawer with chalk in the first, third, and fifth positions, or proceed as follows: clip five pen caps to a ruler at centimeter intervals and secure them with tape. Pen caps are usually of a diameter to hold a piece of chalk securely.



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* These printed grids are packaged under the title "Robot Walk Grid". The title is from lessons in the CSMP Kindergarten Program and has nothing to do with the lessons in this strand.

G1 Introduction to Area

CAPSULE LESSON SUMMARY

Examine many shapes each of which has area ten little squares. Find the area of these shapes in little triangles where the little triangles are exactly half of the little squares. Examine some shapes with area less than ten little squares and more than ten little squares.

MATERIALS

Teacher: Grid board[#]; colored chalk

Student: Colored pencils; Worksheets G1*, **, and ***

DESCRIPTION OF LESSON

Distribute Worksheet G1* and display a grid board.

T: On one side of this worksheet, you are to color two different shapes. Each shape is to have exactly ten little squares in it.

As you say this, hold up a copy of Worksheet G1* and make a sweeping motion with your hand over the grid paper indicating that they are free to use the entire sheet.

While the students work, provide individual help as necessary to correct mistakes. Most mistakes will be in counting and in these cases ask the student to count the number of squares in the shape.

T: How many little squares are there in your shape?

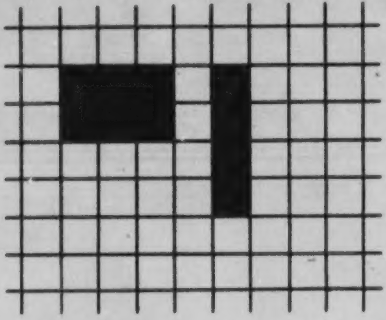
S: Nine (or twelve or some number other than ten).

T: I asked for shapes with exactly ten little squares. Try to fix it so you have (exactly) ten little squares.

[#] See the NOTE ON GRIDS in the introduction to this strand.

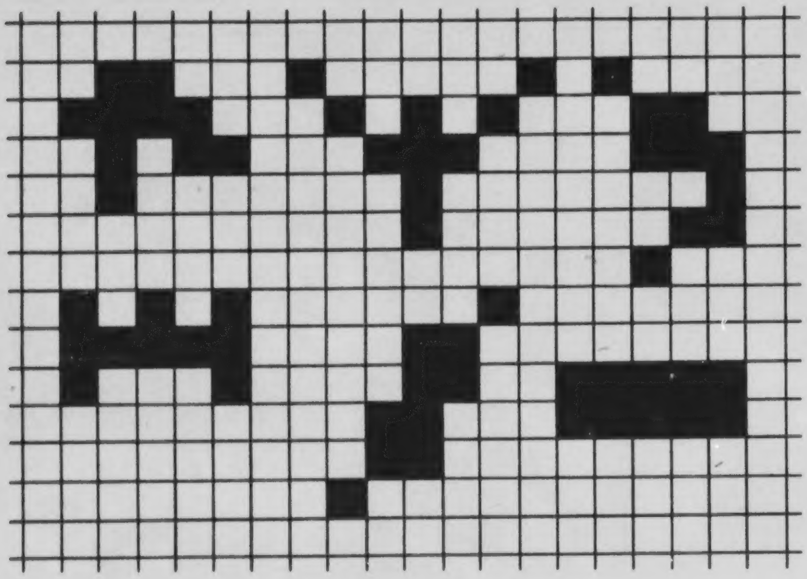
G1

You may find some students have drawn disconnected shapes.



In a case like this, you could point out that although in these two shapes together there are ten little squares, you would like one (connected) shape with ten squares and then a second (connected) shape with ten little squares.

Ask several students who finish quickly and have drawn interesting shapes to copy one of their shapes on the demonstration grid board. The following illustration shows some examples of shapes each of which has area ten little squares.



Let the class react to the shapes on the demonstration grid board.

T: These shapes are really beautiful. Do they all look the same? Do your shapes look like these?

S: No (or some do, some don't).

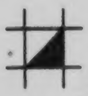
T: But there are ten little squares in each of them. The area of each of these shapes is ten little squares.

Be sure to use the words "little squares". If a unit of area – here the little square – has not been specified, then the expression "the area is 10" has no meaning.

Put this information on the grid board next to each shape.

Area = 10 ■

T: Suppose we cut each little square into two little triangles.



How many little triangles are there in each of your shapes?

S: 20.

T: That's right; can you explain your answer?

S: Each little square has two little triangles and 2 × 10 = 20 (or 10 + 10 = 20).

If this or a similar answer is not forthcoming, suggest that the students draw the cutting lines in one of their shapes and then count the number of little triangles. You may want to do this with one of the shapes on the demonstration grid board.

G1

Next to each shape on the grid board, write the area in little triangles.

Area = 10 ■ or 20 ◢

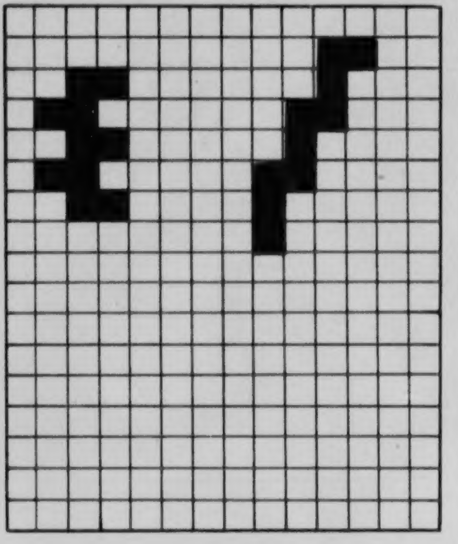
T: On the back of the worksheet, you will again color two shapes. This time one shape should have an area smaller than ten little squares, and the other one should have an area larger than ten little squares.

As you observe your students' drawings, you might suggest that next to each shape they write its area either just in little squares or both in little squares and in little triangles (depending on the abilities of the students).

When students finish these drawings, distribute Worksheets G1** and *** in turn. Be sure the students realize that there are problems on both sides of each worksheet.

Name ______________ G1 *

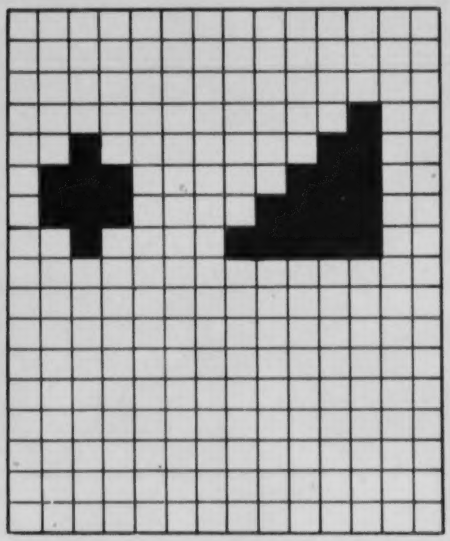
Shapes with area ten little squares



(Many solutions are possible.)

Shape with area smaller than ten little squares

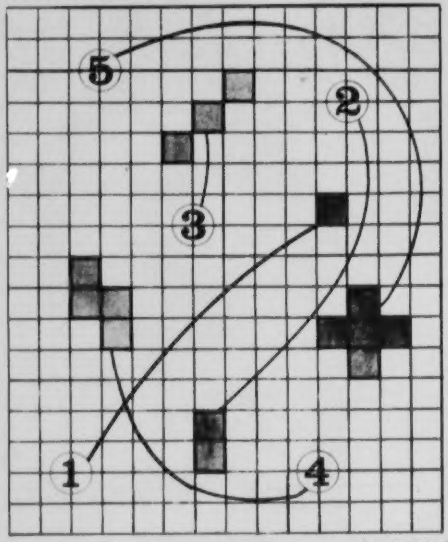
Shape with area larger than ten little squares



(Many solutions are possible.)

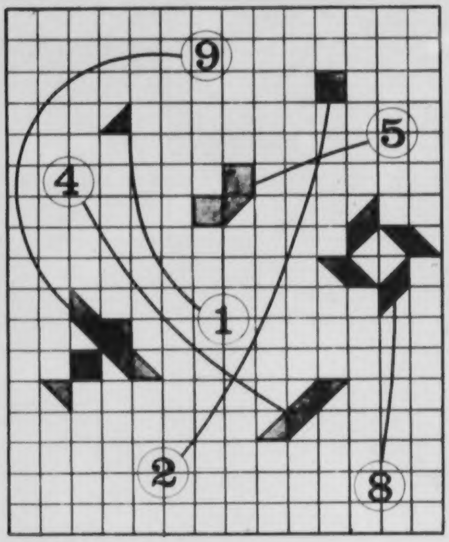
Name ______________ G1 **

Pair the numbers with the shapes.



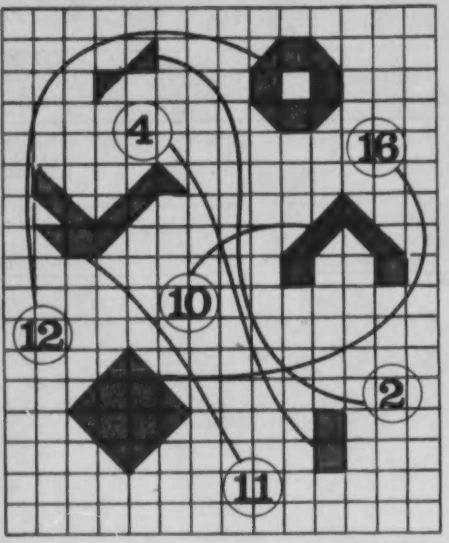
(Turn the page.)

Pair the numbers with the shapes.



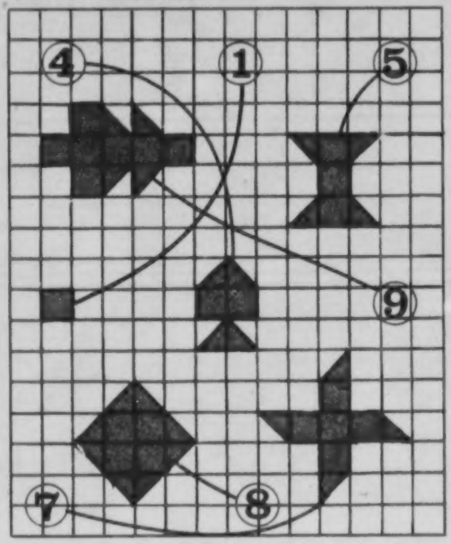
Name ______________ G1 ***

Pair the numbers with the shapes.


4
16
10
12
2
11

(Turn the page.)

Pair the numbers with the shapes.



G2 Taxi-Geometry #1

CAPSULE LESSON SUMMARY

Imagine that the grid is a map of Nora's neighborhood and investigate paths from one point to another on the grid where the paths must follow the lines (streets) of the grid. Discuss the lengths of these paths (taxi measure) and determine which is longest and which is shortest.

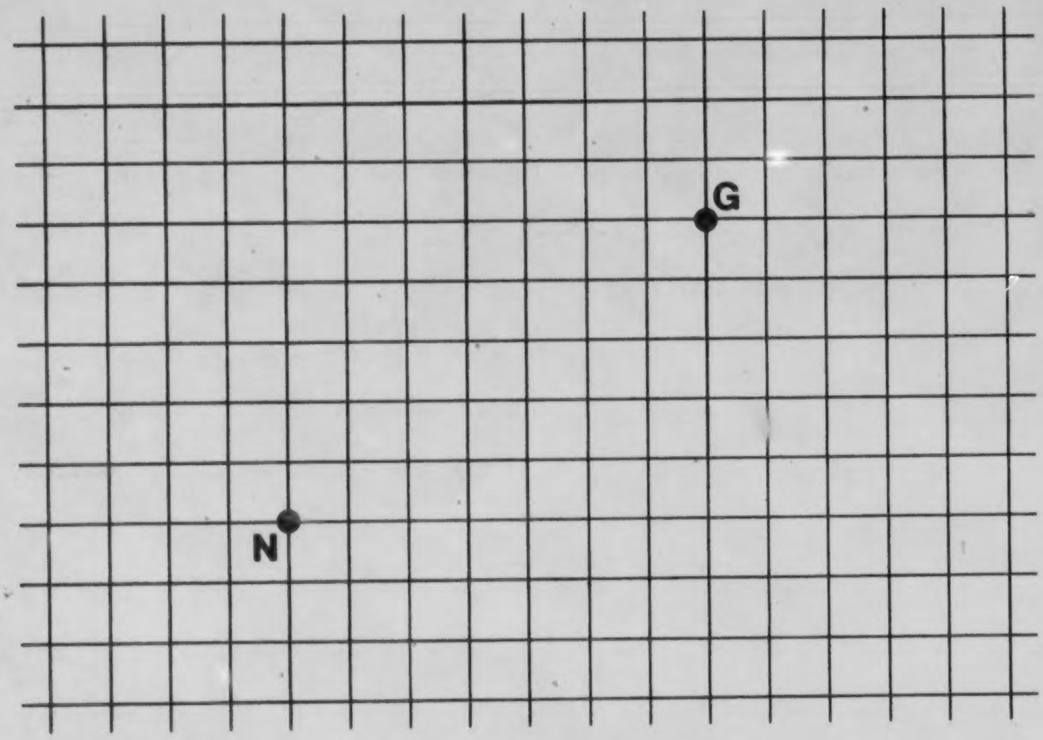
MATERIALS

Teacher: Grid board*; colored chalk

Student: Worksheet G2; colored pencils

DESCRIPTION OF LESSON

Display your grid board. Draw and label two dots as shown below.



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* See the NOTE ON GRIDS in the introduction to this strand.

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G 2

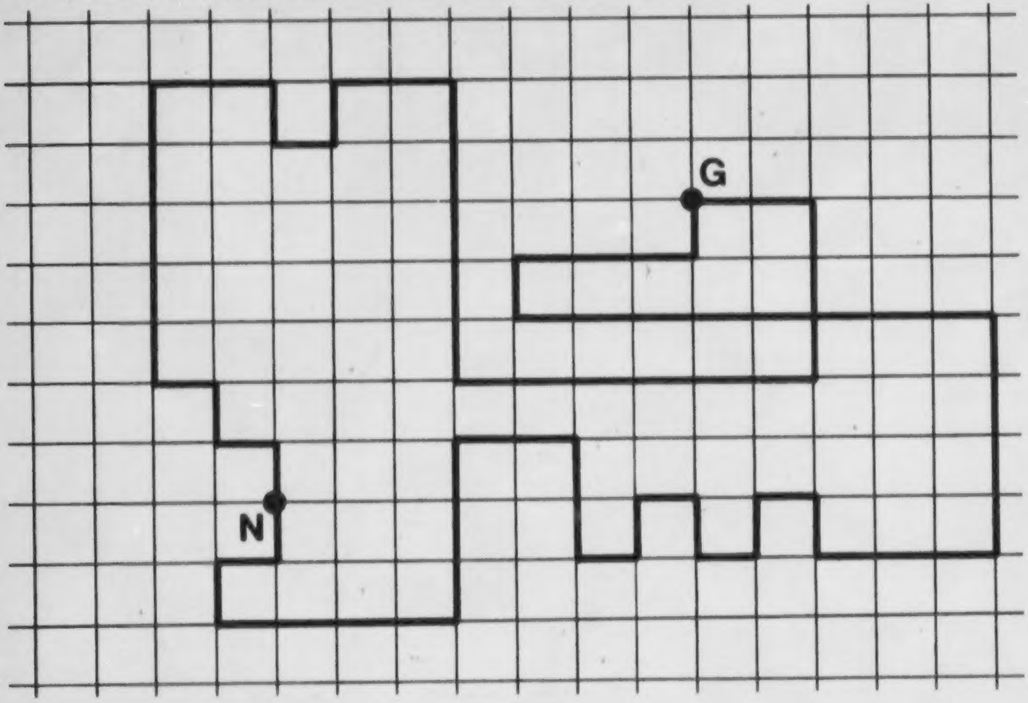
T: This is a map of a city. The lines are for streets. A girl named "Nora" lives in this city. Her house is here (point to "N"). Nora's grandmother lives here (point to "G"). Nora likes to visit her grandmother so she often walks from her house to her grandmother's house.

Nora knows it is not polite to walk across other people's yards, so she always walks on the sidewalks (along the streets). Who can show us a way for Nora to walk from her house (point to "N") to her grandmother's house (point to "G").

Invite a student to show by tracing with a pointer or a finger one path from N to G. Accept any path that follows the lines.

T: Sometimes Nora likes to walk around the city and sightsee on the way to her grandmother's. Can someone show us a long path that Nora could take? Remember, Nora has to follow the streets.

Invite two or three students to show different paths from N to G. Do not draw the paths; just let the students trace the paths with their fingers or a pointer. For your information, two different paths are shown below.



Distribute Worksheet G2 and colored pencils or crayons.

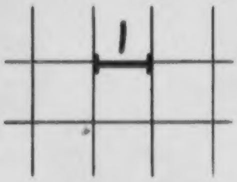
T: On this worksheet you are to draw some long paths from N to G. Use different colors for your different paths. Remember, follow the streets!

As the students work on this problem, give individual help to those who have not understood what is a path from N to G. Allow sufficient time for the students to be free and creative in finding interesting paths. When some students have drawn many paths on their papers, ask several of them to copy one of their paths on the demonstration grid board. Try to get varying lengths of paths and paths going through different parts of the grid on the board. Give students different colors of chalk or crayon (if you have a paper covered grid board) to use.

After a while (perhaps about 15 minutes), call the class' attention to the paths that have been drawn on the demonstration grid board.

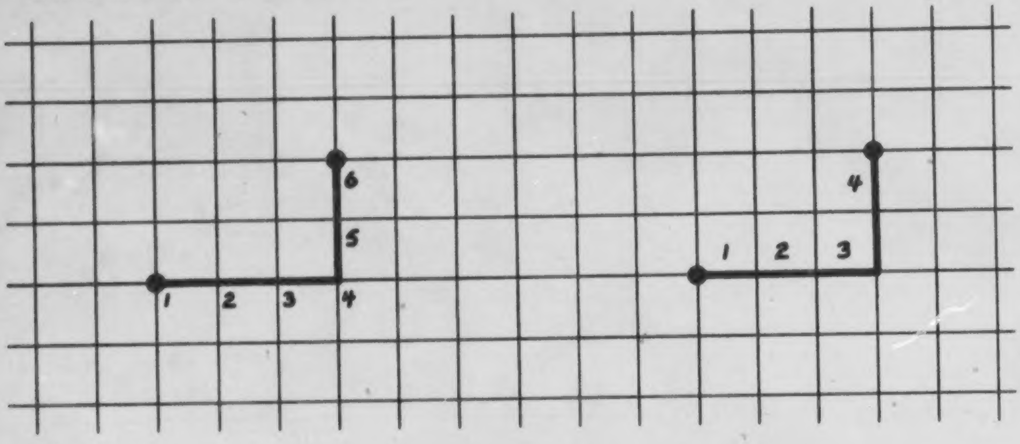
T: Here we have several different paths that Nora could take when she goes to her grandmother's house. Which one do you think is the longest?

Let the students discuss which is the longest path and how to be sure it is the longest. If it is not suggested by a student, suggest yourself that to measure the paths you can count the number of blocks. Indicate one block in an upper corner of your grid board.

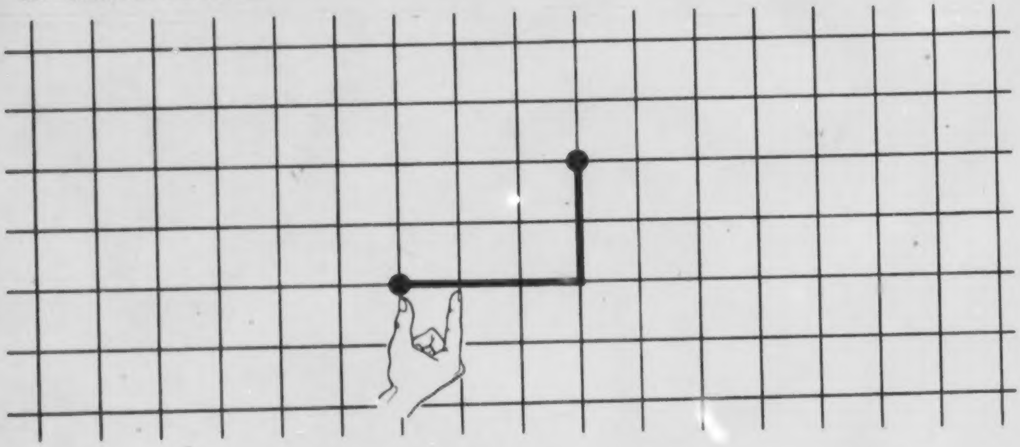


G2

NOTE: Some very common mistakes can occur when students count the blocks in a path, especially at the corner turns. For instance, the next illustration shows two common counting errors.



Both paths in the above illustration are in fact five blocks long. To avoid counting mistakes, it may be helpful to instruct the students to show each block with their thumb and index finger as they count. This technique is illustrated below.

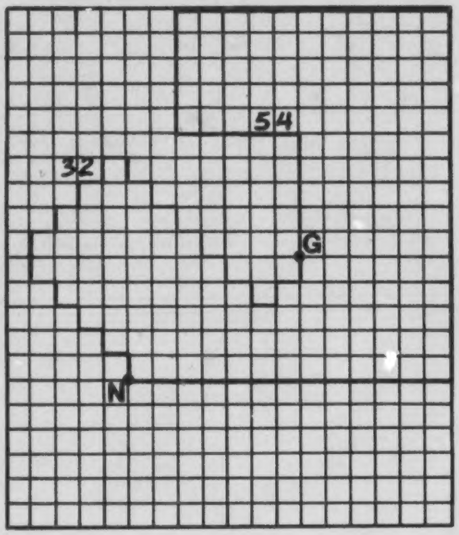


Ask the students to measure the paths on the grid board and to determine which path is the longest and which is the shortest. Pay close attention to each student's counting technique and correct if necessary.

Then ask the students to measure (count the blocks in) the paths that they have drawn. Instruct them to find which of their paths is longest and which is shortest.

Name ____________________ **G2** *

Long paths from N to G.



(Many solutions are possible.)

G3

G3 Booker's Bakery #1

CAPSULE LESSON SUMMARY

Use a story about sharing cakes and cookies to discuss dividing a shape exactly in half. Use the symmetry of a shape to find different ways of dividing it in half. Examine shapes like all the letters of the alphabet to determine which can be easily cut in half.

MATERIALS

Teacher: 25 cm piece of string (optional); colored chalk; meter stick; UPG I-Geometry poster #1; colored pens

Student: Worksheets G3* and **; ruler or straight edge

NOTE: Cut UPG I-Geometry poster #1 into the six indicated pieces so that the letters can be displayed individually in Exercise 2.

DESCRIPTION OF LESSON

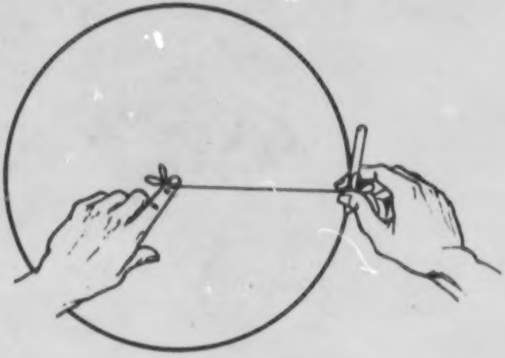
Exercise 1

T: Do any of you know about Booker's Bakery? What kind of things do you get at a bakery?

After a brief discussion, continue:

T: Mr. Booker is very famous in his town, because he is a very good baker and makes delicious cakes and cookies. Next door to the Bakery live twins, Jack and Jill, and they especially like the cakes and cookies that Mr. Booker makes. One day they had saved enough money between them to buy a chocolate fudge cake.

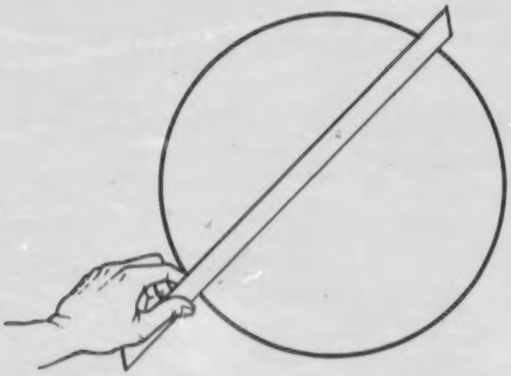
Draw a circle (with radius about 20 cm) very carefully on the board. One way to do this is to use a piece of string about 25 cm long with loops tied at both ends. Put a piece of chalk through one loop and secure it tightly; put an index finger through the other loop and draw the circle as shown in the next illustration.



T: Jack and Jill want to share the cake fairly between the two of them. How can they do this?

S: Cut the cake in two pieces.

T: Like this?



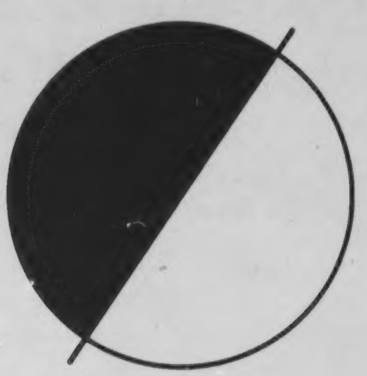
Don't draw a cut line, but demonstrate where it could be with a meter stick.

S: No, in half.

Ask a volunteer to show how to cut the cake. If it is a fairly accurate job and the class is in agreement, draw the cut line.

G3

T: Yes, I think that this is a fair cut. Both pieces look the same; let's agree that they are exactly the same size.



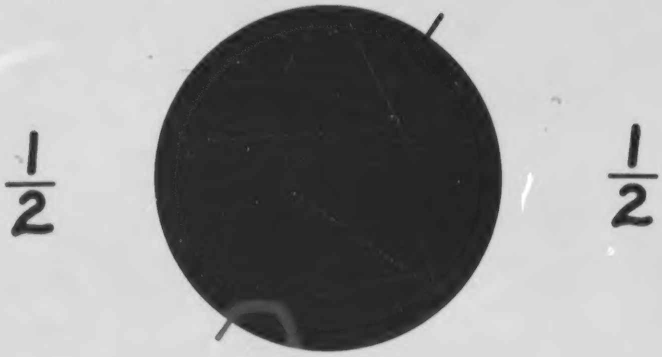
T: This is Jack's share (shaded blue). How much of the cake does he get?

S: One-half.

T: I'll shade Jill's part in red. How much of the cake does she get?

S: One-half.

Write this information on the board.



T: When both Jack and Jill have eaten their halves of the cake, what is left?

S: Nothing; the cake is all eaten.

Write this equation on the board.

$$\frac{1}{2} + \frac{1}{2} = 1$$

T: One-half of the cake plus one-half of the cake equals the whole cake.

Exercise 2

T: On their birthday Jack and Jill went to visit Mr. Booker at the bakery. Mr. Booker had a nice surprise for them; he had baked 26 special cookies just for the twins. Do you have any idea what shapes these 26 cookies were?

Let students make suggestions. Perhaps a student will think of the letters of the alphabet; otherwise, you should tell them that the cookies were shaped liked the 26 letters of our alphabet.

T: Jack and Jill are now faced with a problem; they have to share the cookies fairly between them. How should they do this?

S: Each of them should get 13 cookies.

T: Why 13?

S: One-half of 26 is 13.

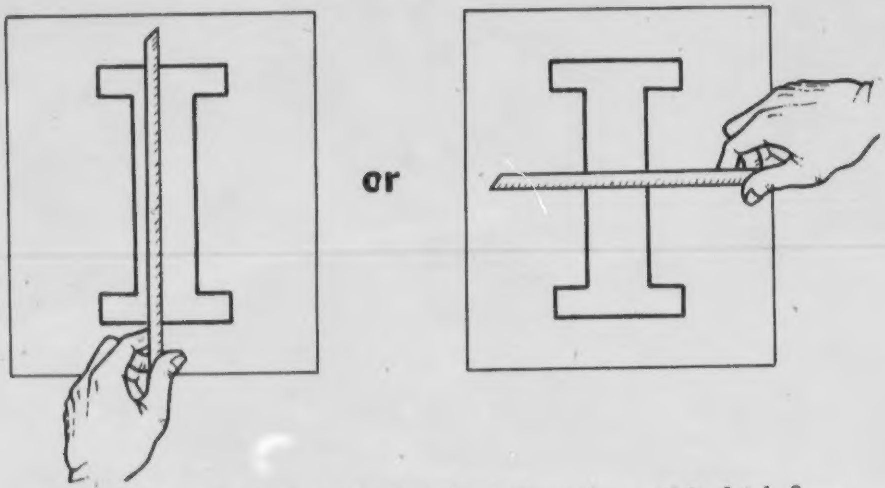
S: 13 + 13 = 26.

T: Jack thought about which 13 cookies he wanted, and Jill thought about which 13 cookies she wanted. It was a hard decision because some of the cookies looked bigger than others. Can you suggest another way for them to share the cookies that would be fair to both Jack and Jill?

G3

S: Cut each cookie in half.

If necessary, suggest this idea yourself. Then tape the letter "I" from UPG I-Geometry poster #1 on the board and discuss how to cut this cookie. Call on a volunteer to demonstrate the cutting. Let the student use a ruler or other straight edge held on edge against the letter to show where it could be cut.



T: Would this cut allow Jack and Jill to share the cookie fairly?

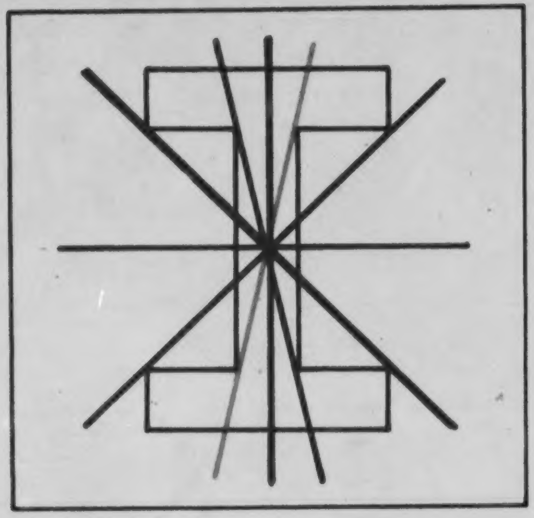
S: Yes.

T: How can you tell?

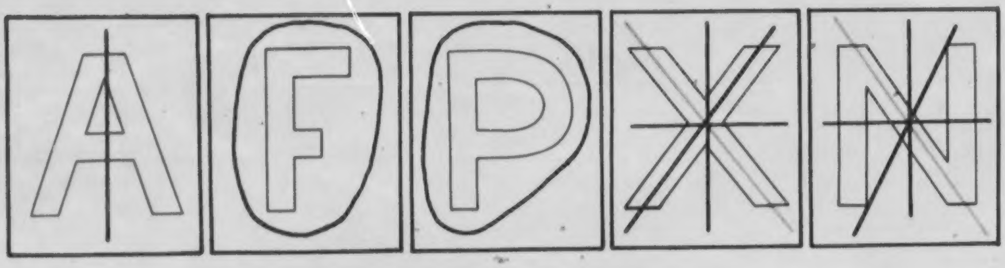
Stress that the cut should be so that both pieces look the same. If the class and you agree that the cut is "fair", draw a colored line to show the cut.

T: Does someone see another way we could cut this cookie?

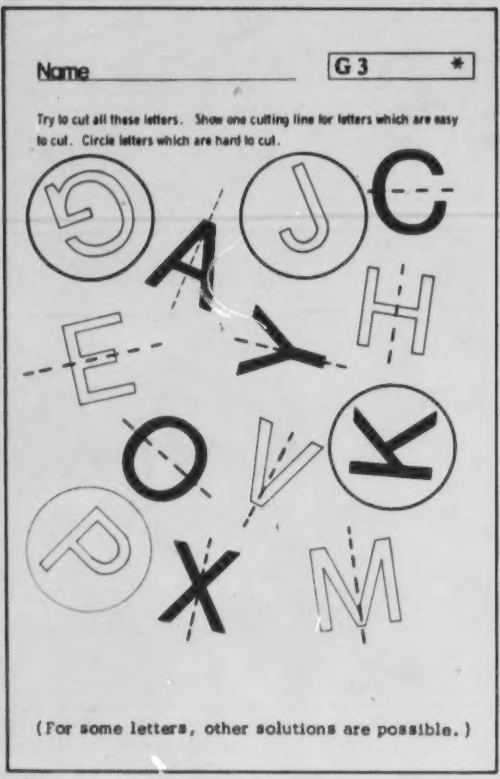
You will likely get many other suggestions, some good and some bad. If, however, you do not, do not insist on finding more than one way. For your information, several possible cutting lines are shown below.



Repeat this cutting activity with several other letters, for example: A, N, P, X, and F. Do at least one letter that has only one cutting line (for example, "A") at least one that has no straight cutting line. For your information, some cutting lines for the letters mentioned above are illustrated below. "F" and "P" are circled to indicate they are hard to cut; that is, there is no straight symmetric cutting line.



Worksheets G3* and ** are available for the rest of the lesson. Explain the instructions to the class and be sure the students understand that if they cannot find a cutting line, then they should circle the letter.



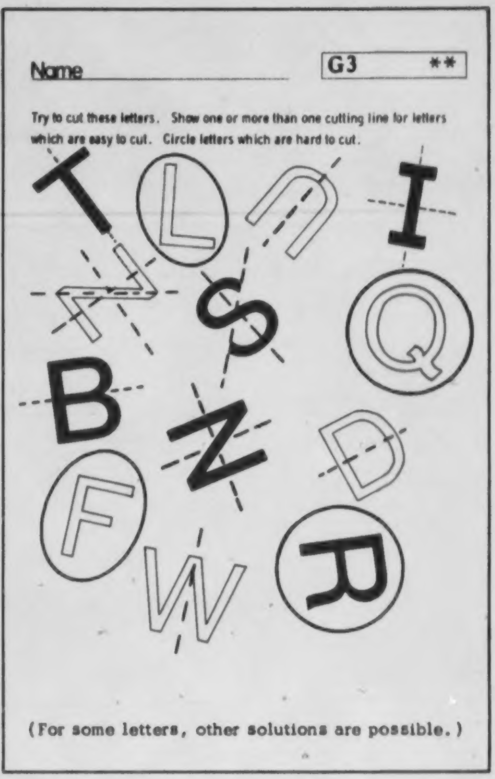
Name

G 3 *

Try to cut all these letters. Show one cutting line for letters which are easy to cut. Circle letters which are hard to cut.

G
J
C
A
H
E
Y
O
V
K
P
X
M

(For some letters, other solutions are possible.)



Name

G3 **

Try to cut these letters. Show one or more than one cutting line for letters which are easy to cut. Circle letters which are hard to cut.

T
L
U
I
Z
S
Q
B
N
D
F
W
R

(For some letters, other solutions are possible.)

G4 Measuring with Shoes and Pencils

CAPSULE LESSON SUMMARY

Measure the length of the classroom with shoes and discuss how the measurements will differ depending on whose shoes are used. Estimate the same length measured with other objects such as a piece of chalk or a pointer. Measure other objects in the room with erasers or smaller things. Measure the length of a desk with a pencil.

MATERIALS

Teacher: None
Student: Pencil; paper

DESCRIPTION OF LESSON

Exercise 1

T: Today, let's measure the length of the wall.[*]

S: With what?

T: That's a good question. What could we measure the wall with? What could we use to find out how long it is?

Accept any plausible suggestions. Allow, if you wish, students to perform the measurements with some of the objects they suggest using. For example,

S: The wall is between nine and ten (meter) sticks long.

S: The wall is between thirty and thirty-one rulers long.

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[*] Choose a wall that you can walk along from corner to corner. Try also to arrange that all of the students are able to see your feet as you step off the distance. If necessary, you could choose to measure the length of a center aisle between two prearranged marks on the floor.

After a short while, suggest:

T: I would like to measure the wall with my shoes. How can I do that?

S: Walk along the wall.

Be sure that someone points out that you should put one foot directly in front of the other as you walk. Measure the wall with your shoes so that the students can observe what you are doing. Ask the class to count with you each step that you take. Announce the result; for example,

T: 35 is too short and 36 is too long. The wall is somewhere between 35 and 36 shoes long.

Write this information on the board.

LENGTH OF WALL: between 35 shoes and 36 shoes

Choose a student whose shoes are clearly shorter than yours to measure the wall with his or her shoes. Ask the class to first predict whether the number of shoes in this measurement will be more or less or the same as yours. Then let the student do the measuring and record the result on the board; for example,

LENGTH OF WALL: between 35 shoes and 36 shoes (teacher's)
between 44 shoes and 45 shoes (Jason's)

T: Why didn't Jason get the same number of shoes as I did?

S: Because his shoes are smaller than yours (or your shoes are bigger than his).

Repeat this measuring activity with a student whose shoes are clearly longer than the first students (and longer than yours, if possible).

Hold up a piece of chalk or an eraser or something obviously shorter than anyone's shoes.

T: If you were to measure the wall with this piece of chalk (or this eraser), would you get a larger number or smaller number?

S: Larger, because the piece of chalk is shorter than a shoe.

T: Suppose I had a stick, and I measured the wall with it and found that the wall was between 12 and 13 sticks long. What could you say about the stick? Is it longer than my shoes or shorter?

S: Longer, because it took fewer sticks to measure the wall.

Find some other things in the room to measure and to measure with. For example, measure the chalkboard with an eraser and with a piece of chalk and with a book. Continue to ask questions as suggested in the following dialogue.

T: The chalkboard is between 19 and 20 erasers long. Suppose we measure it with this piece of chalk (hold up a new piece of chalk). Would it be more or fewer pieces of chalk long than the number of erasers long?

S: More, because the piece of chalk is shorter than the eraser.

T: How many pieces of chalk long do you think the measurement will be?

Record several guesses and then do the measuring. Compare the results with the guesses.

T: Suppose we measure the chalkboard with this book (hold up a book at least twice as long as your eraser). Would it be more or fewer books long than erasers?

S: Fewer.

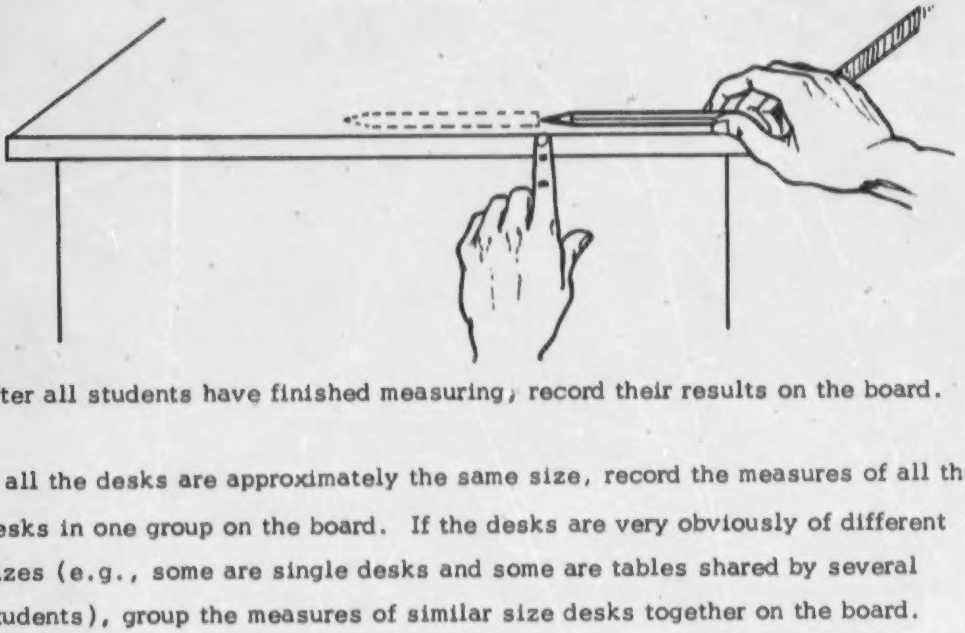
Let the students guess how many books long the chalkboard would be and then do the measuring.

Exercise 2

Ask every student to hold up their pencil. If almost all of the pencils are about the same length, give some very short pencils to some of the students.

T: I want each of you to measure the length of your desk with your pencil.

Pick a desk in a central spot which everybody can see and demonstrate how to measure the length of that desk with your pencil. Use your finger to mark the end point of the pencil and count each time you set the pencil down.



After all students have finished measuring, record their results on the board.

If all the desks are approximately the same size, record the measures of all the desks in one group on the board. If the desks are very obviously of different sizes (e.g., some are single desks and some are tables shared by several students), group the measures of similar size desks together on the board. Point to one of the groups of measures.

T: Why are there so many different numbers?

S: Our pencils are different lengths.

Point to the largest number in this group.

T: Whose measure is this? (Gary) Why did Gary get a larger number when he measured the length of his desk?

S: He must have a short pencil.

Ask Gary to hold up his pencil for the class to see. Point to the smallest number in the same group of measures.

T: Is the pencil that was used to get this length shorter or longer than Gary's? (Longer) How do you know?

Students may have difficulty verbalizing an explanation. Do not insist that their explanations be precise.

T: Whose measure is this? (Kristy)

Ask Kristy to hold up her pencil for the class to see.

If some students believe that the desks are different lengths, ask Kristy to measure Gary's desk with her pencil and compare Kristy's measure to Gary's. Choose two numbers in the same group that are about the same. Point to these numbers.

T: What can we say about the pencils used to get these two measures?

S: The pencils must be about the same length.

Ask the students who had these measures to compare their pencils for the class.

G5 Area with Tangram Triangles and Squares

CAPSULE LESSON SUMMARY

Determine which of two shapes is bigger by comparing their respective areas in Tangram triangles or in Tangram squares. Find the areas of some shapes in both Tangram triangles and Tangram squares.

MATERIALS

Teacher:	Worksheet G5*
Student:	Worksheets G5*, **, and ***; Tangram triangles and squares

NOTE: Prepare for each student a set of ten Tangram triangles and ten Tangram squares. These triangles and squares can be punched out from the Tangram cards (right side) either by the students or by you and stored in envelopes.

DESCRIPTION OF LESSON

Distribute a set of Tangram triangles and Tangram squares to each student. Give the students a few minutes to make their own designs and play with these shapes before proceeding.

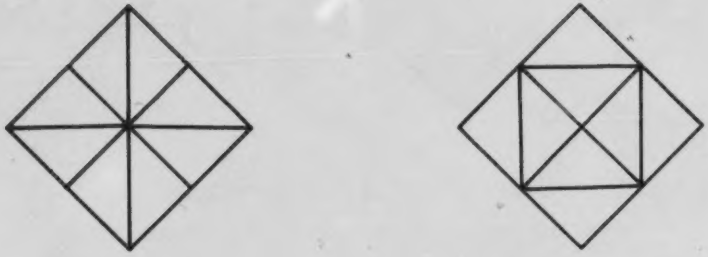
Distribute individual copies of Worksheet G5*. Hold up your copy of the worksheet so that the class knows which side you will look at first.

T: Look at the shapes on this page. Can you guess which shape is bigger?

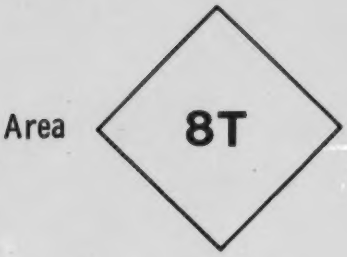
Let students comment on this situation. They may have different ideas of what it means to be bigger. Eventually direct their attention to the area of the figures. Hold up the worksheet and point to the large square.

T: Look at this shape at the top of the page. How many of these little (Tangram) triangles do you think it will take to cover this shape?

Accept several guesses from the students and then ask them to find exactly how many little triangles it takes to cover the shape. Circulate and give help as needed. The next illustration shows two ways to cover the big square with small Tangram triangles.



When students find that the correct number is eight, instruct them to write "8T" on the shape. To illustrate, sketch the shape on the board and write "8T" on it. Write "Area" on the board.



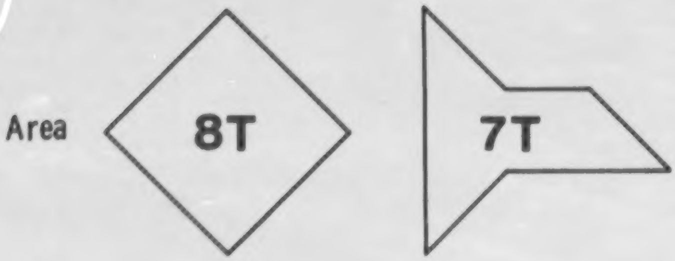
T: The area of this shape is eight little triangles.

Now, look at the shape at the bottom of the page. How many little triangles do you think it will take to cover this shape?

Again, accept some guesses from the students and then ask them to find exactly how many little triangles it takes to cover the shape. When they have found that the correct number is seven, instruct them to write "7T" on the shape.

G5

Sketch the shape on the board and write "7T" on it.



T: The area of this shape is seven little triangles. Which of these two shapes has a bigger area?

S: The first one (or the square or the diamond).

Direct the students to put aside the small Tangram triangles and to take out the Tangram squares. Hold up the worksheet so that the class knows that now you will look at the other side.

T: Turn over your worksheet. This time we will use our little squares to find the areas of these shapes. Which of these shapes do you think is bigger?

Let the class comment on this question without measuring first.

T: How many little squares do you think it will take to cover this shape? (Point to the shape on the left.) How many little squares do you think it will take to cover this shape? (Point to the shape on the right.)

Let the students make some guesses and then let them cover each shape with little squares. When they find that both shapes have area six little squares, instruct them to write "6S" on both shapes.

Your students may be surprised that the areas are the same while the shapes look very different. Let them comment on this situation. They may suggest or you may want to suggest that one shape is "long and narrow" and the other shape is "short and wide".

T: How many little triangles make one little square?

S: Two.

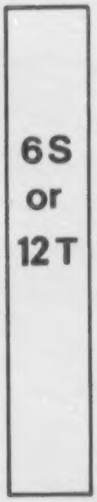
T: Suppose we want to cover this shape (point to the shape on the left) with little triangles. How many little triangles will it take?

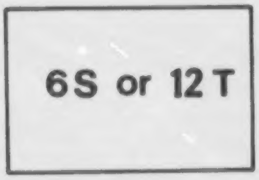
S: 12.

T: How do you know that?

S: Because 2 x 6 = 12 (or 2 + 2 + 2 + 2 + 2 + 2 = 12).

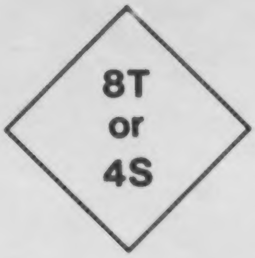
If your students are not certain about this answer, ask them to cover the shape with little triangles and count how many it takes. Repeat the same questions about the other rectangle and then record the information on the board.





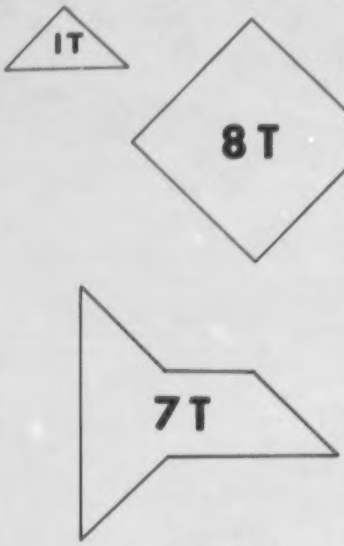
G5

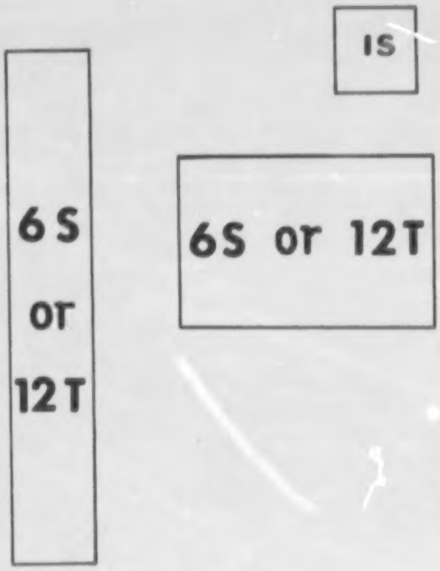
Ask similar questions about the large square on side 1 of the worksheet.



Worksheet G5** and *** are available for individual work. Explain to the students that on side 1 of the ** worksheet they are to find the area of each shape with little squares. Instruct them to write the area on each shape. On side 2 they are to find the area of each shape with little triangles and to write the area on each shape. Students who correctly complete Worksheet G5** should continue with Worksheet G5***. On this worksheet they are to measure the area of each shape with both little squares and little triangles. Be sure that when they write the areas they indicate the units (S or T).

Name ______________ G5 *

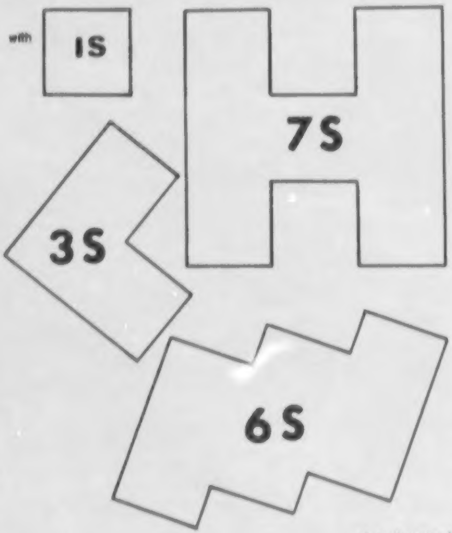



1S
6S
or
12T
6S or 12T

Name ______________ G5 **

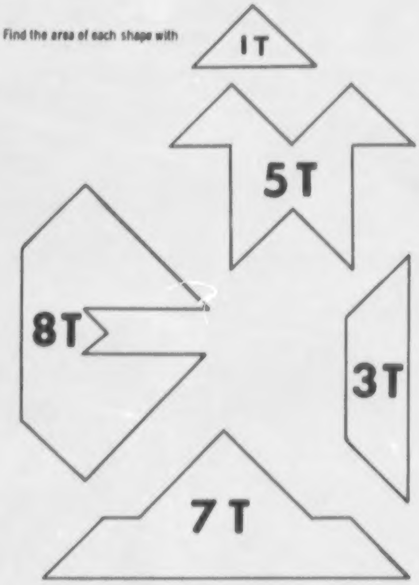
Find the area of each shape

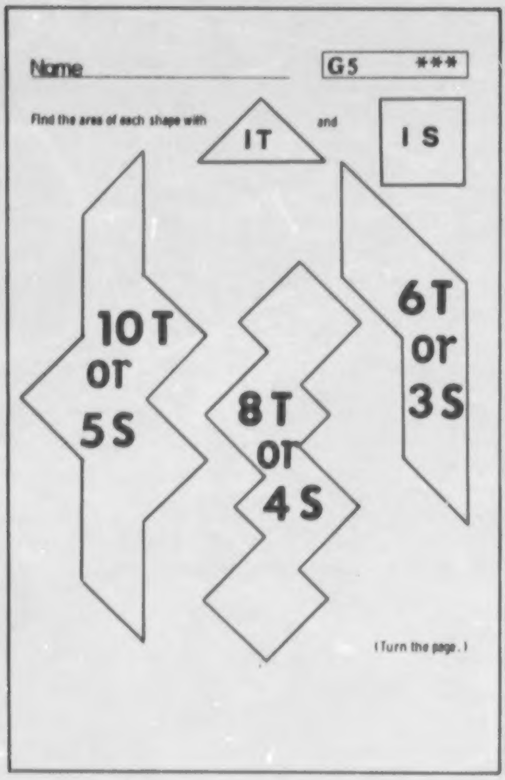
with



(Turn the page.)

Find the area of each shape with

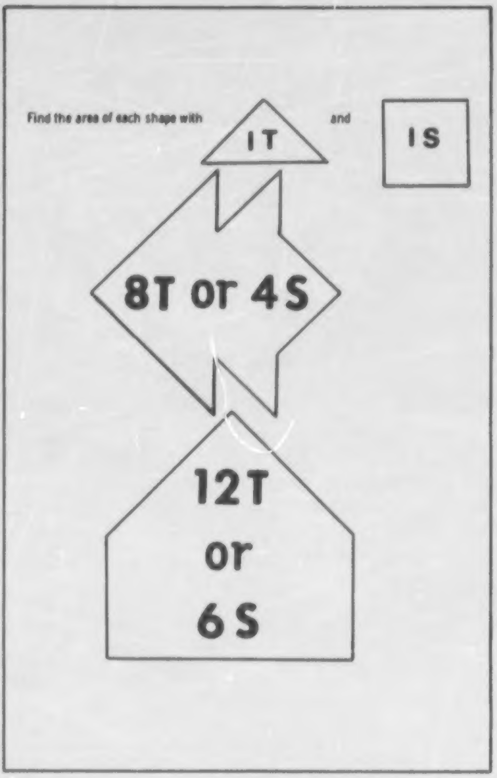

1T
5T
8T
3T
7T


Name ____________________ G5 ***

Find the area of each shape with 1T and 1S

1T
1S
10T or 5S
8T or 4S
6T or 3S

(Turn the page.)


Find the area of each shape with 1T and 1S

1T
1S
8T or 4S
12T or 6S

G6 Taxi-Geometry #2

CAPSULE LESSON SUMMARY

Review the idea of a path which follows the lines on a grid and find the lengths of some paths. Look for shortest paths between two points. Observe the lengths of many paths between two points and look for a pattern.

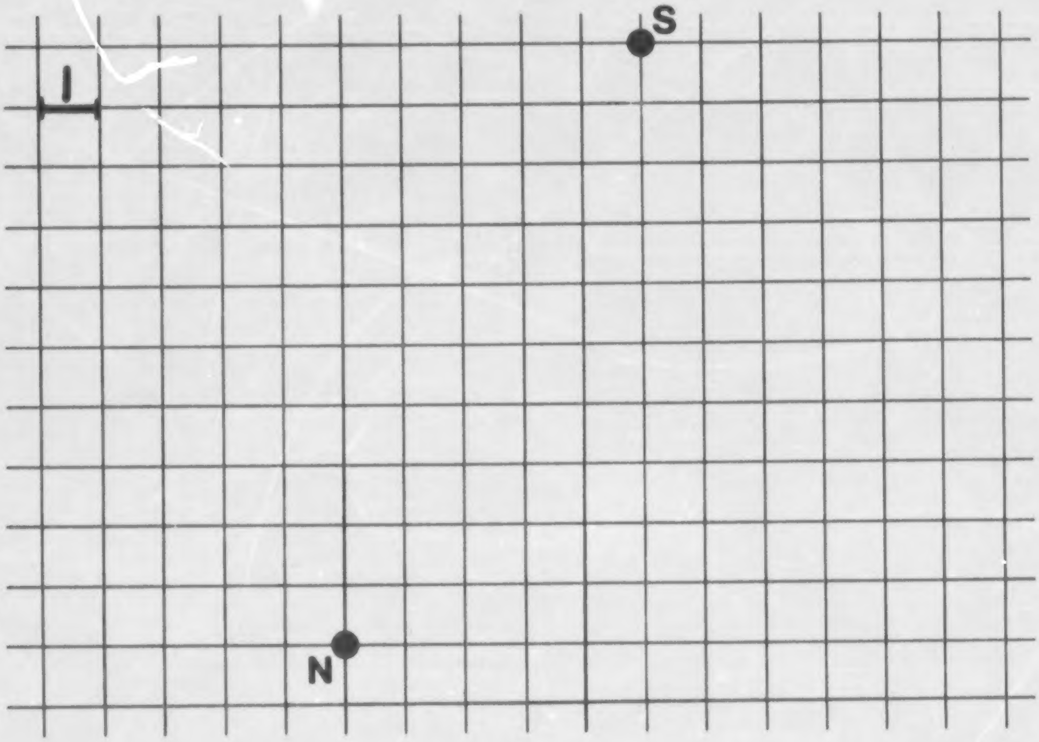
MATERIALS

Teacher: Grid board[#]; colored chalk

Student: Worksheets G6* and **; colored pencils

DESCRIPTION OF LESSON

Display your grid board. Draw and label two dots as in the next illustration. In an upper corner of the board, indicate a one block measure.



[#] See the NOTE ON GRIDS in the introduction to this strand.

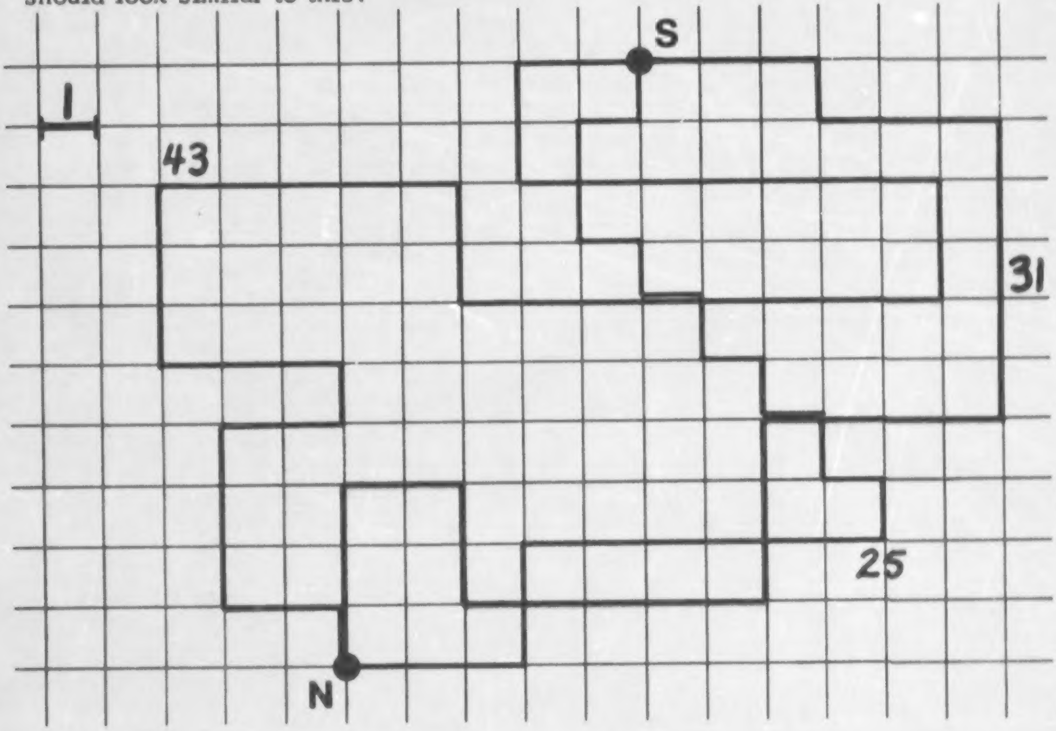
G 6

T: Do you remember our friend Nora? This is a map of the city where Nora lives and here is Nora's house (point to "N"). Today we are going to find paths that Nora could take to school. Here is Nora's school (point to "S").

Some days Nora likes to make a very long walk going to school. Who can show us one way for Nora to go? Remember, Nora follows the streets.

Let two or three volunteers trace with their fingers or a pointer paths from N to S. Accept any path which follows the lines. Then you draw the suggested paths on the grid board using different colors of chalk or crayon (if you have a paper covered grid board).

Ask the students which of the paths is longest. They should suggest counting the blocks in each path; otherwise, you should make the suggestion. Point to "⊢1⊣" to remind the class what is one block. Invite students to count the blocks in each path, and watch that they count correctly. Then label each path as illustrated below. You will have different paths, but your grid board should look similar to this.



T: Some days Nora is in a hurry and wants to take a short way to school. Would someone like to show us a way for Nora to walk to school that is shorter than any of these?

Call on a student to trace a path. You draw it in a new color, and then ask the student to count the number of blocks in his or her path. It is likely that the student will have chosen a 15 block path. Whether or not this is so, ask if someone can find a shorter way. Allow students to make two or three more attempts at finding a shorter path.

It may happen that someone in the class will argue that there is no path shorter than 15 blocks. If the class seems to agree, you should stop trying to find shorter paths. However, the class may not be convinced that a 15 block path is shortest, in which case you should stop this activity after a few attempts and not push for realization.

Distribute Worksheet G6* and colored pencils.

T: On this worksheet you are to draw several paths from Nora's house to school. Try to draw some long paths and some very short paths. Each time you draw a path, count the number of blocks and label the path like we did on the board here.

While the students are working, observe individual work. You can easily catch some counting errors because all paths from N to S should have an odd number of blocks.

After about 10 minutes, make a list on the chalkboard of the lengths of some of the students' paths. Your list should suggest that 15 is the shortest length path and that all paths have odd length, but you should not require the students to make this observation.

Clear your grid board except for the dot labeled "N".

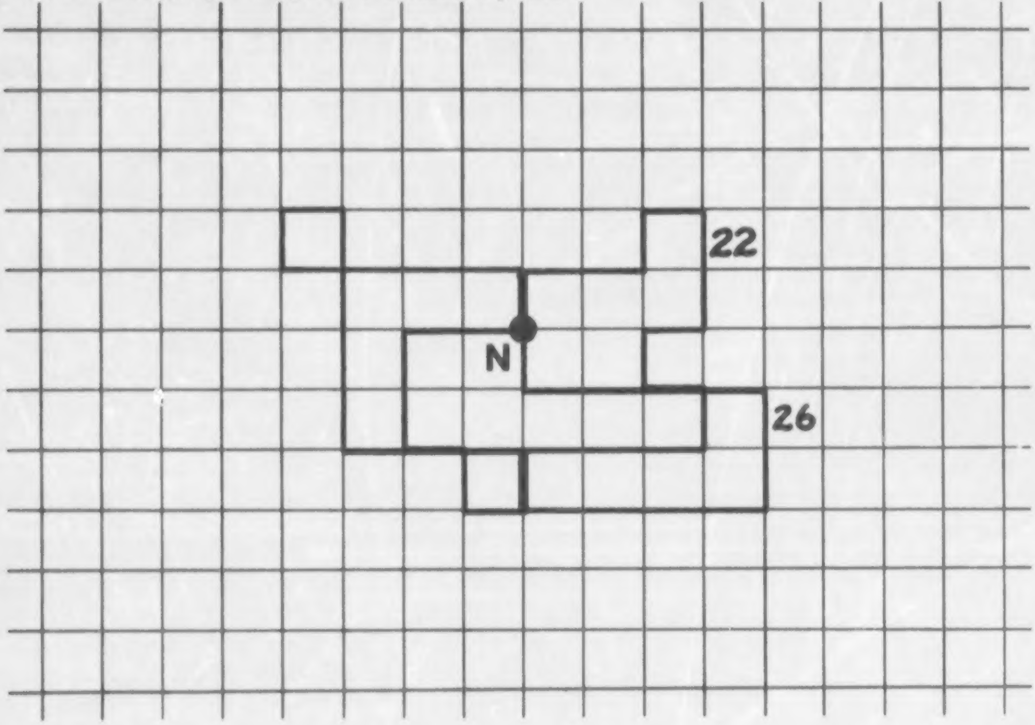
G6

T: How many of you went trick-or-treating on Halloween night?

Let students comment for a minute or two about trick-or-treating.

T: Let's pretend that Nora goes trick-or-treating. She starts at her house and walks up and down the streets, stopping at houses along the way. (Trace a possible path as you say this.) Then, finally she returns to her house. Who can show us another path that Nora could take trick-or-treating?

Let the students show two different paths starting and ending at N. For each path, count the number of blocks and label it. Your board should look similar to this though you may have different paths.



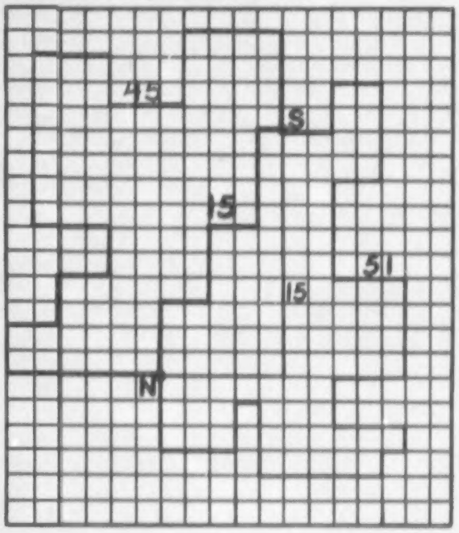
Direct the students to look at Worksheet G6**.

T: On this worksheet you are to draw several different paths that Nora could take trick-or-treating. Each time, count the number of blocks and label the path like we did here.

Again you should observe individual work. Some counting errors will be easy to catch because this time all paths will have even length. After about 10 minutes, make another list on the chalkboard of the lengths of the trick-or-treating paths. This time your list should suggest that all such paths have even length. Your students may observe this fact; otherwise, you could ask if they see anything interesting about these numbers and perhaps lead them to the observation that all of Nora's trick-or-treating paths have even length.

Name________________ G 6 *

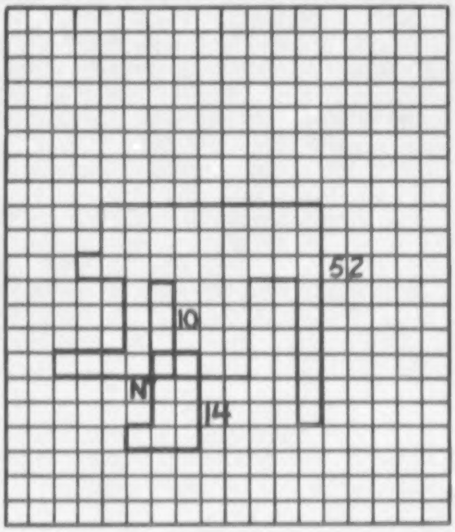
Long and short paths from N to S.


45
S
15
51
15
N

(Many solutions are possible.)

Name________________ G 6 **

Long and short trick-or-treating paths from N to N.



(Many solutions are possible.)

G7 Millions of Dots / Tangram Puzzles

CAPSULE LESSON SUMMARY

Partially fill the chalkboard with uniform size dots and estimate how many of this size dot it would take to completely fill the board. Discuss the question of filling the board with smaller and smaller dots. Individually work on the Tangram booklet to make certain designs with all the Tangram pieces.

MATERIALS

Teacher: None

Student: Tangram; Tangram booklet; pencil

NOTE: For this and subsequent lessons involving the Tangram, each student will need a set of Tangram pieces. The set consists of seven pieces (two large triangles, two small triangles, one medium size triangle, one square, and one parallelogram) and is found on the left side of the Tangram card. Punch out a set for each student and keep each individual Tangram set in an envelope. The frame around the Tangram must be saved for use in Lesson G12 Higgledy-Piggledy Dots.

DESCRIPTION OF LESSON

Exercise 1: Millions of Dots

T: I would like to see if you can guess how long one minute is. When I say "go", you must keep perfectly quiet for what you think is one minute. When you think one minute has passed, raise your hand but do not say anything. Ready? Go!

It is likely students will begin to raise their hands before one minute has passed. After one minute has passed, say:

T: Stop! One minute has passed. Perhaps one minute is longer than you thought. Can someone describe in words how long one minute is?

G 7

Let students respond; reinforce mention of seconds. If necessary make this suggestion yourself.

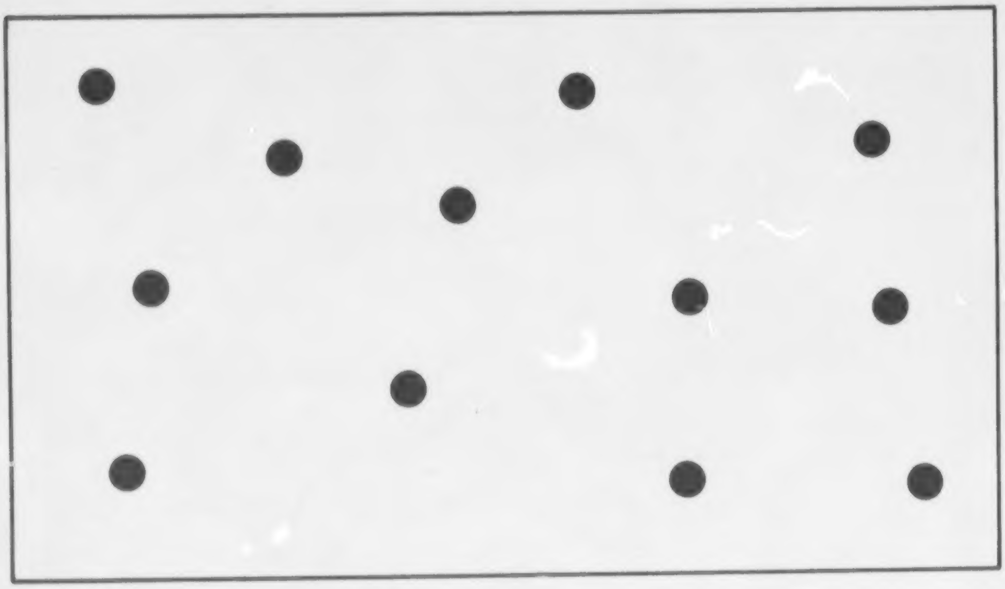
T: One minute is about the time you need to count slowly from 1 to 60, or 60 seconds.

Count a few seconds to give the class the idea of how slow to count.

T: How long is half a minute?

S: It's about the time you need to count slowly from 1 to 30 (or 30 seconds).

Ask your students to count together from 1 to 30 while you draw some rather big dots on the board. When they reach 30, stop drawing. Spread out the dots on the board.



T: How many dots did I draw? I would like to fill the board with dots about this size, and I am going to ask for your help.

Send two students to the board. Tell the students they are allowed to draw dots during half a minute or 30 seconds. Remind them to draw dots about the same size as your dots. You and the rest of the class will count from 1 to 30 and then say "STOP".

T: How many dots are there on the board now?

Repeat this activity of having students draw dots on the board during 30 seconds several times (four or five times, or until the board is quite full of dots). Do not count the number of dots after the first time.

T: How many dots this size do you think we would need to completely cover the whole board?

S: 1,000.

Accept any large number.

T: What if we used smaller dots? (Illustrate a dot about half the diameter of those already drawn on the board.)

S: We would need to draw more dots.

T: What if we used even smaller ones? (Illustrate a smaller dot.)

S: Even more dots.

T: Even smaller ones? (Illustrate a very tiny dot, hardly visible.)

S: Many more dots.

T: Now, close your eyes. (Speak very softly as if you were telling a fairy tale.) Try to think of the smallest dot you can imagine. Try to imagine how many of those there would be on our board.

Let the students comment on this suggestion.

G7

Exercise 2: Tangram Puzzles

Distribute individual Tangram sets and the Tangram booklets.

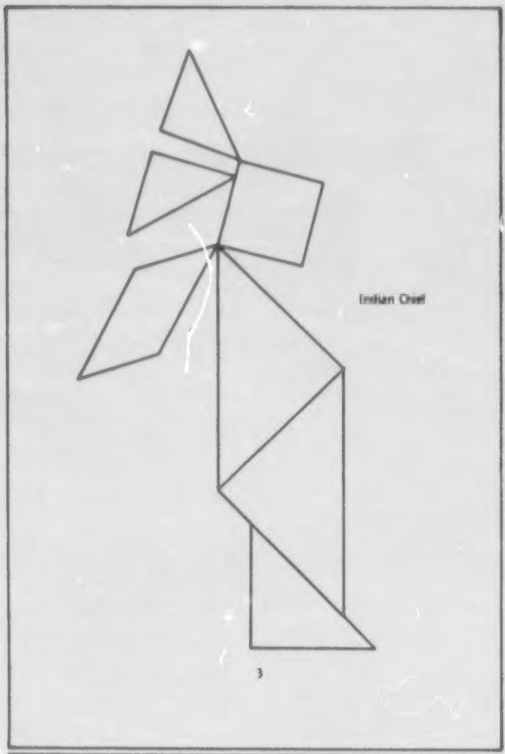
Veteran students should be familiar with the Tangram from the CSMP First Grade Program. For the benefit of students unfamiliar with the Tangram and as a review for the others, spend a minute or two discussing it. For example, ask how many Tangram pieces there are, what shapes are the pieces, and what are some other shapes you can make by putting two or three Tangram pieces together.

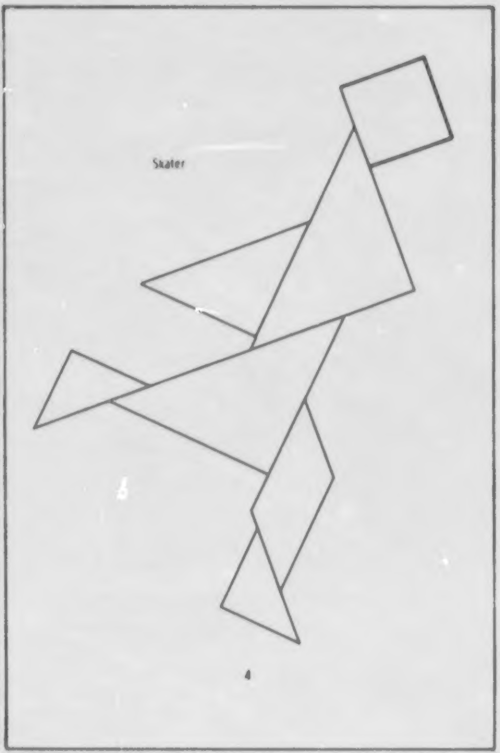
Tell the students that in the Tangram booklet there are many different shapes they can make with all the Tangram pieces. On each page, they must try to cover the shaded shape with the Tangram pieces. When they have done it, raise their hands and you will check it. Each time you check a page, indicate that the student has successfully made the shape with Tangram pieces by drawing a smiling face or star on the page.

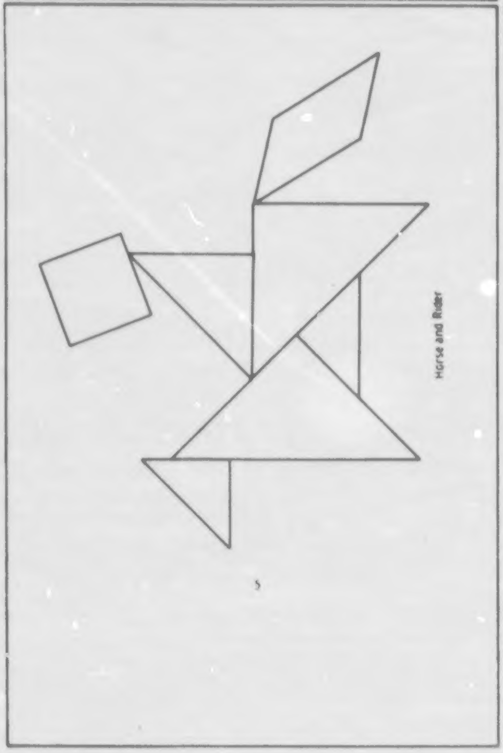
If students become frustrated with some pages, instruct them to go on to other pages and come back to those pages later. If some students think Tangram pieces are missing from their sets, suggest they check by putting all their pieces on the cover square. Students who do not want to wait for you to check their pages can be encouraged to draw dividing lines (as on the cover) to show how the pieces are put together to make a shape.

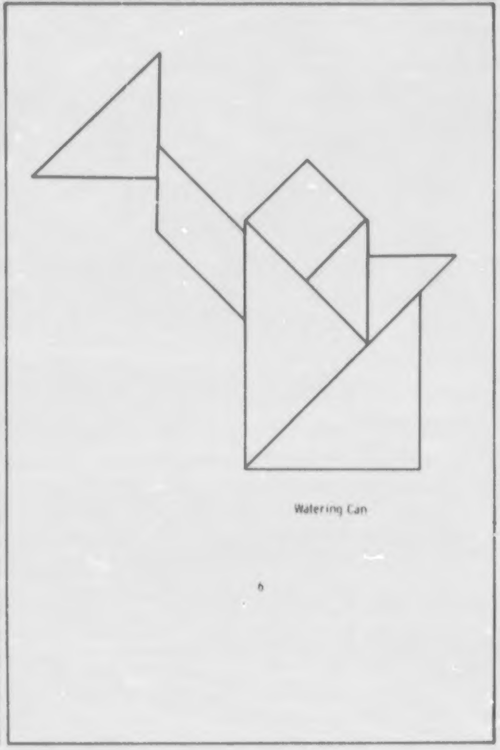
You should allow this activity to continue about 20 to 30 minutes (not more) or until the end of your mathematics class period. Collect the Tangram booklets and the envelopes of Tangram pieces. Also collect the Tangram frames. These materials will be used again for other lessons.

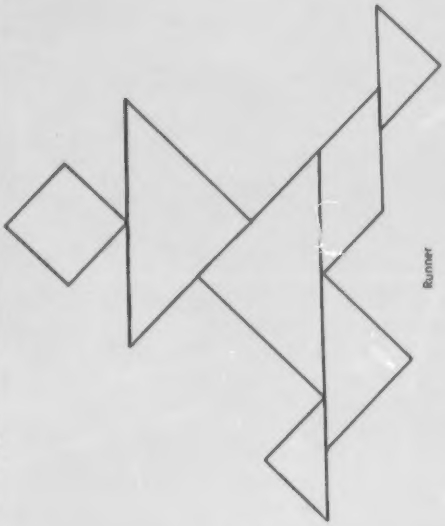
Tangrams


Indian Chief
3



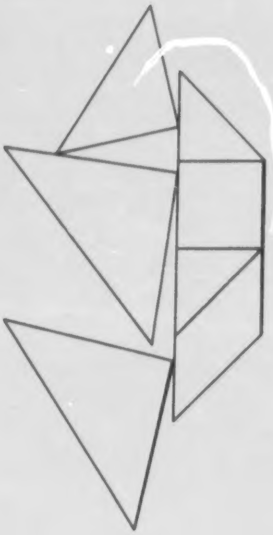

Horse and Rider
5


Watering Can
6



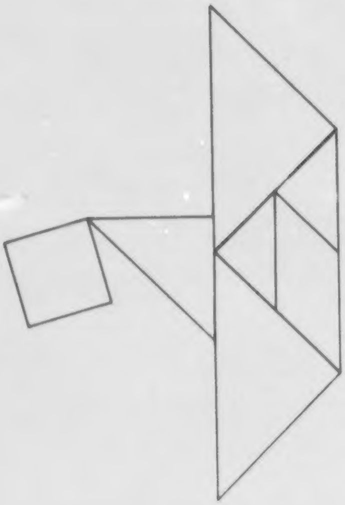
Runner

7



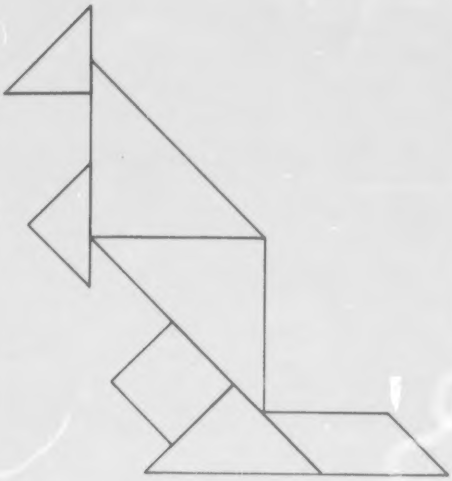
Boat

8



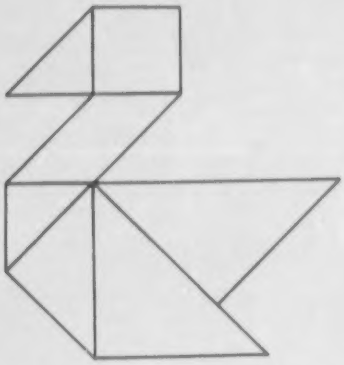
Another Boat

9



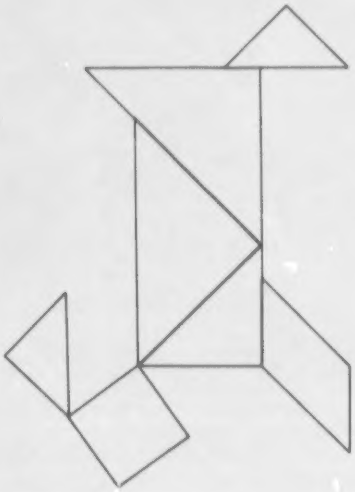
Kangaroo

10



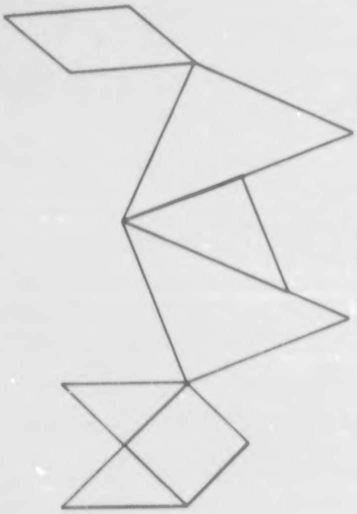
Duck

11



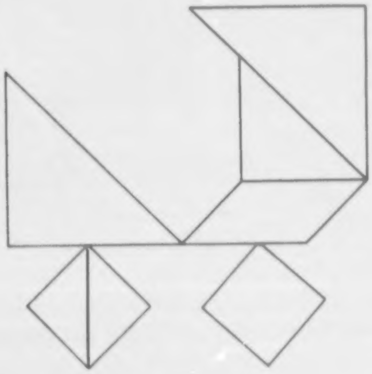
Dog

12



[illegible]

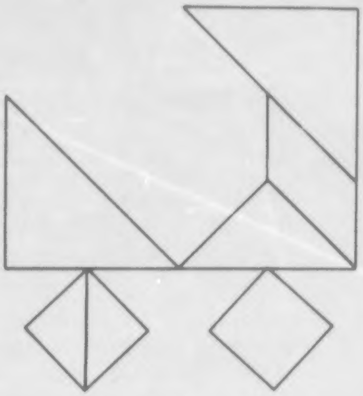
13



Carriage

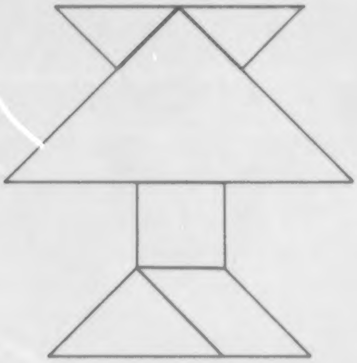
14

182



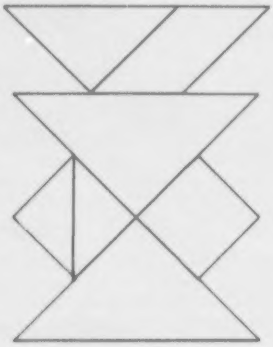
Another Carriage

15



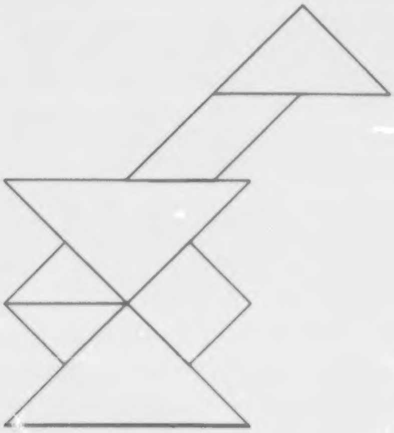
You name it! ______________

16



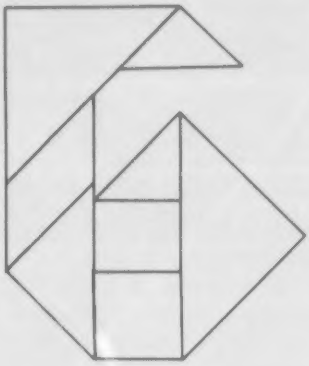
Vase

17



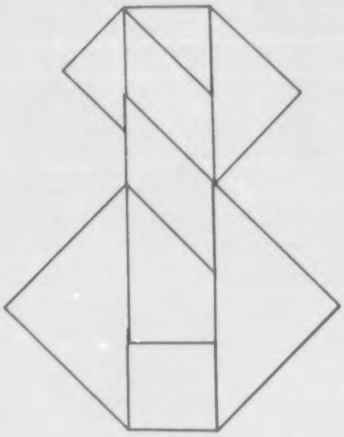
Pail and Shovel

18



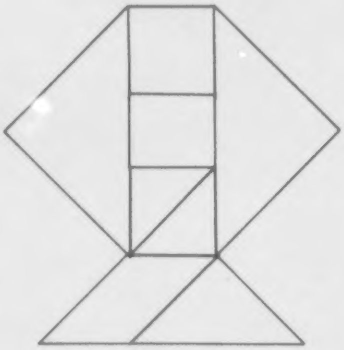
Six

19



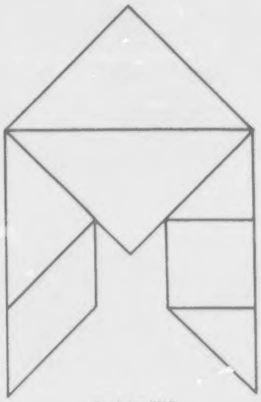
Eight

20



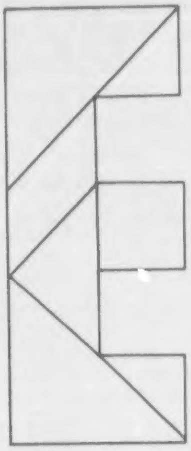
Knight's Helmet

21



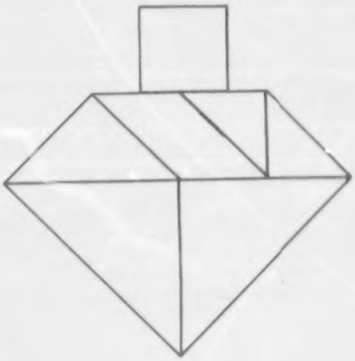
The letter "M"

22



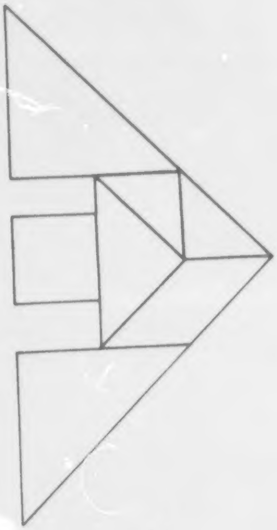
The letter "E"

23



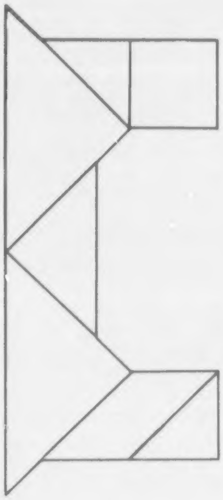
Top

24



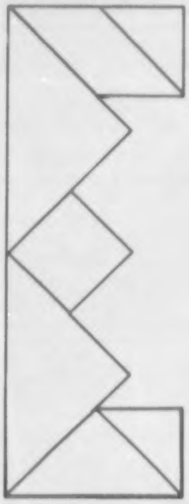
You name it! ____________

25



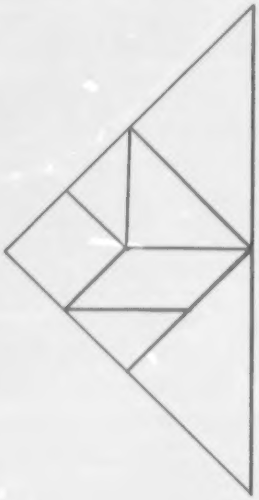
Table

26



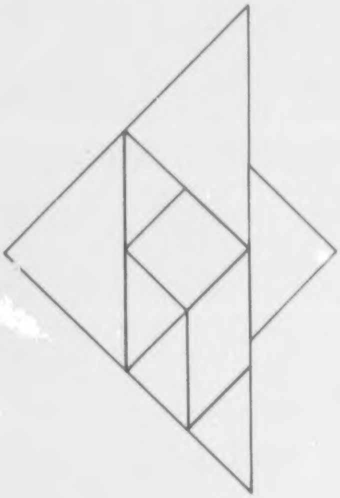
Bridge

27



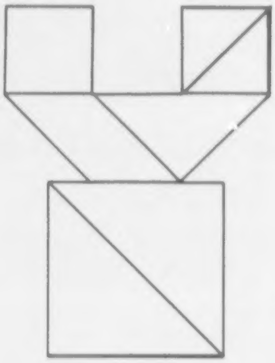
Triangle

28



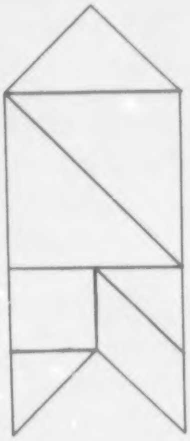
Mask

29



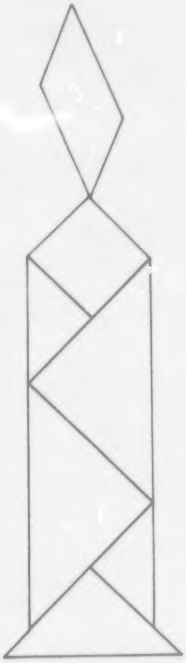
Trophy

30



Arrow

31



The End

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G8 Measuring with Centimeter Rods

CAPSULE LESSON SUMMARY

Measure many different objects with centimeter rods or some other uniform measuring instruments.

MATERIALS

Teacher:	Centimeter rods (optional); other measuring instruments such as paper clips, toothpicks, or straws
Student:	Centimeter rods (optional); other measuring instruments such as paper clips, toothpicks, or straws; pencil

DESCRIPTION OF LESSON

This lesson has been titled "Measuring with Centimeter Rods", because we suggest that if your classroom has available sets of centimeter rods (Cuisenaire Rods, Numerods, Color Factor Rods, or one of the many other kinds), it would be desirable for you to give the class opportunities to measure with them. Since CSMP does not provide such items in the classroom set of materials, no specific activities have been described using centimeter rods. We suggest, however, that measuring and other activities with centimeter rods may be very exciting and beneficial for your class, and leave it to the teacher to structure them as seems appropriate.

If your classroom does not have available sets of centimeter rods, we suggest that you repeat Lesson G4 Measuring with Shoes and Pencils using various uniform measuring instruments such as paper clips or toothpicks or straws. Vary the exercises by having the students measure different objects in the classroom or that you bring in specifically for measuring.

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G9

G9 Comparison of Area

CAPSULE LESSON SUMMARY

Find the area of a rectangular shape in a given size square, in triangles half the size of the square, and in other given figures as units. Without measuring, compare the areas of many other shapes to the area of the rectangle.

MATERIALS

Teacher: UPG I-Geometry Poster #2

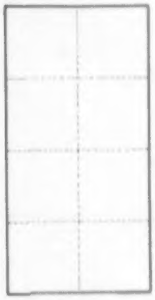
Student: Tangram triangles and squares set; Worksheets G9*, **, ***, and ****

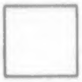
NOTE: The shapes on UPG I-Geometry Poster #2 must be cut out in preparation for this lesson. As indicated on the poster, some shapes are used for Exercise 1 and some for Exercise 2. The two 4 by 2 rectangles are used for both exercises. When you cut out the shapes, mark the backs to recall in which exercise the shapes will be used. These shapes will be used again in a lesson on estimation of area in UPG-II, so do not dispose of them after this lesson.

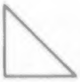
DESCRIPTION OF LESSON

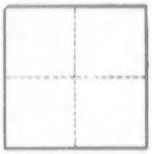
Exercise 1

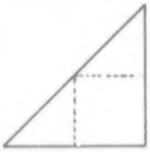
Have the following eight cut out shapes ready for display. The labels are only for reference in the description of the lesson.

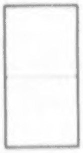

I-1

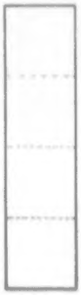

I-2

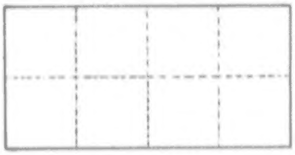

I-3


I-4


I-5


I-6


I-7


I-8

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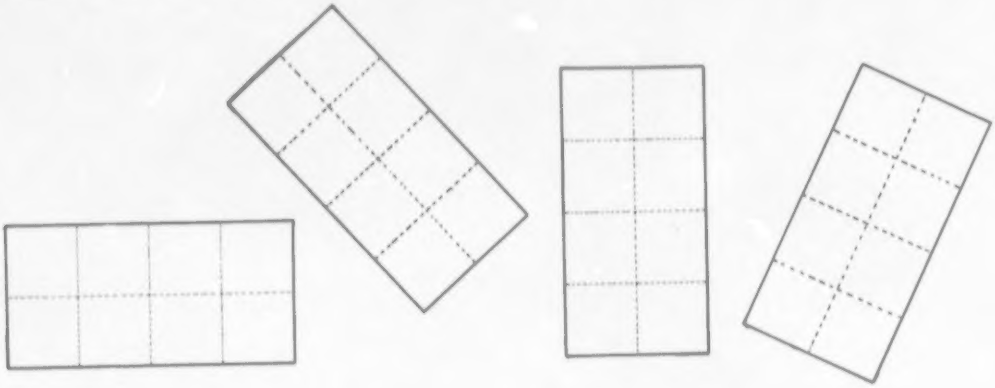
Conduct this exercise at a rather brisk pace so as not to bore the class. However, if at any time many students appear to be confused, ask for an explanation or allow class discussion of an answer.

Display the rectangle I-1 in different positions.

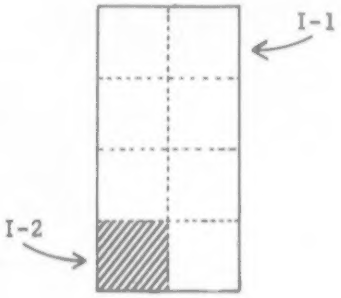


Tape the rectangle I-1 to one side of the board. Then hold up the square I-2 and ask:

T: How many little squares like this one do we need to cover the rectangle?

S: 8.

Place the square I-2 on top of the squares of the rectangle I-1 to illustrate that eight of them would be needed to cover the rectangle.



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Then tape the little square I-2 to the board and write the area of the rectangle in terms of this square.

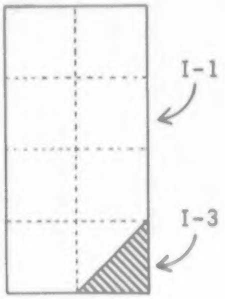
Area of the rectangle = 8 □

Hold up the triangle I-3 and ask:

T: How many of these little triangles like this do we need to cover our rectangle?

S: 16.

Place the triangle I-3 on top of the rectangle I-1 to illustrate that two triangles (I-3) are needed to cover each square of the rectangle I-1. Conclude that sixteen triangles would be needed to completely cover the rectangle.

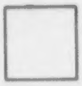


Then tape the triangle I-3 to the board and write the area of the rectangle in terms of this little triangle.

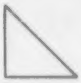
Area of the rectangle = 16 ◺

Repeat this activity for each of the remaining shapes I-4 through I-8. Record the answers on the board.

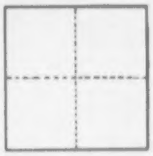
Area of the rectangle = 8



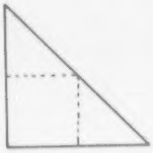
Area of the rectangle = 16



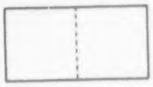
Area of the rectangle = 2



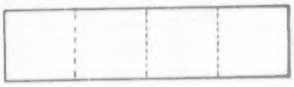
Area of the rectangle = 4



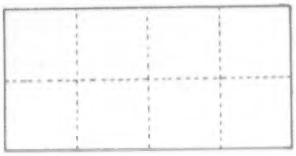
Area of the rectangle = 4



Area of the rectangle = 2



Area of the rectangle = 1



Before going on to Exercise 2, erase the board and take down all demonstration shapes except the rectangle I-1.

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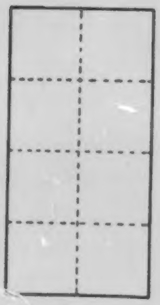
Exercise 2

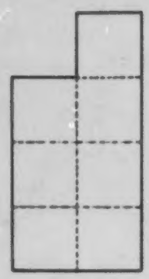
Next to the rectangle I-1 make a table as shown below. Be sure your table is large enough to tape several shapes in each row.

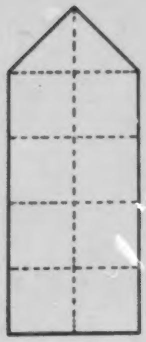


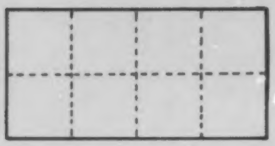
I-1

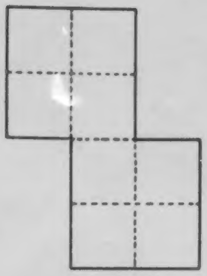
Smaller	
Same	
Bigger	

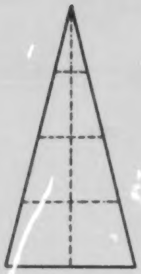
Be ready to display one at a time the shapes for Exercise 2.

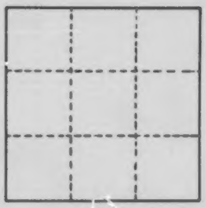

II-1

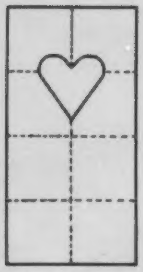

II-2

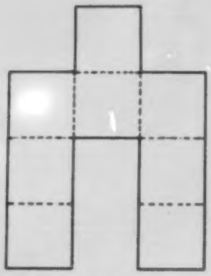

II-3

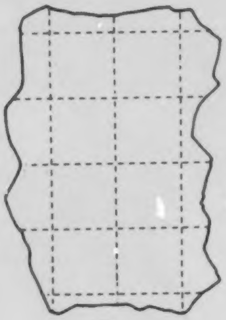

II-4

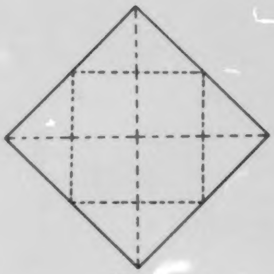

II-8


II-9


II-5


II-6


II-7


II-10

Display the shape II-1 shown above and ask:

T: Does this shape have a smaller area or a bigger area or the same area as our rectangle?

S: Smaller.

Let the class discuss why the area is smaller. Then tape the shape in the correct row of the table.

Encourage students to make different observations leading to the same conclusion. For example,

S: It is smaller because part of the rectangle is missing.

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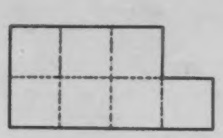
S: The rectangle will more than cover this shape, so this shape is smaller.

S: It takes 7 little squares to cover this shape and 8 little squares to cover the rectangle; so this shape is smaller.

Smaller	
Same	
Bigger	

Continue this activity for the remaining shapes. Be sure to discuss where to put each shape in the table.

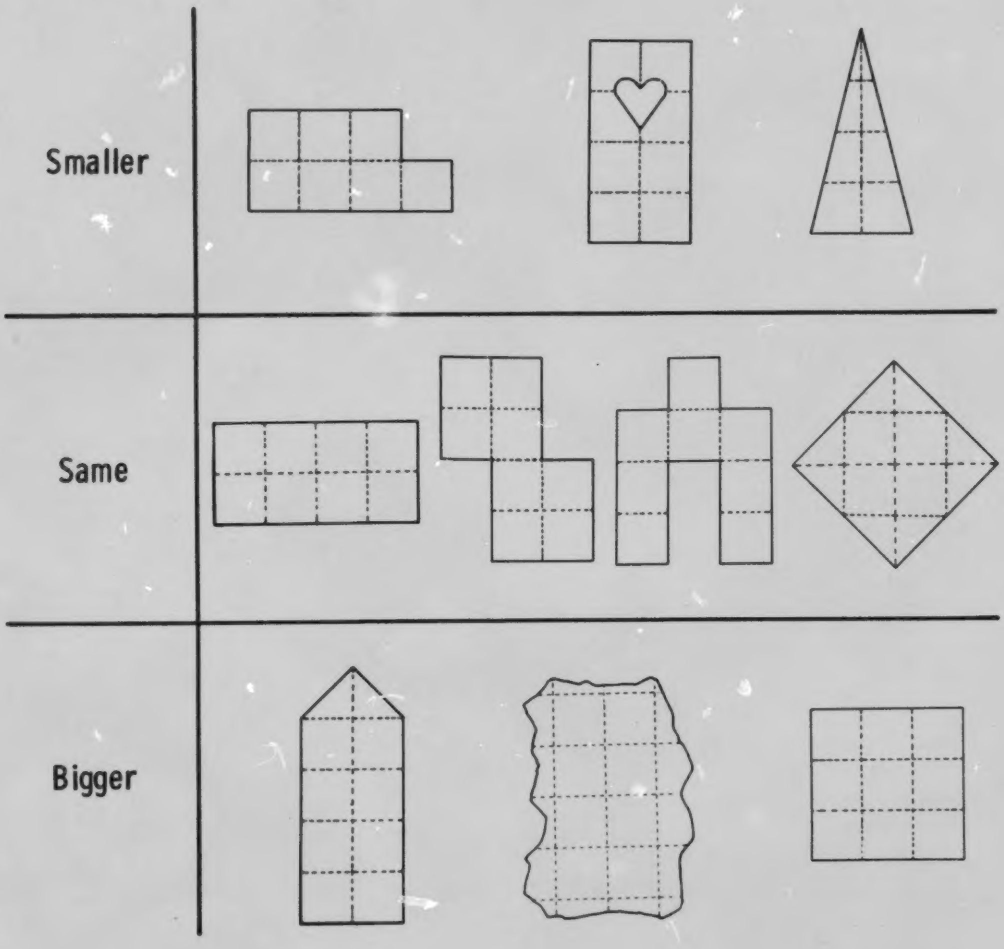
One way to get the entire class involved in this activity is to make it into a game. For example,

T: This is a game. I will show you some new shapes and you will have to compare them with our rectangle. Don't say your answer out loud. If the area of the new shape is smaller, hold up one hand.

If the area of the new shape is the same, keep your hands down.
If the area of the new shape is bigger, hold up both hands.

Such a game will allow you to see the answers of all the students. After the area (smaller, same, bigger) of each shape has been decided, indicate what the students should have done (held up one hand, no hands, or both hands) as a reminder.

At the end of the game or the activity, your board should have all the shapes for this exercise taped to it.



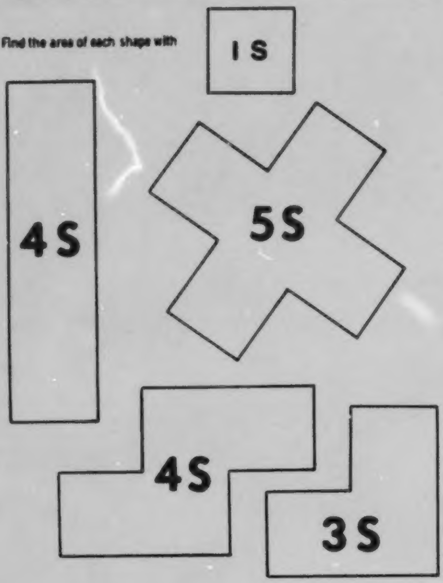
G9

T: Remember, the area of our rectangle (point to I-1) is exactly eight little squares. Can you show me which of the shapes on the board have an area of exactly eight little squares? Which shapes have an area more than eight little squares? Which shapes have an area less than eight little squares?

Distribute the Tangram triangles and squares set (ten little triangles and ten squares) and Worksheets G9* and **. Students who finish these worksheets should continue with Worksheets G9*** and ****. Be sure they understand the directions. You could ask students to check their answers by measuring the area of each shape with Tangram squares or little Tangram triangles.

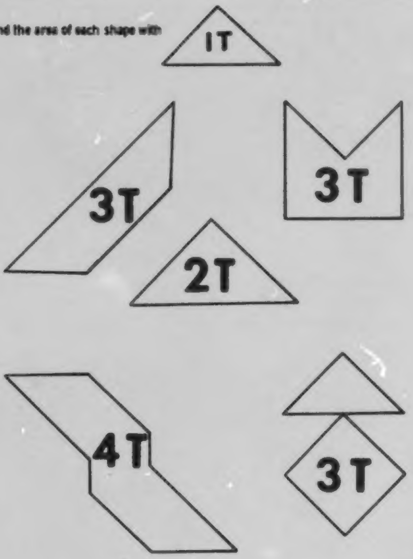
Name ____________ G 9 *

Find the area of each shape with

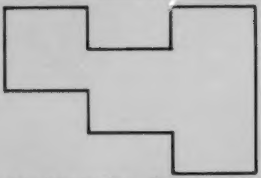


Name ____________ G 9 **

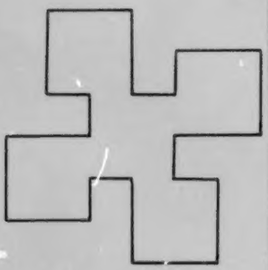
Find the area of each shape with



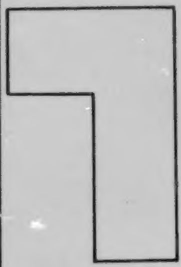
Name ____________ G 9 ***



Compare these shapes to the one above. Circle your answers.

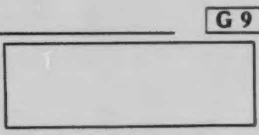


Smaller [✓ Bigger] Same



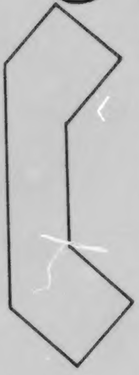
Smaller Bigger [✓ Same]

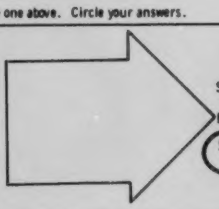
Name ____________ G 9 ****



Compare these shapes to the one above. Circle your answers.

Smaller [✓ Bigger] Same





Smaller
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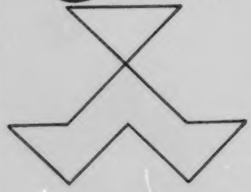


G10 Measuring with a Tape Measure

CAPSULE LESSON SUMMARY

Introduce a tape measure for measuring the lengths of certain objects. If appropriate, compare the separation of marks on the tape measure to centimeter rods. Measure many different objects in centimeters and decimeters (orange rods).

MATERIALS

Teacher: Centimeter rods (optional); tape measure

Student: Tape measure; paper; pencil

DESCRIPTION OF LESSON

NOTE: If you have available a set of Cuisenaire rods or other centimeter rods (see the note at the beginning of Lesson G8), begin with Exercise 1A. Otherwise start with Exercise 1B.

Exercise 1A

Exhibit one white rod (or a one centimeter rod).

T: I would like to measure this side (point out the width) of my desk[*] with white rods. About how many white rods long do you think this side is?

Let many students give estimates and list them on the board for later discussion and comparison with the actual measurement.

T: How shall we make this measurement?

Discuss with the class the merits of the different ways suggested by students.

---

[*] You could substitute any other suitable object in your classroom. Try to choose something between 50 and 100 white rods (centimeters) long.

If the students are quite familiar with Cuisenaire rods, it is likely that someone will suggest using the long orange rods. For example,

S: Use orange rods and count ten white rods for each orange rod.

Call on some students to actually make the measurement with orange rods and white rods, or use any other method the class agrees upon. Record the result on the board. For example,

7 orange rods + 6 white rods = 76 white rods

Compare the actual measurement with the students' estimates.

T: Now I would like to measure the side of my desk with red rods. Do you know an easy way to do this?

S: A red rod is the same as two white rods; we need half as many red rods.

Accept any reasonable suggestions. You could direct the discussion so as to use one of the following methods. The first method suggests counting by fives because five red rods make one orange rod.

7 orange rods + 6 white rods = 35 red rods + 3 red rods = 38 red rods

76 white rods = 38 white rods + 38 white rods = 38 red rods

If the class is enjoying this activity and is quite familiar with the Cuisenaire rods, you could continue by asking for the same measurement with yellow rods and with orange rods. For (this) example,

between 15 and 16 yellow rods

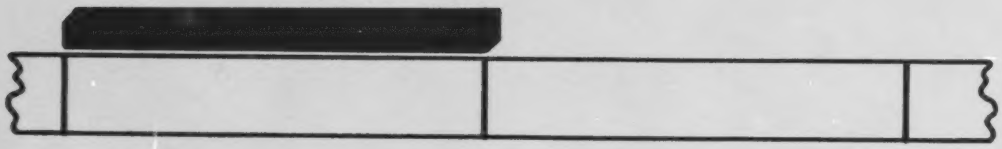
between 7 and 8 orange rods

G10

Display one of the tape measures as you say:

T: Measuring my desk by putting rods end to end wasn't so easy. But I have something here that may make the job of measuring a little easier. It's called a "tape measure". On one side there are orange marks and between two orange marks we can fit an orange rod.

Demonstrate this fact.

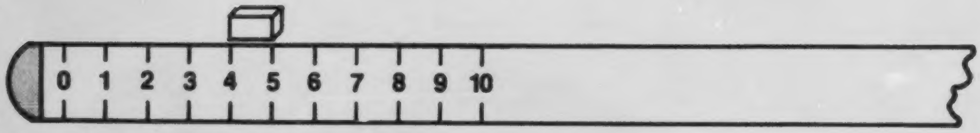


T: So we could measure the desk with this tape measure.

Demonstrate measuring the desk with the tape measure and ask a student to count the spaces between orange marks. Be careful to put one end, the "0" mark of the tape measure at one edge of the desk. For our example, a student would count seven spaces and the other edge of the desk would land within the eighth space between orange marks.

T: On the other side of the tape measure a white rod will fit between the marks.

Demonstrate this and then measure the desk with this side of the tape measure.



Exercise 1B

Display one of the tape measures.

T: I would like to measure this side (point out the width) of my desk with this tape measure. On one side of the tape measure the marks are for centimeters (write the word on the board and show the class that side of the tape measure). About how many centimeters long do you think this side of my desk is?

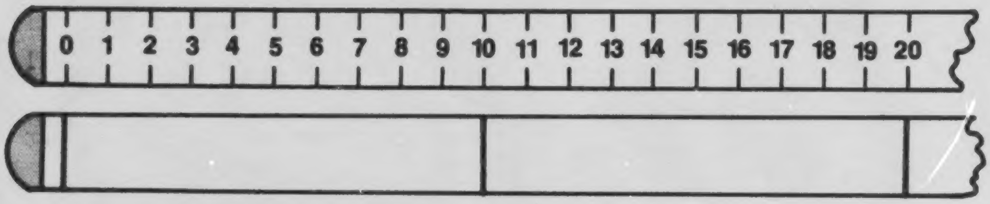
Let many students give estimates and list them on the board. Then, with a couple students assisting you, measure the width of your desk. Record the result on the board, for example it might be about 76 centimeters. As you record the result point out that "cm" is an abbreviation for "centimeters".

76 cm

Compare the actual measurement with the students' estimates.

T: Now I would like to measure the side of my desk with the other side of the tape measure. There are orange marks on this side of the tape measure. Between two orange marks there are ten centimeters; we will call this an orange length.

Demonstrate this to the class.



T: How long is this side of my desk if I measure with the orange lengths?

S: More than seven orange lengths.

S: Between seven and eight orange lengths.

Do not be concerned if your students do not give these responses; let them make guesses and then measure the width of your desk. Ask a couple students to assist you and to count the orange lengths. Record the result on the board. using "or" as an abbreviation for "orange".

76 cm
between 7 or and 8 or

T: We could be more exact if we counted as many orange lengths as possible and then measured the remaining part with centimeters.

Ask a couple of students to measure the desk again in this way, and then write the measurement on the board.

76 cm
between 7 or and 8 or
7 or + 6 cm

Exercise 2

Distribute tape measures and paper.

T: I am going to write a list[*] of objects on the board and I would like for you to copy this list on your paper. Make a table like mine. You will have to measure these objects with your tape measure and write the measurements on your paper.

---

[*] This is only a suggested list; you may think of other objects in your classroom that would be more interesting for the students to measure.

Draw the table on the board and first list whatever you measured in Exercise 1.

	ORANGE (rods)	CENTIMETERS (white rods)
Teacher's Desk	Between 7 or and 8 or	76 cm
My Pencil		
My Paper (long side)		
My Paper (short side)		
My Shoe		
My Desk		
My Arm		
My Height		

Instruct the students to find the measurements and write them on their papers. Give assistance to students who have difficulty using the tape measure.

Attach a tape measure on the wall somewhere in the classroom so that you can measure the height of each student. Call them one at a time to be measured. Students who finish measuring everything on the list could be paired and asked to measure some bigger objects such as the (width or length of the) chalkboard or a bulletin board.

When most of the class have finished measuring everything on their lists, collectively discuss some of the measurements. It may be interesting for the class to know who is the tallest and who is the shortest, for example. If some students measured the dimensions of a chalkboard, you might ask the rest of the class to predict how long it is before the result is announced.

G11

G11 Taxi-Geometry #3

CAPSULE LESSON SUMMARY

Review the situation from previous Taxi-geometry lessons. Introduce a hide-and-seek game to find all the points which can be reached with a four (five) block path.

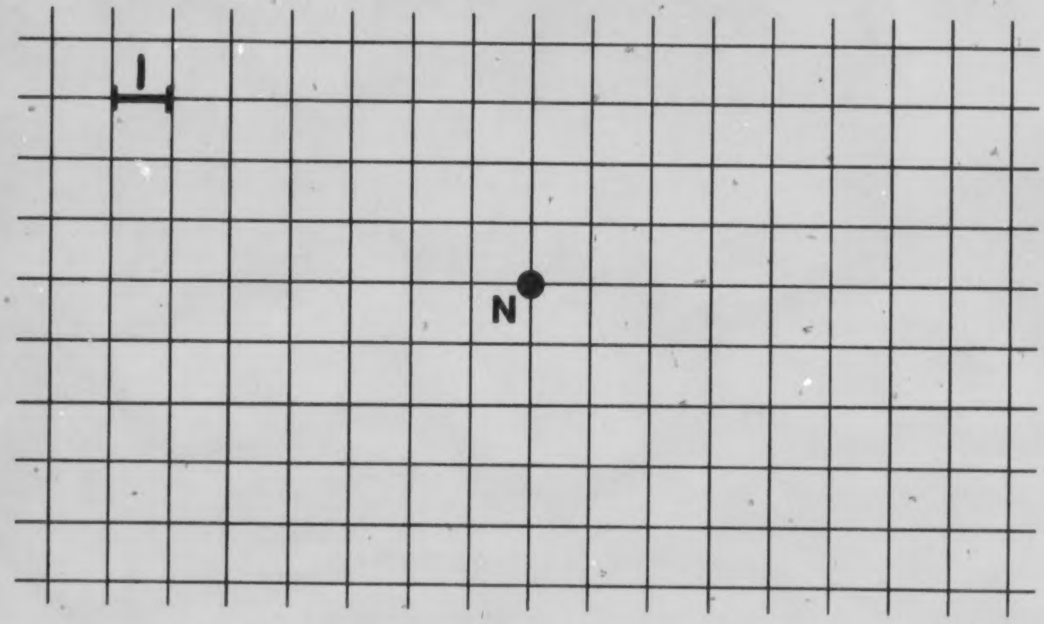
MATERIALS

Teacher: Grid board[*]; colored chalk

Student: Worksheet G11; colored pencils

DESCRIPTION OF LESSON

Display your grid board. Draw a dot and label it "N". Indicate a one block measure in an upper corner.



T: This is a map of the city where our friend Nora lives and here is Nora's house (point to "N"). Do you remember something about Nora?

---

[*] See the NOTE ON GRIDS in the introduction to this strand.

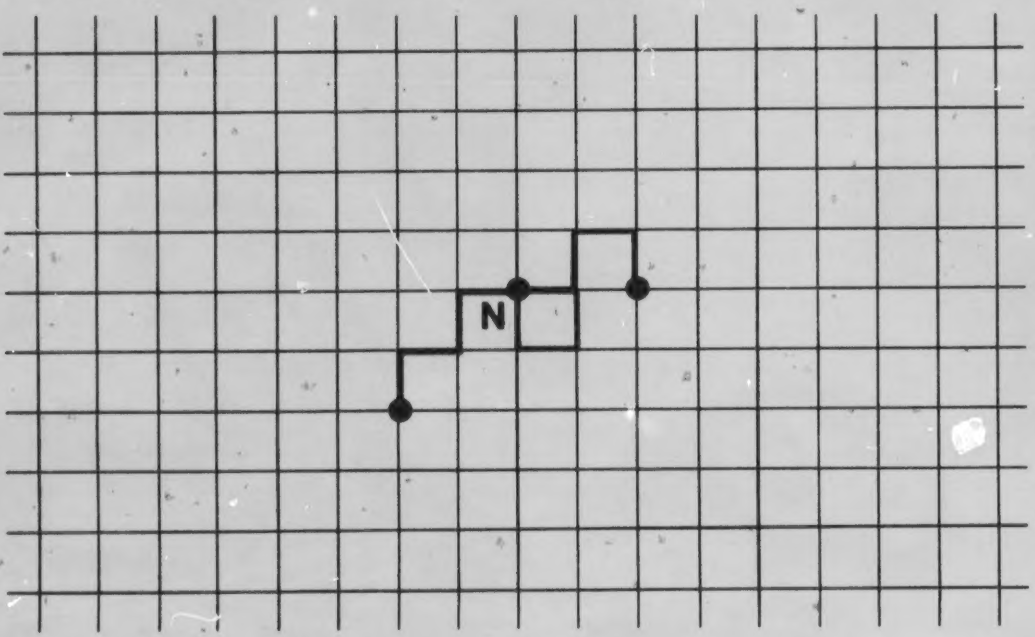
Ask students to recall some episodes from previous lessons about Nora. Encourage them to mention long and short walks that Nora can take, and to mention counting the number of blocks in a path.

Distribute Worksheet G11 and colored pencils.

T: Today Nora is playing a kind of hide-and-seek game with her brother, Nick. She tells Nick that she is going to take a four block walk and then hide. Nick has to try to find her. Where should Nick look?

Let the students come to the board and point to places where Nora could be hiding. Each time a place is correctly located, draw a red dot there. After two or three correct places have been located, ask the students to try to find on their papers all the places where Nora could be hiding and draw a red dot at each one. Remember, Nora can take any four block walk. Observe and help students who are having difficulty. When you see a new place on a student's paper, ask the student to draw the red dot at that place on the demonstration grid board. Occasionally, ask a student to explain how he or she found a place Nora could be hiding. For example,

S (tracing the blue path shown below): She could take this path.

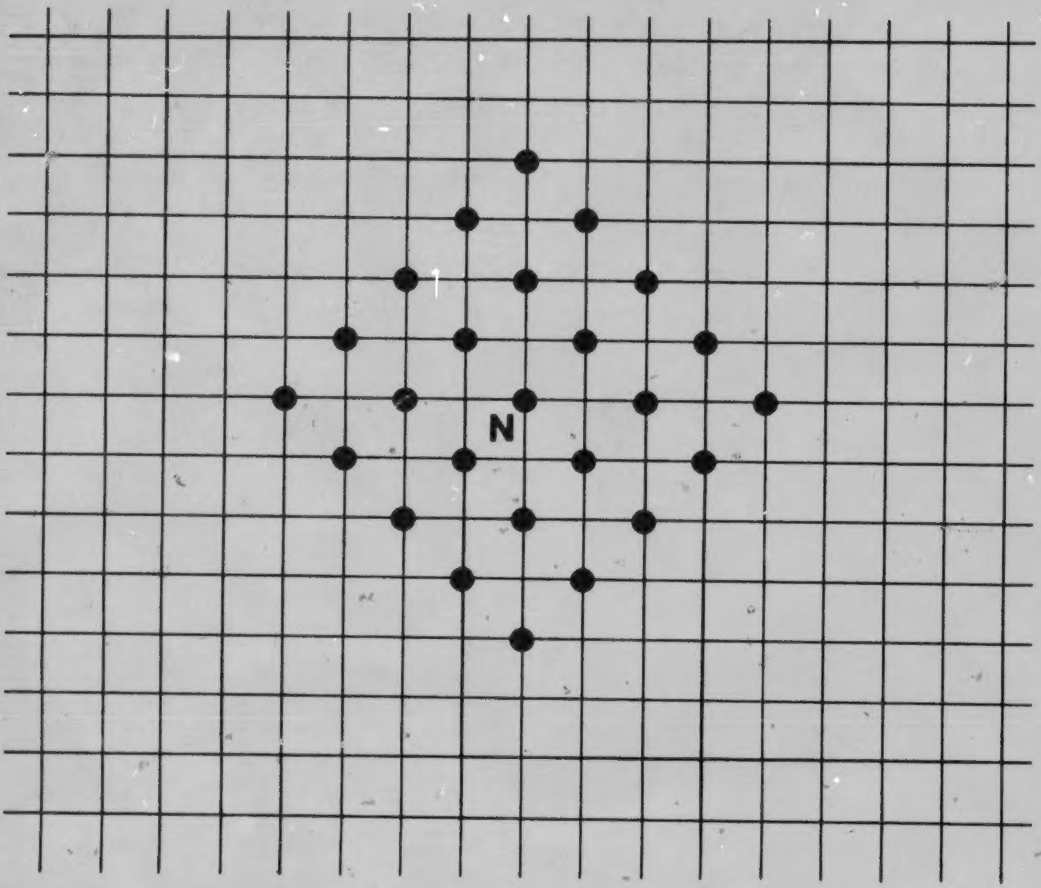


G11

S (tracing the green path): She could take this path.

S (tracing the red path): She could be hiding at her house.

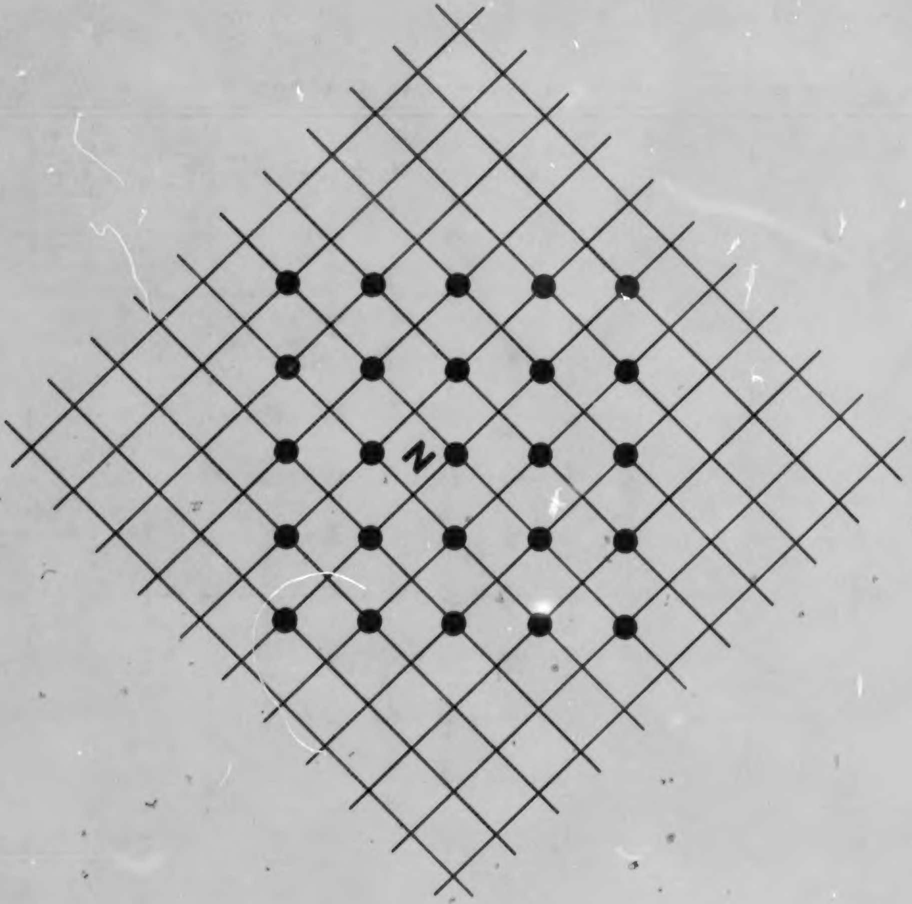
A completed picture should have 25 red dots as shown here.



This problem will not be easy at first. Do not be concerned if students make many counting errors in the beginning; just encourage them to be careful. You may have to remind the class several times that Nora can take any four block path. Soon some students should observe a pattern. Encourage them to describe any patterns they see in the picture.

After all the red dots have been drawn, ask a student to find the number of places where Nora could be hiding. (25)

The students may notice that if you "tilt" the picture, you can easily see five rows of five dots each.

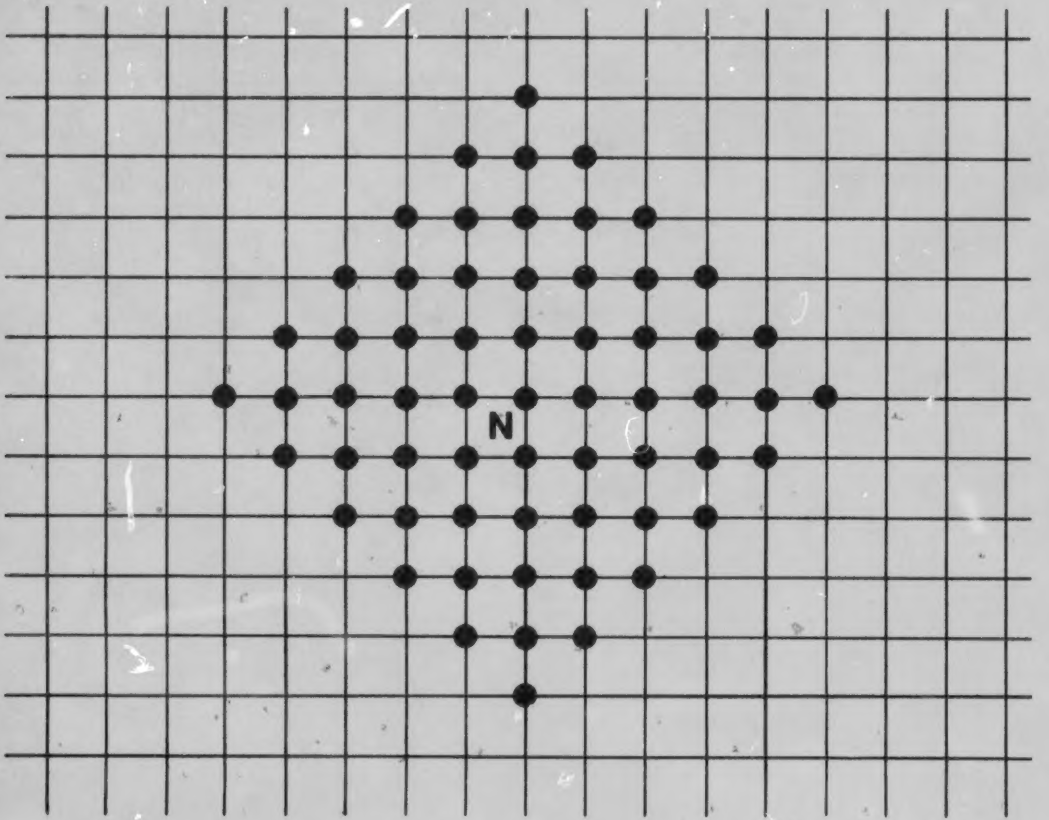


This observation makes counting the dots much easier: either count by fives or calculate "5 x 5". You should suggest looking at the picture this way if it escapes the notice of your students.

T: Now it is Nick's turn to hide. He tells Nora he will take a five block walk and then hide. Where are the places that Nick could hide?

First, let several students locate some points on the demonstration grid board and when a correct place is located, draw a blue dot there. Then ask the students to try to find on their papers all the places where Nick could hide and draw a blue dot at each one. As new places are found, invite students to draw the blue dots on the demonstration grid board. Occasionally, ask students to explain how they found places where Nick could hide. Once the pattern is observed, one student could draw several blue dots at one time.

A completed picture should have 36 blue dots as shown here.



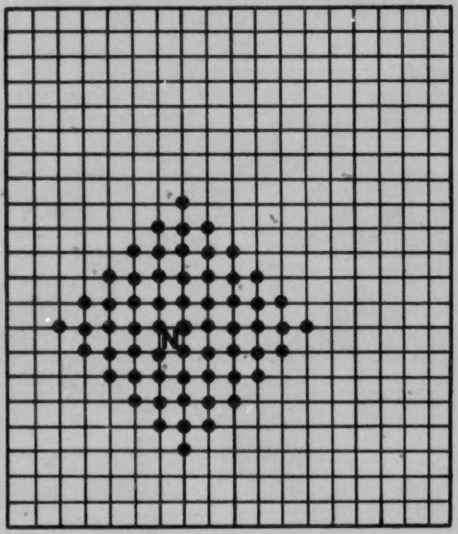
Encourage the students to comment on the pattern in the picture and to notice that Nick's hiding places are all different from Nora's. Call on a student to count the blue dots. (36)

Again, if you tilt your view of the picture you will notice six rows of six blue dots. This observation should not be too difficult if you previously observed five rows of five red dots this way. Then to count the blue dots, calculate "6 x 6" or count by sixes.

T: How many more places does Nick have to hide than Nora? (11)

Name ______________________ G II

25 red dots 36 blue dots



G12 Higgledy Piggledy Dots

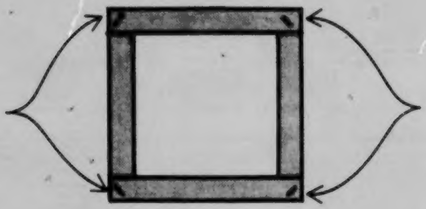
CAPSULE LESSON SUMMARY

Separate a collection of dots so that half of the dots are inside a string (on one side of a line or inside a square frame) and half of the dots are outside of the string (on the other side of the line or outside of the square frame).

MATERIALS

Teacher:	Colored chalk; meterstick or straight edge; UPG I-Geometry Poster #3; cardboard frame
Student:	Unlined paper; colored pencils; ruler or straight edge; Worksheets G12*, **, and ***; Tangram frame

NOTE: Construct a cardboard square frame from the four pieces of cardboard provided in your classroom set of materials. This can be done by stapling the cardboard pieces together to form corners.



Check that the square frame has (inside) side length of 45 cm and test that you are able to enclose exactly ten dots on UPG I-Geometry Poster #3 (as illustrated on page 77). Tangram frames for the students' individual use were on the Tangram cards and saved from Lesson G7.

DESCRIPTION OF LESSON

Exercise 1

Distribute unlined paper (two or three sheets per student) and colored pencils or crayons.

T: Draw 32 dots on one piece of paper. Don't make them too big or too small, and try to spread them out all over your paper. Remember: exactly 32 dots. Count them in your head as you draw them.

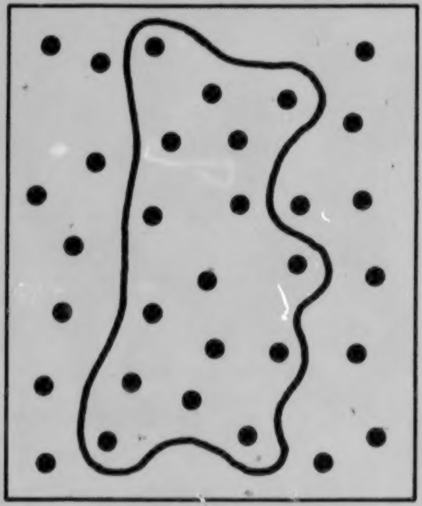
Give the students a couple minutes to perform this task. Observe some of their drawings and help those students who have difficulty drawing and keeping count of the dots. You might have to make the drawing for a student who cannot manage to both draw and count 32 dots.

T: What is one-half of 32? (Ask students who will probably be able to perform this calculation in their heads.)

S: 16.

T: One-half of 32 is 16, because 16 + 16 = 32. Draw a red string around one-half of your 32 dots. You should have 16 dots inside the string and 16 dots outside of the string.

This is a difficult task for a young child, so do not expect your students to solve it immediately. Observe their efforts and keep emphasizing that they draw a string with 16 dots inside and 16 dots outside. When you observe a mistake (say 12 dots inside and 20 outside), ask the student to check the solution with you. Then suggest that the student make another attempt. Many different drawings are possible; here is one that is nicely laid out.



If necessary, encourage students to make several attempts or suggest students try to adjust their pictures. Do not provide a solution yourself. What is important for the individual student in this kind of activity is to try to play around with the dots in a plane, not to stare at 16 dots in a beautifully drawn string.

If some students do not find a solution after two or three attempts, you could draw 10 or 12 dots on a paper and help them solve the problem of putting one-half of them in a string. Then ask them to try the original problem again.

Some students will find a solution very quickly. For these students, you could suggest that they continue with their pictures and in another color (say blue) they draw a string around one-half of the 16 dots inside the red string. Then in a new color draw a string around one-half of the dots inside the blue string.

After most of the students have solved the original problem, get the class' attention and proceed with the lesson.

Exercise 2

Distribute rulers or straight edges for drawing line segments and Worksheet G12*. Note that the same picture is on both sides of the worksheet for students who need a second try.

T: How many dots are there on this paper? Count them.

S: 24.

T: What is one-half of 24?

S: 12.

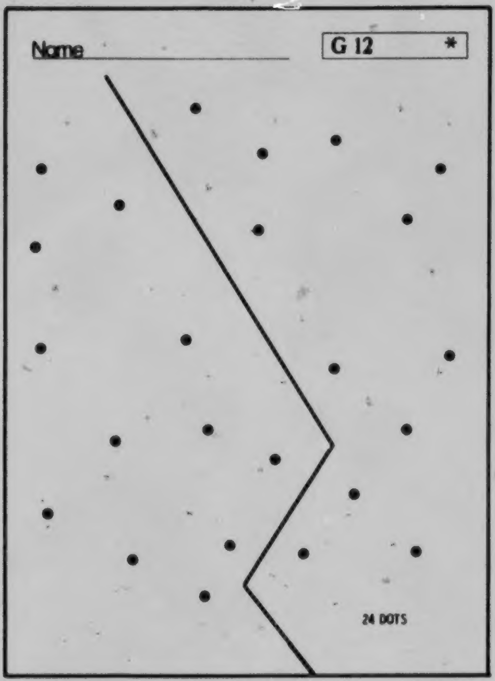
T: One-half of 24 is 12, because 12 + 12 = 24. Using your ruler, draw a straight line so that one-half of the dots are on one side of the line and the other one-half of the dots are on the other side of the line. Exactly 12 dots should be on one side and exactly 12 dots should be on the other side.

The same remarks apply as in Exercise 1. Furthermore, you might have to make it clear that the straight line does not have to be horizontal or vertical.

Students who find a solution very quickly could be asked to find another (or several other) line(s) which will also solve the problem. A more difficult problem would be to find a second line which together with the first line divide the paper into four parts so that in each part there are exactly 6 (or one-fourth of the) dots.

Many solutions are possible. A "technique" which usually will solve the problem is to start at one edge or corner of the paper and move your ruler parallel to itself until you observe one-half of the dots on one side. If it does not work the first time, just twist the ruler a little and start again. Again, it is important for the students to play around with these dots and some lines. Do not explain this technique to them.

If a student suggests:



do not say it is wrong, but ask the student to try to find a solution using just one straight line.

G12

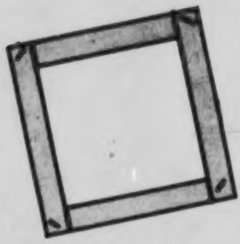
Exercise 3

Tape UPG I-Geometry Poster #3 to the board. Display the large cardboard square frame.

T: What shape is this frame?

S: Square.

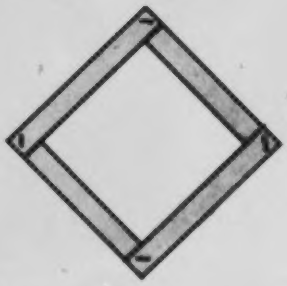
Tilt the frame slightly.



T: What shape?

S: Still a square?

Hold the frame on a forty-five degree tilt.



T: Is it still a square?

S: Yes.

Some students will want to call the last figure a "diamond". Point out that sometimes it is called a "diamond", but it still remains a square no matter how you move it around.

Point to the poster taped on the board.

T: How many dots?

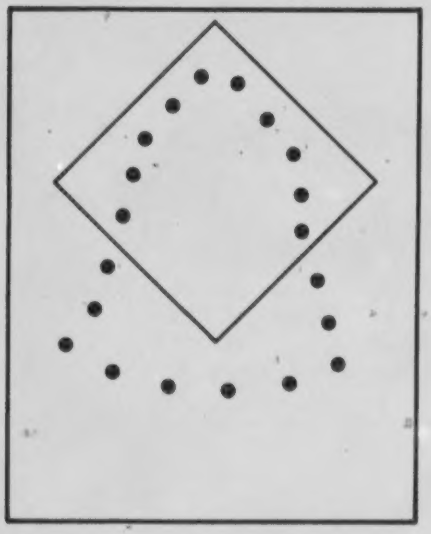
S: 20.

T: What is one-half of 20?

S: 10.

T: I would like to use my square frame to draw a square around exactly one-half of these dots. Who can help me?

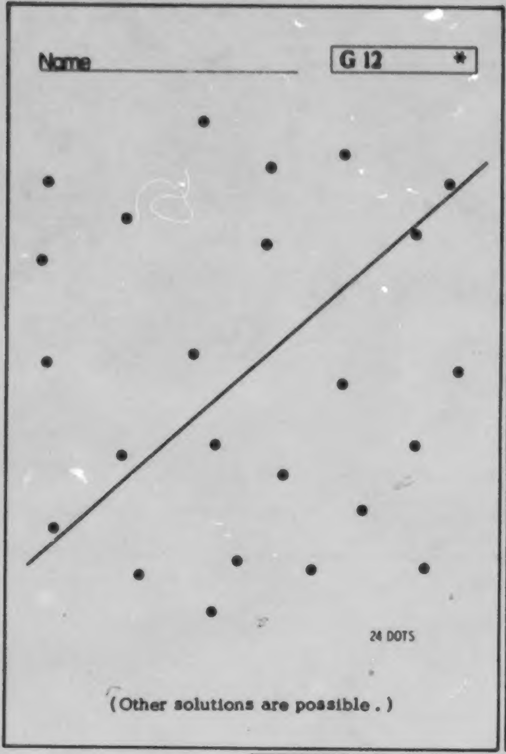
Call two students to the board and ask them to work together to position the frame so that exactly 10 of the dots are visible inside the square frame. After they have positioned the frame correctly, ask them to hold it steady while you draw the square. Then, take the frame away and have the class check that exactly 10 dots are inside the square and exactly 10 dots are outside.



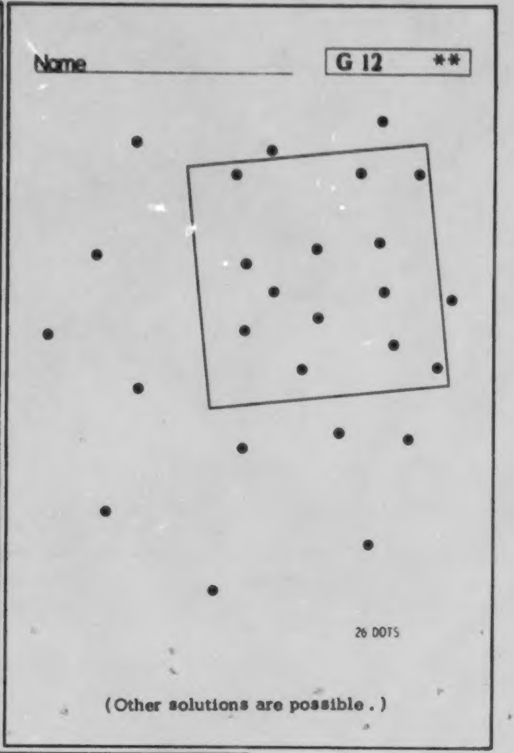
G12

Distribute Tangram frames and Worksheets G12** and ***. Explain to the students that on both worksheets they should first decide how many are one-half of the dots. Then, use the Tangram frame to draw a square around one-half of the dots. Encourage students to position their Tangram frames carefully before drawing a square. The same picture of dots is on both sides of a worksheet for students who need a second try. Help students who have difficulty deciding how many dots should be inside the square to compute one-half of the number of dots.

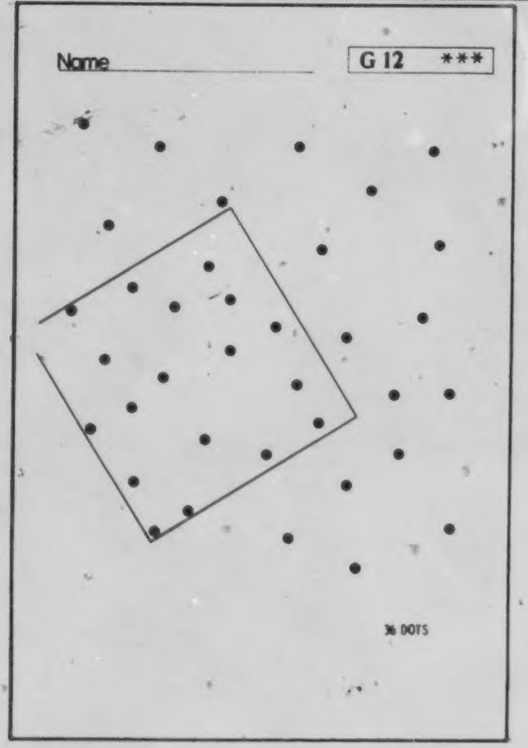
Note that in order to solve the problem on Worksheet G12***, it will be necessary to "extend" the paper either physically by taping an additional piece of paper to the worksheet or mentally. Most students will do this mentally, but you should be prepared with tape and additional pieces of paper.


Name
G 12 *
24 DOTS

(Other solutions are possible.)


Name
G 12 **
26 DOTS

(Other solutions are possible.)


Name
G 12 ***
36 DOTS

G13

G13 Booker's Bakery #2

CAPSULE LESSON SUMMARY

Divide a shape into halves, thirds, and fourths. Write number stories about $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. Use the symmetry of a shape or the number of squares in the shape on a grid to divide it into halves, thirds, and fourths. Find the number of squares in one-half, one-third, and one-fourth of the shape.

MATERIALS

Teacher: Colored chalk; UPG I-Geometry Poster #4 (two copies)

Student: Colored pencils; Worksheets G13*, **, ***, and ****

NOTE: Before the lesson begins, cut each of the two copies of UPG I-Geometry Poster #4 in half. The four shapes (four copies of the same shape) will be used in Exercise 2.

DESCRIPTION OF LESSON

Exercise 1

T: Do you remember the twins Jack and Jill who live next door to Booker's Bakery?

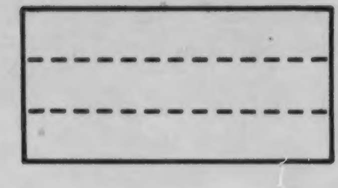
Allow a couple minutes for the class to recall that Jack and Jill often share cakes or cookies from the bakery. Discuss that sharing equally means that each person gets the same amount.

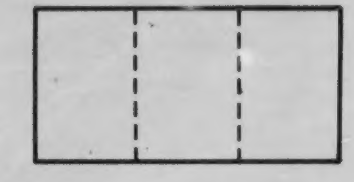
T: Sometimes when Jack and Jill have a friend visiting them, their mother will buy a small cake for the children to share. Today Jack and Jill have one friend visiting them, and their mother has bought a cake. I will draw a picture of the cake.

Draw a rectangular shape on the board.

T: There are three children and they would like to share this cake fairly. Who can show us how they could cut the cake?

Invite a student to trace the cutting lines. There may be several different suggestions; draw dotted lines each time a correct suggestion is given. Then erase the cutting lines and ask for another way to cut the cake. Two divisions you are likely to receive are illustrated below. Do not insist on other suggestions but accept them if offered. Leave one of these divisions on the board.





Choose three colors of chalk, say red, blue and green.

T: I would like to color one-third of this cake red, one-third blue, and one-third green to show the three pieces go to the three children.

Invite students to do the coloring, but encourage them to do it quickly.

G13

Write the fractions under each colored piece of the cake.

$$\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}$$

Write the number story for this situation on the board as you say:

T: One-third of the cake plus one-third of the cake plus one-third of the cake is the whole cake.

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

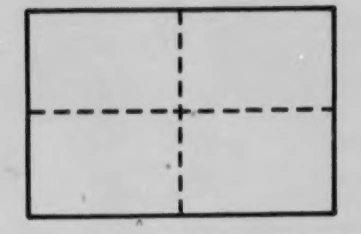
Erase the board and continue with a new situation.

T: Suppose another friend came to visit Jack and Jill today, so that there are four children. Then the four children would have to share the cake equally.

Draw a picture of the cake on the board again.

T: Who can show us how to cut the cake so that four children will all get the same amount?

Again, invite students to trace cutting lines. Draw dotted lines each time a correct suggestion is given. Leave one "simple" division on the board. For example,



Choose four colors of chalk and ask the students to color one-fourth of the cake in each color. Write the fractions near the colored pieces.

Write a number story for this situation as you say:

T: One-fourth of the cake plus one-fourth of the cake plus one-fourth of the cake plus one-fourth of the cake is the whole cake.

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$$

T: Suppose Jack gets this red piece and Jill gets this blue piece. What part of the cake do Jack and Jill get together?

S: One-half.

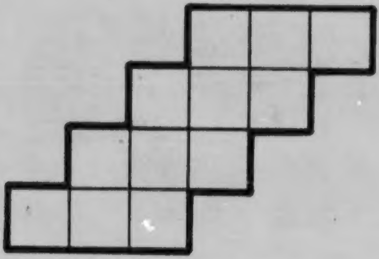
You may have to rephrase the question and indicate in the picture that you want the class to look at the red piece and blue piece together. Then write this number sentence on the board.

$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Erase the board before going on to the next exercise.

Exercise 2

Tape one copy of this shape on the board.



T: One day Jack and Jill got a strange shaped cake from Booker's Bakery. This is the shape (point to the board). Also, the cake was already cut into many pieces. How many pieces?

S: 12.

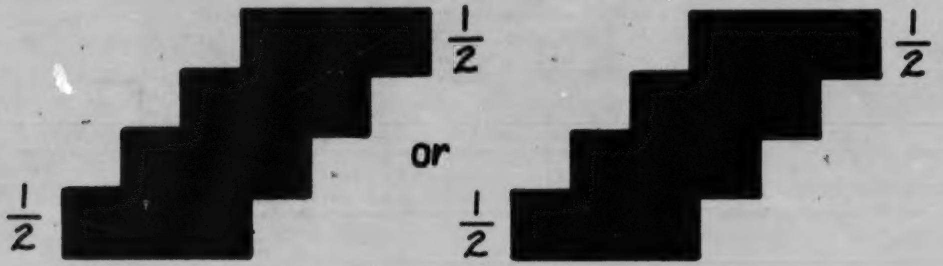
T: Yes and they are all the same size. Jack and Jill would like to share the cake fairly between them. How can they do it?

S: Each one gets half of the cake.

S: Jack takes six pieces and Jill takes six pieces.

T: Which pieces should we give to Jack and which to Jill?

Let students suggest several different divisions and then color half of the shape red and half of the shape blue. Choose a suggestion from one of your students that illustrates some symmetry in the shape. For example,



T: The red pieces could be Jack's and the blue pieces could be Jill's. Is it easy to see that both children get the same amount of cake?

Encourage students to discuss this question briefly in terms of symmetry (both pieces of the cake look exactly the same) and in terms of numbers (both children get six pieces).

T: How many pieces are in the whole cake?

S: 12.

T: How many pieces are in one-half of the cake?

S: 6.

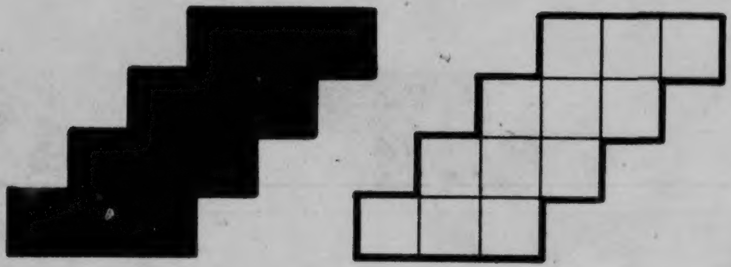
G 13

Write this equation to one side on the board as you say "one-half of twelve is six."

$$\frac{1}{2} \times 12 = 6$$

Leave this copy of the shape and the equation on the board. Tape up a new copy of the shape.

$\frac{1}{2} \times 12 = 6$



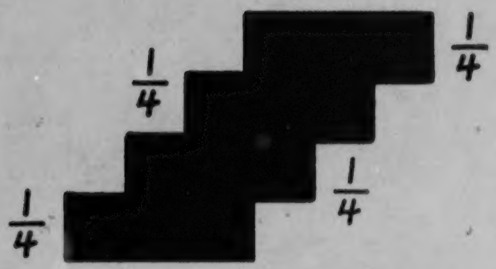
T: Before Jack and Jill could divide the cake, their cousins Judy and Jim came to visit. Now there are four children and they have to divide the cake fairly among them. How can they do it?

S: Each one gets one-fourth of the cake.

S: Each one gets three pieces.

T: Which pieces should we give to Jack, which to Jill, which to Judy, and which to Jim?

Again, let students make several suggestions. Choose one suggestion that illustrates symmetry, and color the cake with four colors.



T: Is it easy to see that all four children get the same amount of cake?

Again encourage discussion which suggests both symmetry and counting.

T: How many pieces in the whole cake?

S: 12.

T: How many pieces in one-fourth of the cake?

S: 3.

Under the first equation write a second as you say "one-fourth of twelve is three."

$$\frac{1}{2} \times 12 = 6$$

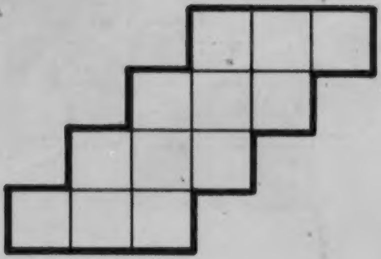
$$\frac{1}{4} \times 12 = 3$$

G13

Take down the shapes and erase the board except for these two equations. Tape up another copy of the shape.

$\frac{1}{2} \times 12 = 6$

$\frac{1}{4} \times 12 = 3$



T: Suppose that Jack and Jill had just one friend visiting them. Then there would be three children. How could they divide this cake fairly among three children?

S: Each one gets one-third.

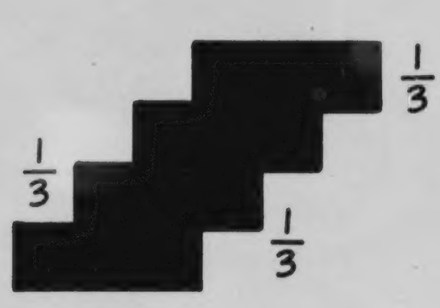
S: Each one gets four pieces.

As before, let students suggest which pieces would go to each of the three children and color the shape for one of the suggestions. Then, as before, observe that one-third of twelve is four and write the equation on the board.

$\frac{1}{2} \times 12 = 6$

$\frac{1}{4} \times 12 = 3$

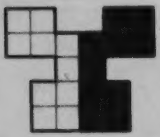
$\frac{1}{3} \times 12 = 4$

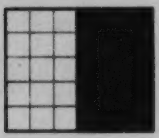


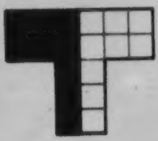
Worksheets G13*, **, ***, and **** are available for independent work.

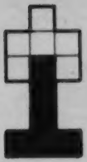
Name ______________________ G 13 *

Color one-half of each shape red.





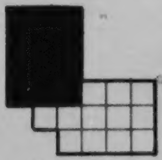


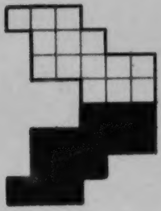


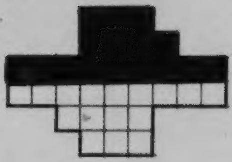
(Many solutions are possible.)

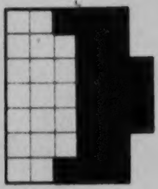
Name ______________________ G 13 **

Color one-half of each shape red.





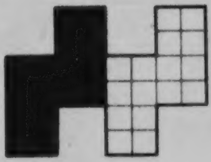


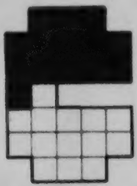


(Many solutions are possible.)

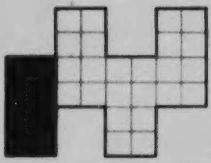
Name ______________________ G 13 ***

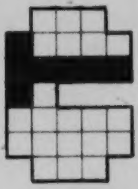
Color one-half of each shape red.





Color one-fourth of each shape blue.

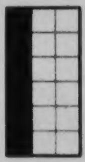


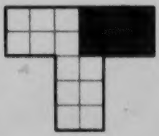


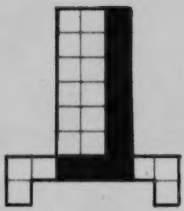
(Many solutions are possible.)

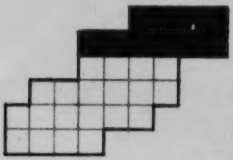
Name ______________________ G 13 ****

Color one-third of each shape green.









(Many solutions are possible.)

WORKBOOKS

Comprehensive School Mathematics Program

CSMP Mathematics for the Upper Primary Grades

Part I

Workbooks

CEMREL, Inc.

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WORKBOOKS

INTRODUCTION

There are many opportunities for the student to work individually during the course of the lessons described in the other content strands. In the Workbook strand, however, it is this individualized work which becomes the chief end of the majority of lessons. It has been our goal in this strand to provide the students with opportunities:

- to review many of the ideas they have met in other content strands;
- to apply their acquired knowledge to new situations requiring various kinds of strategic thinking;
- to learn how to read and use mathematics workbooks.

The following workbooks are provided.

7 - 12
A Catalog of Problems #1
8? - 15?
A Catalog of Problems #2
Fishing for Numbers

These five workbooks are written with problems of varying levels of difficulty all in one booklet. Approximately the first ten pages of each booklet are easy problems, the next ten to twelve pages are average level difficulty, and the last ten pages are more challenging problems. For each workbook we suggest that all students start work at the easiest level of difficulty (i.e., on page 1) and then work through as many pages as they can handle during the two lessons scheduled for that workbook. We estimate that, in an average class, about two-thirds of the students will correctly finish the first ten pages, about one-third will finish the first twenty pages, and a few will finish all or most of the booklet. These proportions will vary from class to class.

This guide contains an answer key for each workbook. The key follows an introduction to the workbook and suggested collective lessons which present the workbook to the whole class or provide a warmup activity on a problem similar to one found in the workbook.

The first lesson in this strand is based on a game, the Number Line Game. The game provides a situation where students are obliged to locate numbers on the number line and to develop a strategy for finding a mystery number given a relative location for it on the number line. This game can be played often during the year even though it appears explicitly only once as a lesson.

Several lessons present detective stories where there is a secret number and several clues which lead to the number. The clues provide review of many ideas from the other content strands as well as involve students in new situations involving patterns and counting techniques. Each detective story is paired with an episode in the continuing story of Eli the Elephant. The Eli the Elephant episodes present the students with a model of the negative integers and extend some operations on whole numbers to the integers.

Use of the Workbook Strand for Evaluation Purposes*

The workbooks provide an excellent instrument to assess the progress of your students on a weekly basis. You may not feel it is necessary to check every page and problem for each student. You should develop the sort of procedure for checking students' work with which you are most comfortable. These may include checking one or more specific pages, discussing some particular mistakes with individual students and letting them correct their work, or just looking carefully at the first few pages to be sure the students have understood the general idea of the problems in that particular book.

A progress chart is provided on pages viii and ix to help you keep track of

1) where each of your students is on any particular workbook day,
2) how much each student has completed under each title,

---

* See also Role and Use of Workbooks in Section 1 of the General Introduction.

3) your personal degree of satisfaction with the student's work on each title,

4) which students are showing improvement from one workbook to the next,

and

5) which students are having difficulty with even the easiest problems and would benefit from more individual attention during the lessons.

Here is a list of some of the important points to bear in mind for workbooks.

- All students start at the beginning of each workbook and progress as far as they can.
- All students begin a new title on the same day, even if some students have not finished work on the previous title.
- Not all students are expected to complete a given workbook. Only the faster students will reach the most challenging problems. The slower students may succeed only to do the easiest problems, although you should not assume that this will automatically be the case--surprises are not at all uncommon in this area.
- Read the introductory material for each workbook and give the short introductory collective lesson(s).

Workbook Progress Chart

Student's Name	7 - 12	A Catalog of Problems #1

Workbook Progress Chart

8? - 15?	A Catalog of Problems #2	Fishing for Numbers

W1 The Number Line Game

CAPSULE LESSON SUMMARY

Locate numbers on the number line. Define the distance between two numbers using the number line. Introduce the Number Line Game.

MATERIALS

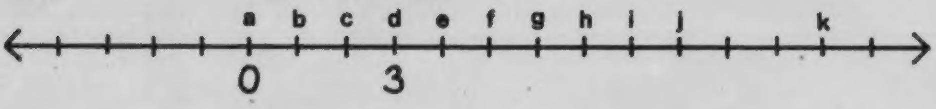
Teacher: None

Student: Worksheets W1*, **, ***, and ****; pencil

DESCRIPTION OF LESSON

Exercise 1: Introduction to the Number Line

Draw a line on the board. Graduate it into intervals, about six centimeters in length. Locate 0 at one of the marks and locate the number 3, three marks to the right.



Do not write these letters on the board. They are here just to make the description of the lesson easier to follow.

Point to some of the marks on the number line asking students to label them. A sample dialogue is given below and on the next page.

T: I have shown you where some of the numbers are on this number line. If 0 is here (point to a) and 3 is here (point to d), what number is here (point to c)?

S: 2.

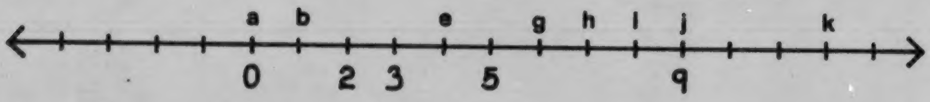
T: What number is here (point to f)?

S: 5.

W 1

T: What number is here (point to j)?

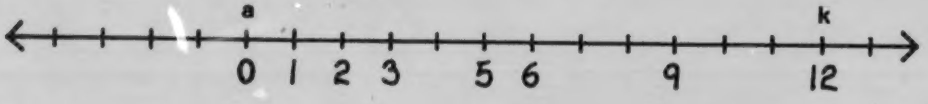
S: 9.



Do not write these letters on the board. They are here just to make the description of the lesson easier to follow.

After each of these questions, let a student point to the appropriate mark and label it.

T: Where is 1 on this number line? (At b)
Where is 6 on this number line? (At g)
Where is 12 on this number line? (At k)



Do not write these letters on the board. They are here just to make the description of the lesson easier to follow.

Point to the remaining marks between a and k, ask the class for the appropriate numbers, and label the marks yourself.

Some students may suggest erasing the line to the left of "0". If this happens, say, "There are many numbers which are less than 0. You will meet them another day."

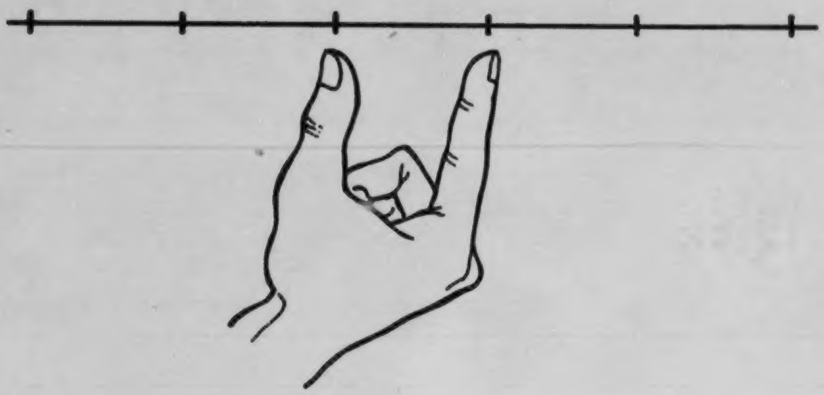
T: The number line goes on and on in both directions. That is what the arrowheads mean. Point to where 20 would be on this line.

Some students may protest that 20 is not on the part of the line drawn, but most will point to the right-hand side of the picture.

T: Point to where 500 would be on this line. (To the right)
Think of a very large number. Point to where it would be on this line.

T: The distance between 0 and 1 is 1.

Use your thumb and index finger to indicate one space.



T: The distance between 4 and 6 is 2.

Use your thumb and index finger to count two spaces.

T: What is the distance between 9 and 12 ?

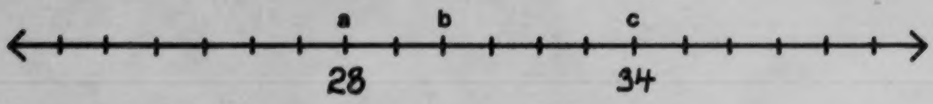
S: 3.

Ask for a volunteer to count the spaces. Do not insist that the students use the "thumb and index finger" method shown in the previous diagram so long as their methods are accurate. Continue this activity with the following or similiar questions.

T: What is the distance between 9 and 4? (5)
What is the distance between 5 and 12? (7)

W 1

Erase the numerals but not the line. Label one of the marks "28" and locate 34 six marks to the right.



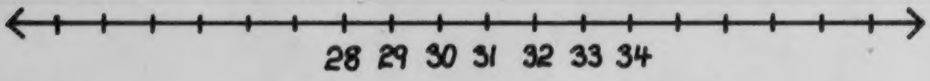
Do not write these letters on the board. They are here just to make the description of the lesson easier to follow.

T: If 28 is here (point to a) and 34 is here (point to c), what number is here (point to b)?

S: 30.

Let a student label the mark "30".

Continue in this manner until all the marks between "28" and "34" are labeled.



T: What are some numbers which will be to the left of 28 on this number line?

S: 27.

S: 15.

T: What are some numbers which will be to the right of 34?

S: 100.

S: 36.

S: 40.

T: Where is 20 on this number line?

S: To the left of 28.

T: What are some numbers between 32 and 27?

S: 30.

S: 28.

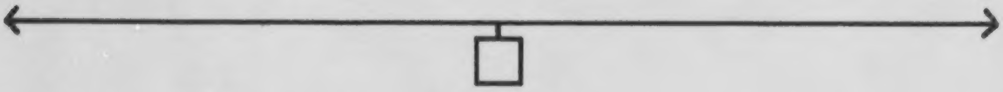
Continue with similar questions as long as the students are interested. Lessons on the number line should be light and move briskly. Students will become accustomed to the number line as they do more activities with it.

Exercise 2: The Number Line Game

The Number Line Game is one which may be played frequently throughout the year whenever you finish a lesson early or have some extra time. Each game takes approximately five to ten minutes. Several variations of the game are given at the end of this lesson. Use these variations when your class is ready for them.

T: I'm going to show you a game you can play on the number line. Some of you may have played it in the first grade. I'm thinking of a number between 0 and 50. I'll show you where it is on the number line; my number is hidden in this box.

Draw a line on the board and draw a box for the secret number.



NOTE: Begin with the box near the center of your line. For convenience in locating numbers, you may wish to extend the line during the game.

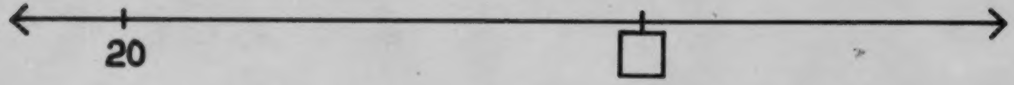
W 1

T: Guess what my secret number is.

This is a description of a possible game. Suppose the secret number is 37.

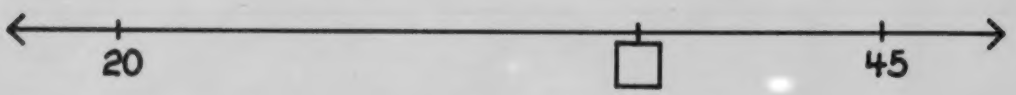
S: 20.

T: I'll show you where 20 is on this number line. It might not be exactly in the right place, but I'll try to make it as close as possible.



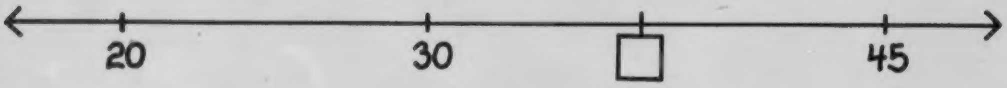
T: 20 is less than my number.

S: 45.



T: 45 is more than my number.

S: 30.



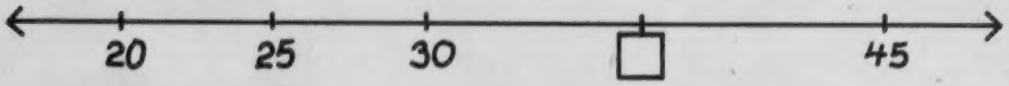
T: 30 is less than my number.

S: 50.

T: My number is between 0 and 50. 50 cannot be my number.

Any time the guess is a number which is not between 0 and 50, do not write anything. Just remind the students that your number is between 0 and 50, and ask someone else to guess.

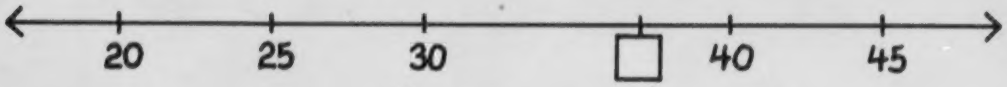
S: 25.



T: 25 is less than my number.

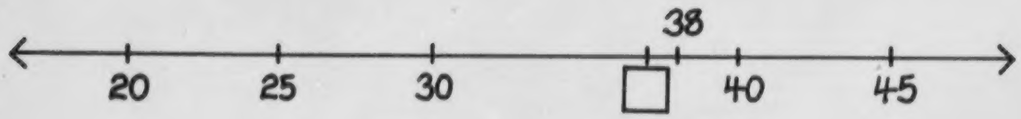
NOTE: Although a guess may give no new information, such as 25 in this sample game, do not criticize or comment except to say that it is more or less than your number.

S: 40.



T: 40 is more than my number.

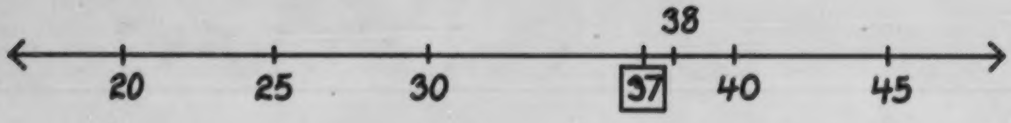
S: 38.



T: 38 is more than my number.

W 1

S: 37.



T: You have discovered my secret number.

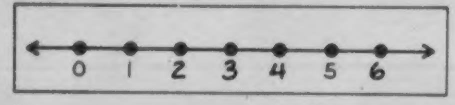
Play the game a few more times with secret numbers between 0 and 50. Let the student who guesses your number in a game choose the secret number for the next game and whisper it to you. Then ask the student to keep the number secret and not to play in that game.

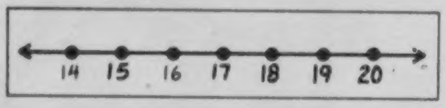
Worksheets W1*, **, ***, and **** are available for independent work.

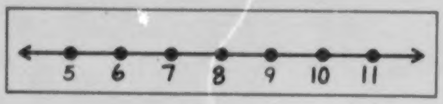
Name

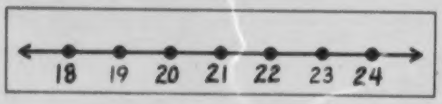
W1 *

Label the dots on these number lines.




14 15 16 17 18 19 20

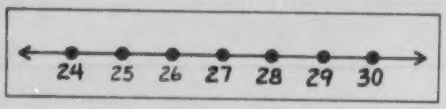


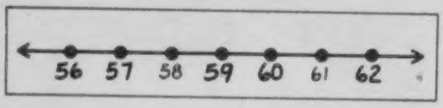

18 19 20 21 22 23 24

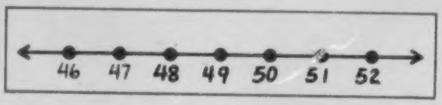
Name

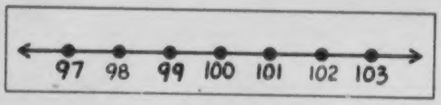
W1 **

Label the dots on these number lines.


24 25 26 27 28 29 30


56 57 58 59 60 61 62

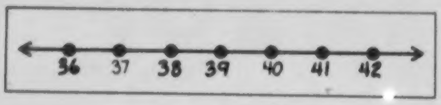

46 47 48 49 50 51 52

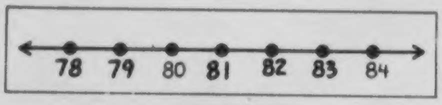

97 98 99 100 101 102 103

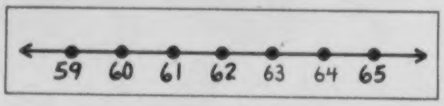
Name

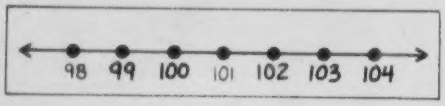
W1 ***

Label the dots on these number lines.


36 37 38 39 40 41 42


78 79 80 81 82 83 84

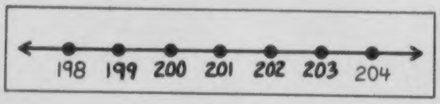

59 60 61 62 63 64 65

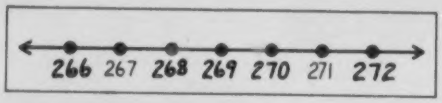

98 99 100 101 102 103 104

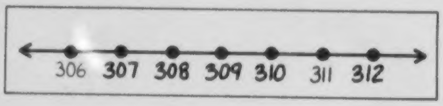
Name

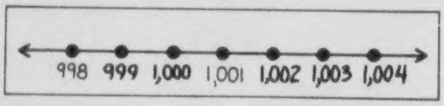
W1 ****

Label the dots on these number lines.


198 199 200 201 202 203 204


266 267 268 269 270 271 272


306 307 308 309 310 311 312


998 999 1,000 1,001 1,002 1,003 1,004

VARIATIONS OF THE NUMBER LINE GAME

Variation 1

When your class is comfortable with the game using numbers between 0 and 50, you can expand the range of possible numbers. Some possibilities are:

- numbers between 0 and 100;
- numbers between 100 and 200;
- numbers between 0 and 200;
- numbers between 0 and 500;
- numbers between 500 and 1,000.

Variation 2

As the students become familiar with the number line game and negative numbers, you can expand the range of possible numbers to include negative numbers. Some possibilities are:

- numbers between $\widehat{50}$ and 50;
- numbers between $\widehat{100}$ and 100.

Variation 3

As the students become familiar with numbers such as $2\frac{1}{2}$, occasionally you can choose a non-integer number as your secret number. It is best to limit the range of possible numbers to those between 0 and 10 when you use this variation.

Variation 4

When you are using only whole numbers for your range of possible numbers, ask a student to choose a secret number and to judge the guesses of the other students. The student who chooses the secret number tells the class whether a guess is more or less than the secret number. You label a mark on the number line for each guess and, if necessary, help the student to judge. Remind the student judges not to give additional clues.

Variation 5

Divide the class into two teams. The teams alternate turns and the members of each team take turns guessing the secret number.

W 2

W2 Detective Story #1 / Introduction to Eli's Magic Peanuts

CAPSULE LESSON SUMMARY

Solve a detective story for which the secret number is an odd number and where the other clues involve an arrow picture and the Minicomputer. Tell the story of Eli the Elephant who discovers magic peanuts; when a magic peanut meets a regular peanut, both disappear much to Eli's confusion.

MATERIALS

Teacher: Colored chalk; Minicomputer set

Student: Paper; pencil; Worksheets W2* and **

DESCRIPTION OF LESSON

Exercise 1: Detective Story

Before the lesson, write a large "21" on a slip of paper and fold the paper so that the "21" is hidden.

Ask the class what a detective does. Guide the discussion to include the idea that a detective tries to uncover secrets by following clues. Tell the students that they are going to be detectives today, and they will be able to discover a secret number if they follow the clues.

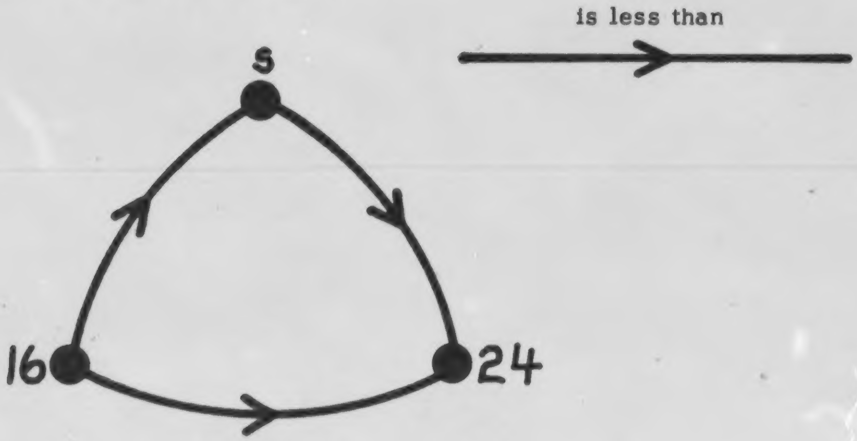
T: The name of a secret number is written on this paper. Listen carefully to my story and you can discover what it is. The first clue I will give you is that the secret number is an odd number. What are some odd numbers?

You may need to explain odd and even numbers briefly. When most of the class understands what an odd number is, begin listing the positive odd numbers in order on the board with the students' assistance. Start with 1 and continue to at least 25. Mention that you could continue writing odd numbers all day, but instead you will put three dots at the end of your list meaning "and so on".

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, ...

T: Now we know that the secret number is one of these numbers, but we do not know which one. We need another clue.

Draw this arrow picture on the board.



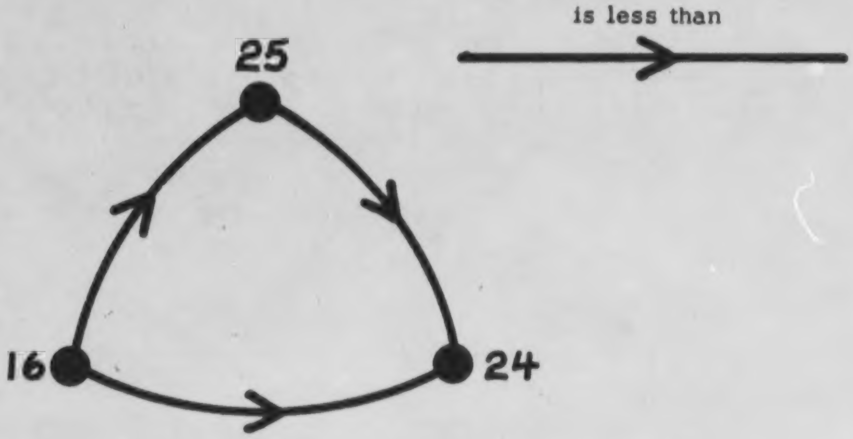
T: The secret number is here (point to "s") in this arrow picture. What are the red arrows for?

S: Is less than.

Point to the dot labeled "s" and remind the class that the secret number is an odd number. Choose an odd number between 0 and 26 and ask if it could be the secret number. If the class disagrees on whether a number could be the secret number or not, put that number in the arrow picture and ask a student to read the number story told by each arrow and its starting and ending numbers.

W 2

For example, suppose the class is not sure whether s could be 25 or not.



Trace the arrow from 16 to 25.

S: 16 is less than 25.

T: Is that true?

S: Yes.

Trace the arrow from 16 to 24.

S: 16 is less than 24.

T: Is that true?

S: Yes.

Trace the arrow from 25 to 24.

S: 25 is less than 24.

T: Is that true?

S: No.

T: So our secret number cannot be 25.

Erase "25" from the arrow picture and cross it out of the list of odd numbers.

As the class discovers which numbers in the list of odd numbers cannot be the secret number, cross them out. Continue until every odd number in your list has been considered. Ask if there are any other odd numbers which could be the secret number. Commend a student who concludes that none of the numbers (indicated by the three dots at the end of the list) could be the secret number because they are all more than 24. If necessary, supply this information yourself and cross out the three dots at the end of the list.

~~1~~, ~~3~~, ~~5~~, ~~7~~, ~~9~~, ~~11~~, ~~13~~, ~~15~~, 17, 19, 21, 23, ~~25~~, ~~27~~, ~~...~~

Erase everything which has been crossed out. Display three Minicomputer boards.

T: Now we know that the secret number is 17 or 19 or 21 or 23. Listen carefully because I will give you only one more clue. The secret number can be written on the Minicomputer with exactly two checkers.

Ask the students to write the secret number on a piece of paper and check a few of their answers.

T: Which is the secret number?

S: 21.

Invite a student to put 21 on the Minicomputer with two checkers. Ask how many checkers would be needed to put each of the numbers 17, 19, and 23 on the Minicomputer. Show the class that "21" is written on your paper.

W 2

Exercise 2: Eli's Magic Peanuts

Veteran students will have met Eli in the first grade. They can help you tell this story.

T: There is an elephant named "Eli". Eli is always very hungry. What do you think is Eli's favorite food?

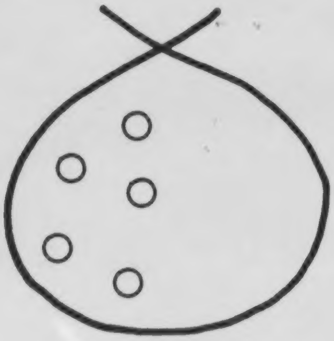
Accept some answers from the class and then tell them that Eli's favorite food is peanuts.

T: Eli likes peanuts so much that he always keeps a little bag of peanuts with him wherever he goes. One day as Eli was walking through the jungle, he found a plant that looked a lot like a peanut plant, but not exactly like a peanut plant. Eli pulled up this strange plant and discovered five peanuts. Eli didn't know it but these were magic peanuts. He picked those five peanuts and put them in his bag with five regular peanuts. What do you think those magic peanuts did?

Discuss possibilities, encouraging the students to use their imaginations.

T: Whenever a magic peanut came near a regular peanut it would do something very strange. All of a sudden both peanuts would magically disappear.

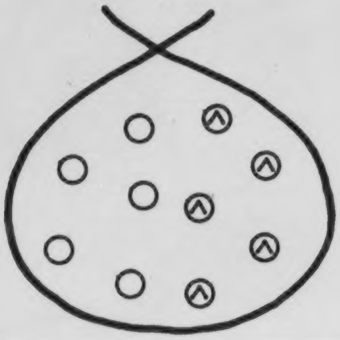
Draw this picture on the board using white chalk.



T: This is Eli's bag of peanuts with five regular peanuts in it. How many magic peanuts did Eli put in his bag?

S: Five.

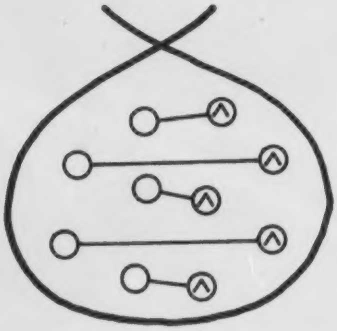
Draw five " Ⓐ " for the magic peanuts using white chalk, saying, "These are for the magic peanuts."



T: When Eli arrived home he was really hungry from walking all day in the jungle. He decided to eat some peanuts. When he took out his bag of peanuts, he had a big surprise waiting for him. How many peanuts do you think he found?

S: None.

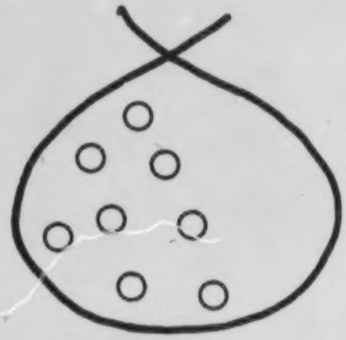
Ask several students to explain their answers. Be sure the class understands that all the peanuts will have disappeared because there was one magic peanut for every regular peanut. Draw connecting lines between the regular and magic peanuts to show how they could be paired up.



W 2

T: Poor Eli. He was very puzzled. He didn't know what we know about magic peanuts. He couldn't figure out where his peanuts had gone.

Erase the board and draw this picture.



T: Here is a bag of peanuts. How many regular peanuts are in it ?

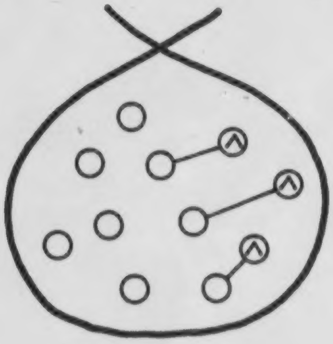
S: Eight.

T: Now Eli is going to put three magic peanuts into his bag. What will happen?

S: The three magic peanuts and three of the regular peanuts will disappear.

S: There will be three peanuts fewer in the bag then Eli started with.

Draw the magic peanuts and lines connecting the magic peanuts with three of the regular peanuts.



T: Now we are going to write the calculation for this story. How many regular peanuts did Eli have?

S: Eight.

Write "8" on the board.

T: And how many magic peanuts did he put in the bag?

S: Three.

T: To show that they are three magic peanuts, I will draw a hat on the "3".

$$8 + \hat{3}$$

T: When Eli opens the bag, how many peanuts will he see?

S: Five.

T: Regular or magic?

S: Regular.

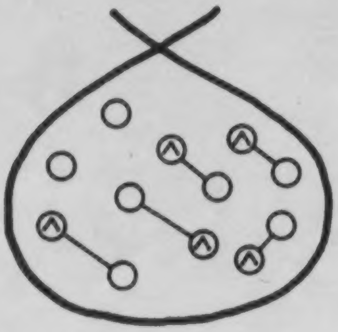
T: So having eight regular peanuts and three magic peanuts is the same as having five regular peanuts in the bag.

Finish the number story on the board.

$$8 + \hat{3} = 5$$

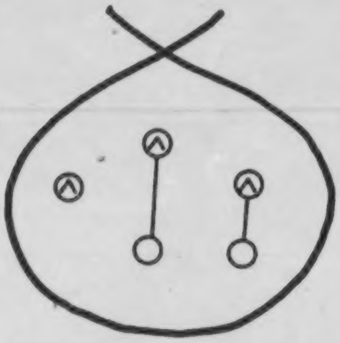
W 2

Now draw a new picture on the board and ask the same series of questions. Use the following picture or make up one of your own.



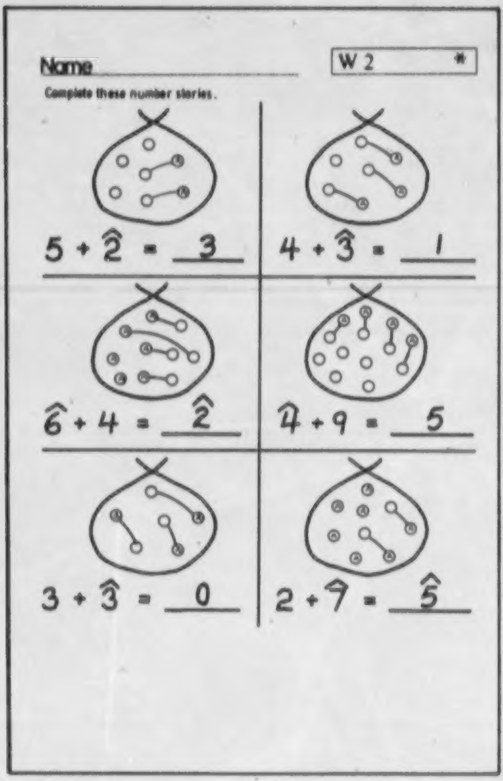
$$7 + \hat{5} = 2$$

Do another example in which Eli finds magic peanuts when he opens the bag. For example,



$$\hat{3} + 2 = \hat{1}$$

Worksheets W2* and ** are available for independent work. Tell the students to finish the calculations for each of Eli's bags of peanuts. The students should draw lines connecting regular and magic peanuts themselves. Ask students who finish quickly to draw their own stories about Eli's magic peanuts.



Name

W 2 *

Complete these number stories.

$5 + \hat{2} = \underline{\ 3\ }$

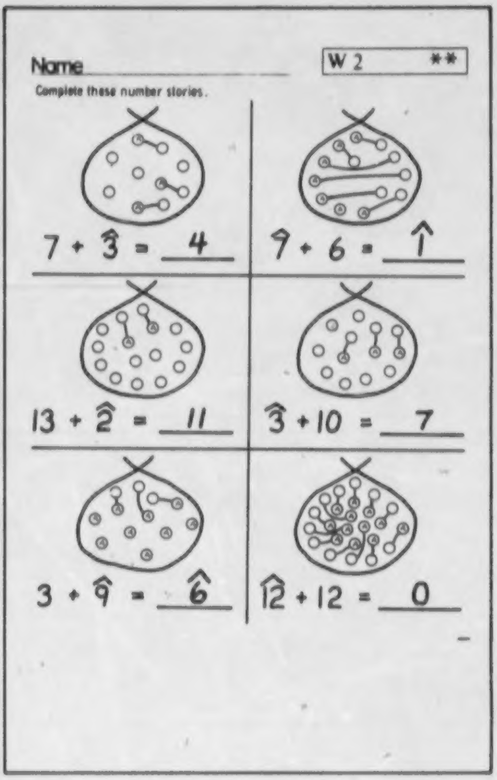
$4 + \hat{3} = \underline{\ 1\ }$

$\hat{6} + 4 = \underline{\ \hat{2}\ }$

$\hat{4} + 9 = \underline{\ 5\ }$

$3 + \hat{3} = \underline{\ 0\ }$

$2 + \hat{7} = \underline{\ \hat{5}\ }$



Name

W 2 **

Complete these number stories.

$7 + \hat{3} = \underline{\ 4\ }$

$\hat{7} + 6 = \underline{\ \hat{1}\ }$

$13 + \hat{2} = \underline{\ 11\ }$

$\hat{3} + 10 = \underline{\ 7\ }$

$3 + \hat{9} = \underline{\ \hat{6}\ }$

$\hat{12} + 12 = \underline{\ 0\ }$

W 3

W3 7 - 12 (Lesson 1)

CAPSULE LESSON SUMMARY

Find many names for the number 12, using the Minicomputer to suggest some of them. Pose the problem: Is the number 12 on a +2 road which contains the number 15 or on a +2 road which contains the number 16? Begin the Workbook "7 - 12". In the first section of this workbook, find the number 7 on each page. In the second section, find the number 12 on each page.

MATERIALS

Teacher:	Minicomputer set; colored chalk
Student:	Workbook "7 - 12"; pencil

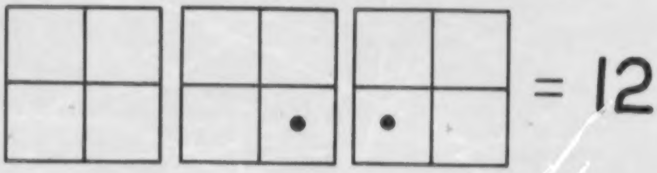
DESCRIPTION OF LESSON

Call on several students to respond to each of these questions:

T: 12 is more than . . . ?

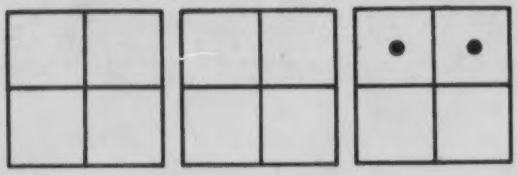
12 is less than . . . ?

Display three Minicomputer boards and ask someone to put 12 on the Minicomputer.



T: Who can put 12 on the Minicomputer in a different way?

Perhaps a student will suggest this configuration.



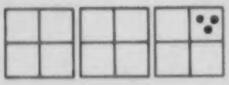
T: How do you see 12 on the Minicomputer?

S: As "8 + 4".

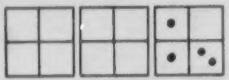
Write an appropriate number story on the board.

$$8 + 4 = 12$$

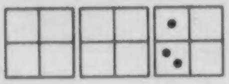
Continue asking for other ways to put 12 on the Minicomputer until you have at least four number stories about 12 on the board. Several examples are given below.



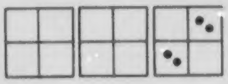
$= 3 \times 4 = 12$



$= 8 + 2 + 1 + 1 = 12$



$= 8 + 2 + 2 = 12$



$= 2 \times 6 = 12$

Put away the Minicomputer boards.

T: What are some other names of 12?

S: "15 - 3".

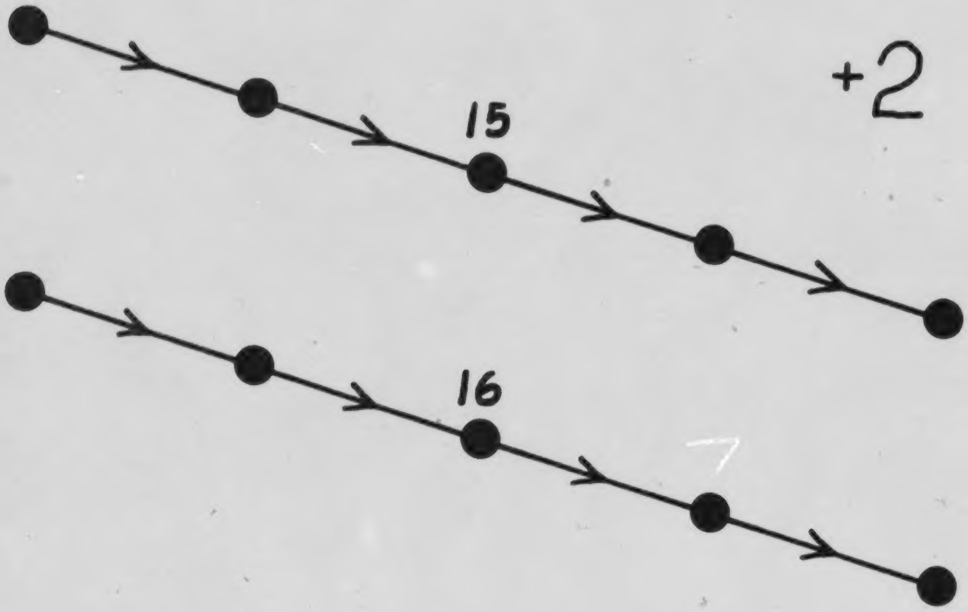
W 3

S: "(2 x 3) + 6".

Encourage the students to suggest many names for 12 and record them on the board. If expressions involving subtraction or multiplication are not suggested, suggest some of these yourself.

T: You have found many names for the number 12. Can you find 12 in this arrow picture?

Draw this arrow picture on the board.



T: Do you think the number 12 is in this arrow picture? If you think it is, which road is it on?

Ask a few students to explain their choice of either the top or the bottom road. If many students believe that 12 is in the top road, collectively label the dots of this road and discover that 12 is skipped.

When your class has decided that 12 cannot be in the top road, direct their attention to the bottom road. Ask whether 12 would be to the right or to the left of "16" and let a few students explain their answers. Collectively label the dots until 12 has been found.

It is possible that your class will need to label only two dots to find 12, but they may wish to label the other dots. Make it clear that it is alright to label the other dots, but not necessary because your objective was only to find 12.

Distribute copies of the Workbook "7 - 12".

Explain to the students that the workbook has two sections. In the first section they are to find the number 7 on each page. In the second section they are to find the number 12 on each page.

NOTE: Once students have found and have labeled the dot for 7 (12 in the second section) on a page, they do not need to label any other dots on that page but may do so if they wish.

At the end of the class period, collect the workbooks for use in Lesson W4.

W 4

W4 7 - 12 (Lesson 2)

CAPSULE LESSON SUMMARY

Continue working in the Workbook "7 - 12". (This is the second of two lessons using this workbook.) For entry classes, introduce the symbols " > " and " < " with a story of a fish named "Goldy".

MATERIALS

Teacher: (If needed) Minicomputer set; colored chalk
Student: Workbook "7 - 12"; pencil

DESCRIPTION OF LESSON

If you have an entry class which has not been exposed to the symbols " > " and " < ", present a story about a fish named "Goldy". The description of this activity follows. Afterward, distribute copies of Workbook "7 - 12" and let the students work independently for the remainder of the lesson.

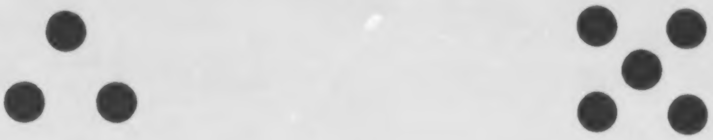
Veteran classes should begin with the Workbook "7 - 12". Allow about 30 minutes for independent work. Spend the remaining time playing the Number Line Game.

At the end of the lesson, collect the workbooks for your review.

Introduction to " < " and " > " (for entry classes)

T: Goldy is a very large fish who lives in a very large aquarium with smaller fish. The smaller fish swim together in "schools". Since Goldy is such a large fish, she gets very hungry and eats the small fish.

Draw this dot picture on the board.



T: Each dot is for a fish. How many fish are in this school (point to the dots on the left)?

S: Three.

T: How many fish are in this school (point to the dots on the right)?

S: Five.

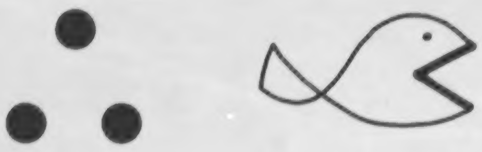
T: If Goldy heads toward one school of fish, the other school swims away. So, if you were Goldy and very hungry which school would you go after?

S: The school with five fish.

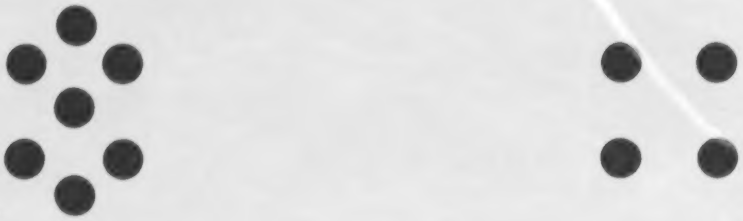
T: This is how we'll show that Goldy goes after that school.

W 4

Draw a simple picture of Goldy as in the next picture.



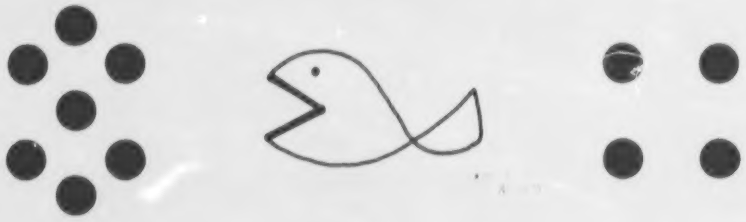
Draw this picture on the board.



As you point to each school of fish, ask, "How many fish are in this school?"

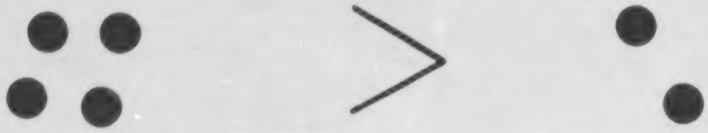
T: Would someone come to the board and point to the school of fish that Goldy would eat?

Draw Goldy with an open mouth ready to eat the larger school.



Tell the students that from now on you are just going to draw Goldy's mouth.

Do as many examples like the following as you feel are necessary, each time inviting a student to draw Goldy's mouth.



T: From now on, instead of drawing dots for the fish, I will write a numeral to show how many fish there are in each school.

Write these numerals on the board.

7 5

T: How many fish are in this school? (7) How many fish are in this school? (5) Who would like to come up and draw Goldy's mouth? Goldy is going to eat up the school with more fish. 7 is more than 5.

Do several examples like the above, asking the students to read the number story each time. For example, "2 < 4" is read "Two is less than four", and "12 > 10" is read "Twelve is more than ten."

Now write "9" on the board twice.

9 9

T: Goldy can't decide which school to eat, so we'll write:

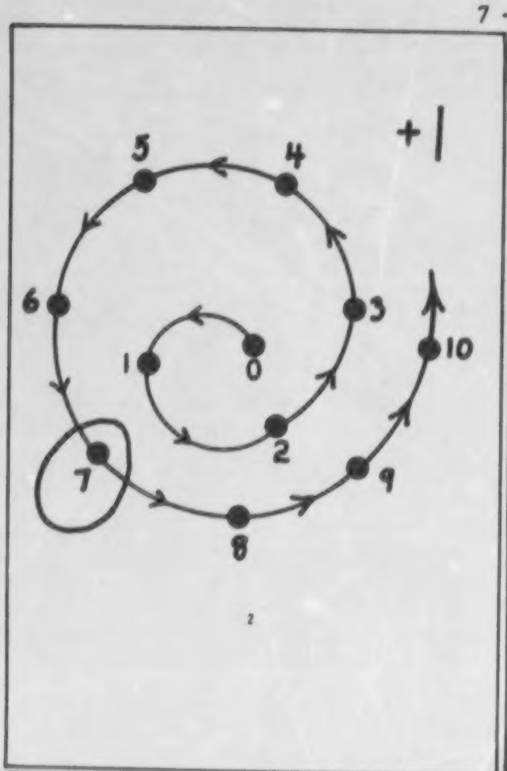
$$9 = 9$$

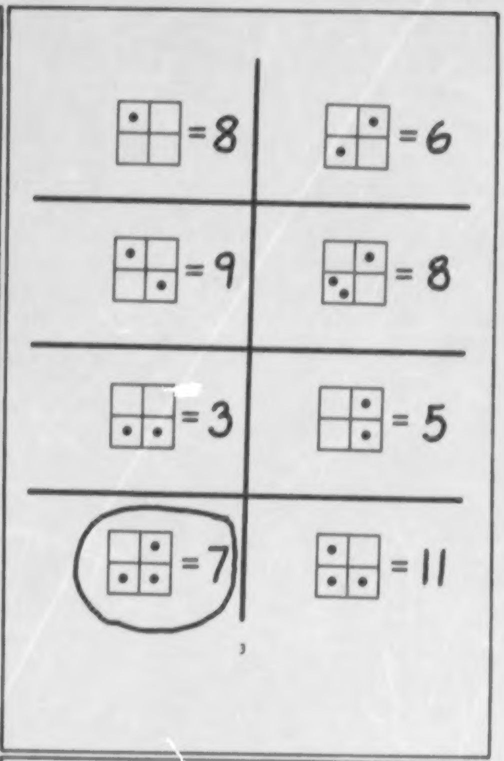
W 4

Do a few more examples including at least one of the type "7 + 2 > 8". No worksheets are included for this lesson, but you may make some if you wish. We suggest that you write these problems on the board and ask the students to copy them onto their papers. Tell them to be sure to leave enough space between the numerals in each pair so as to be able to insert the appropriate symbol.

8	10	1 + 1	3
7	9	2	2 + 1
3	6	2 + 3	5
5	5	2 + 3	1
10	2	6	2 + 4
4	1	9	8
5	8	6 + 3	6 + 1

7 - 12


+1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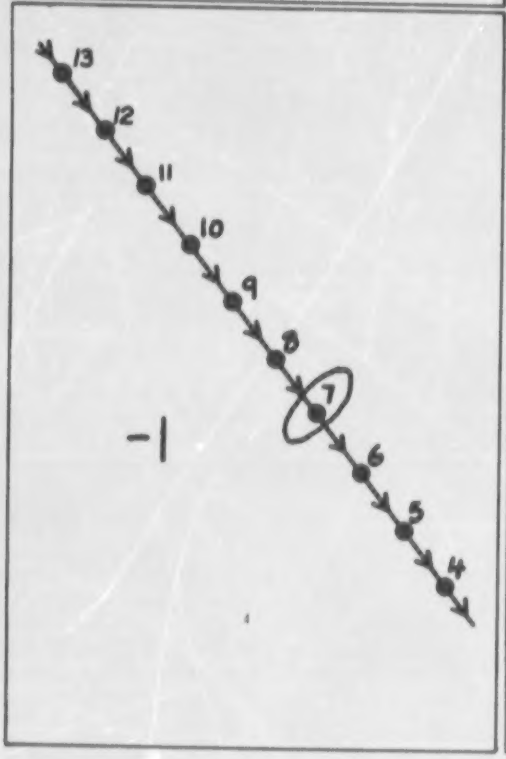
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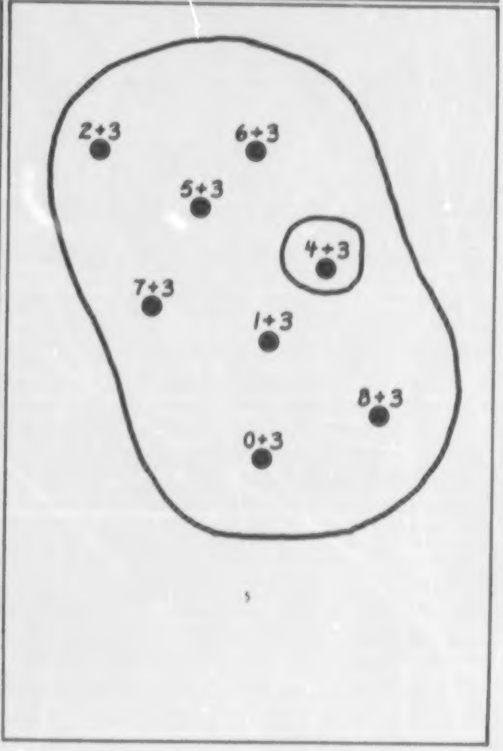
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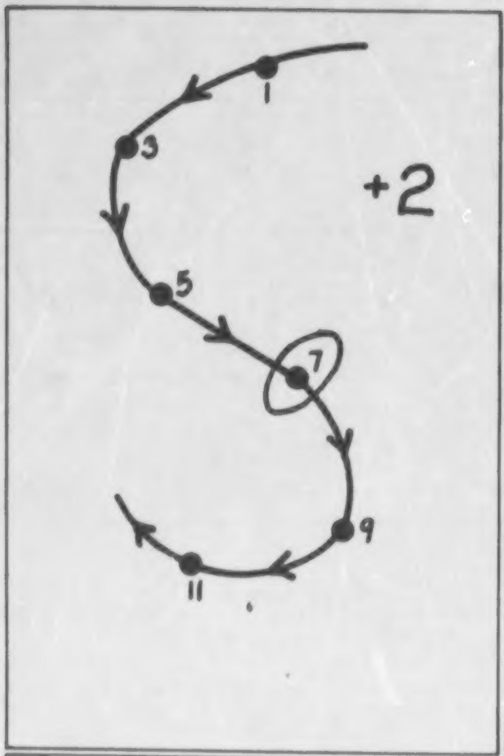
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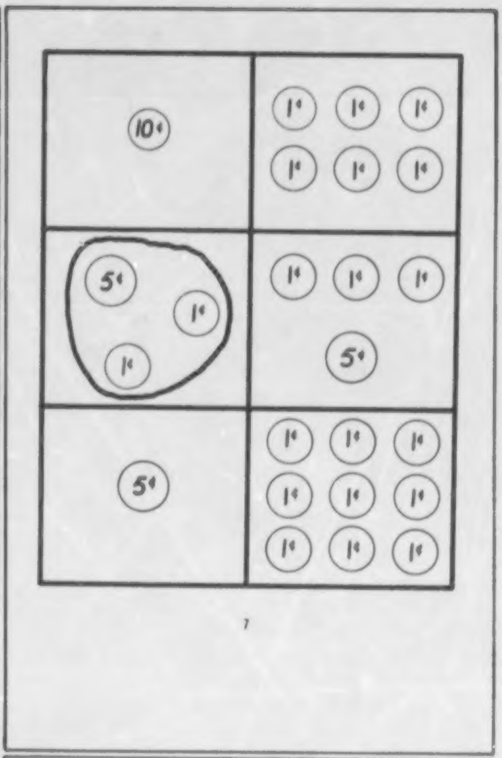
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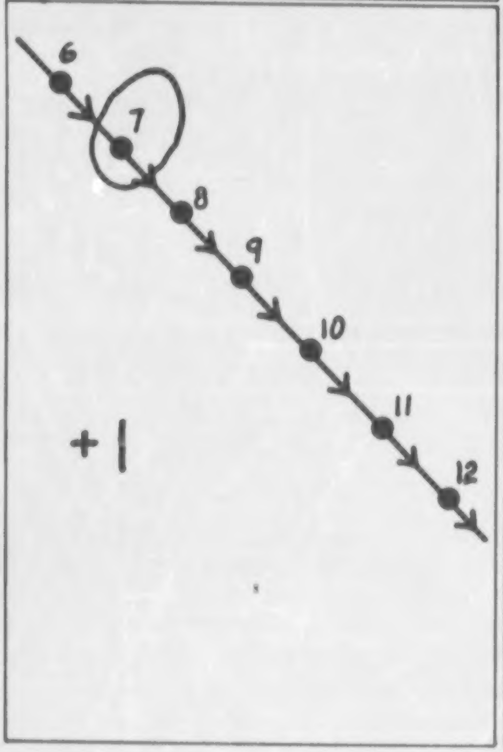
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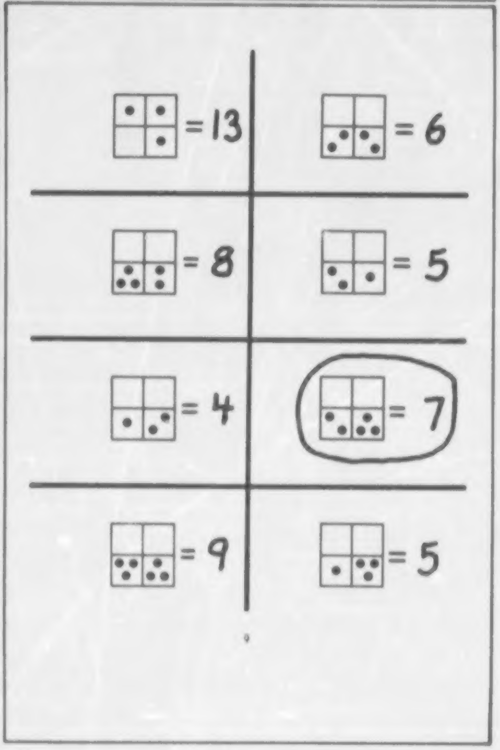
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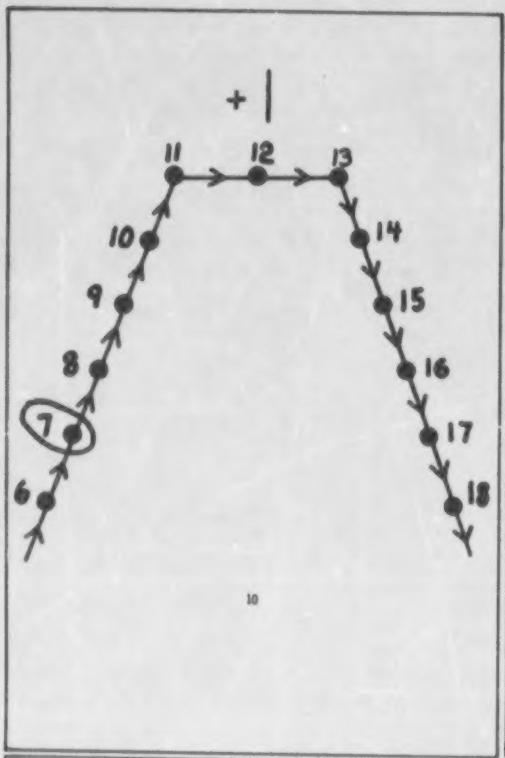
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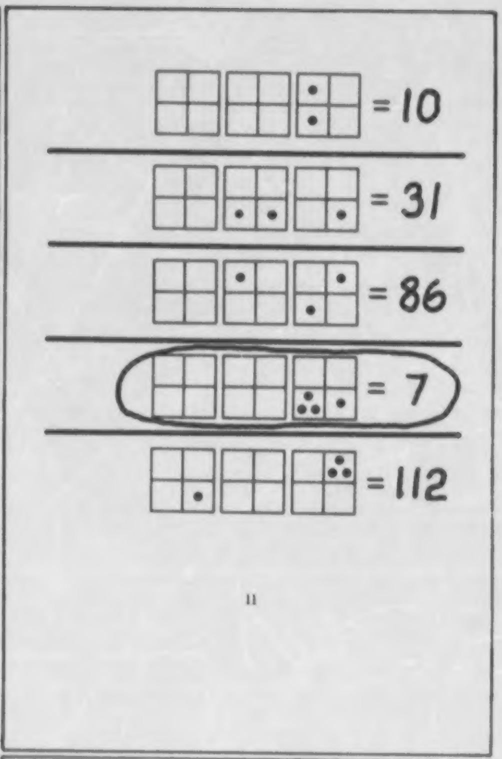
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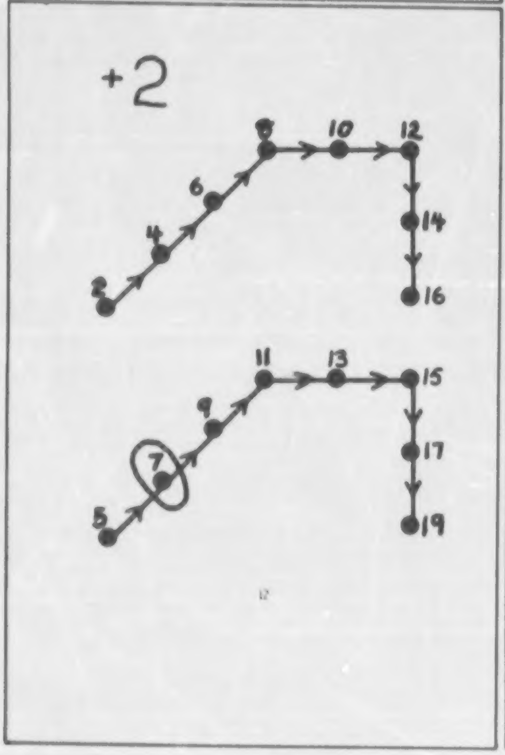
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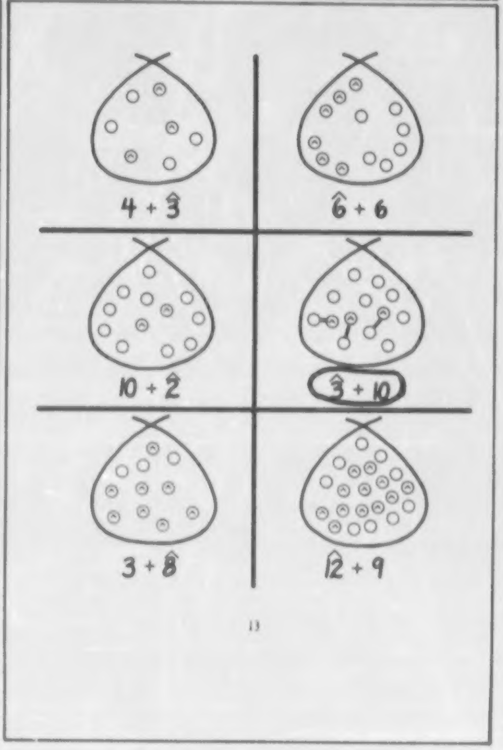
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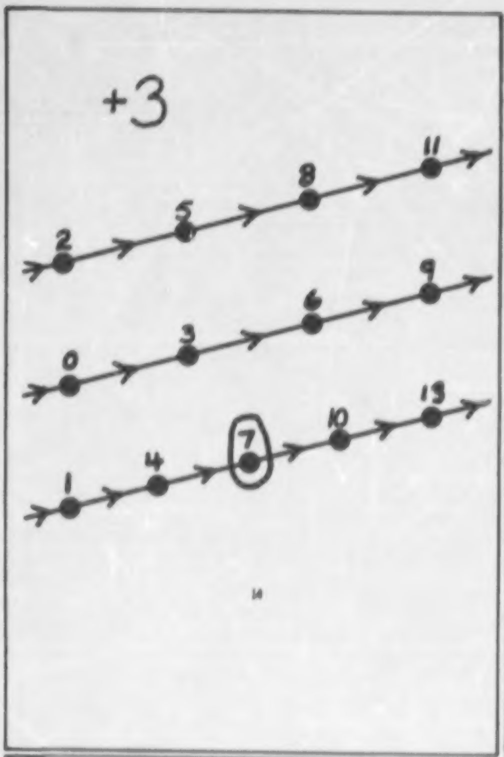
13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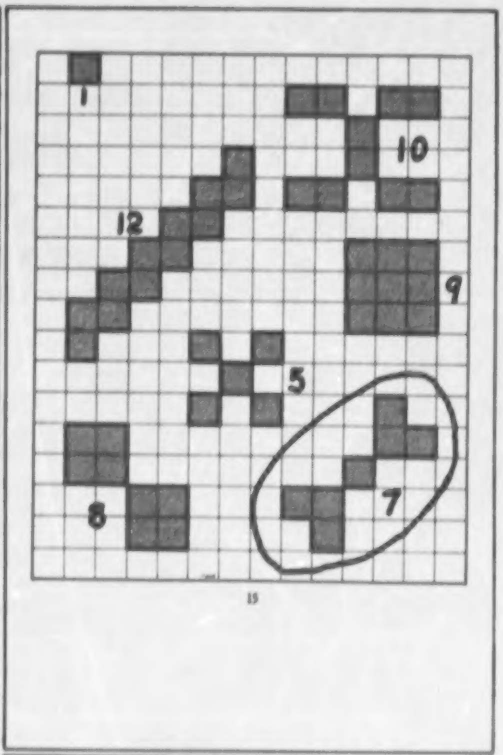
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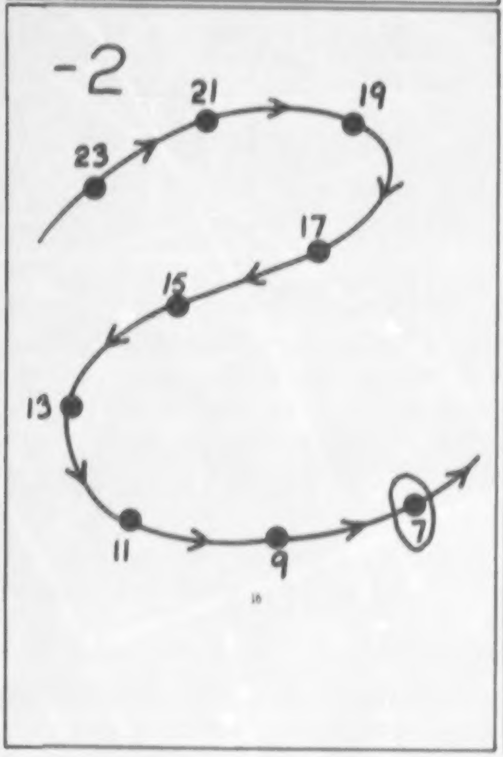
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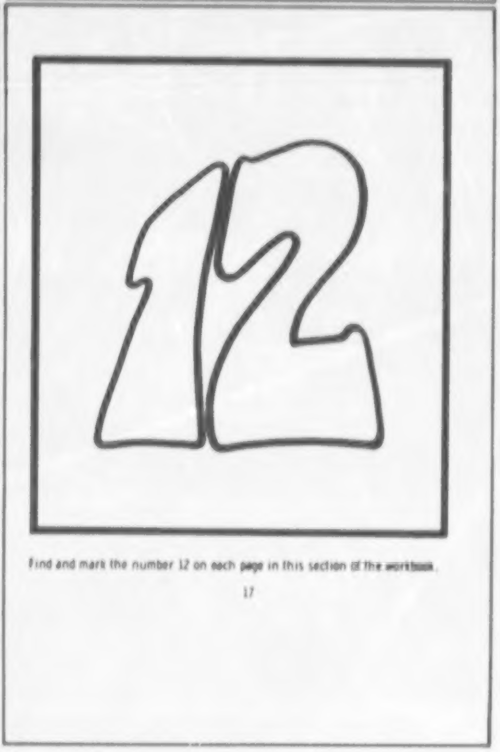

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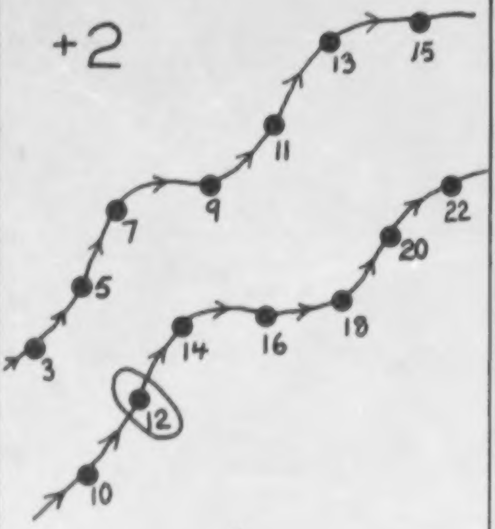
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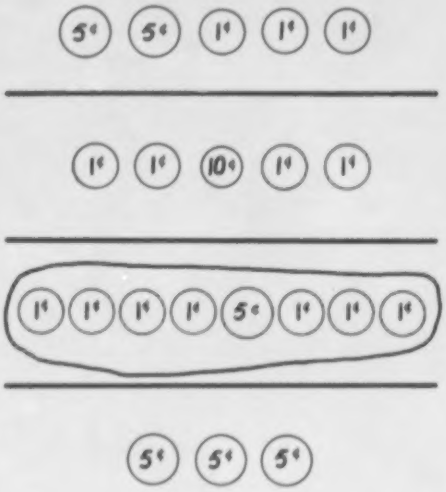
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1¢ 1¢ 1¢ 1¢ 5¢ 1¢ 1¢ 1¢
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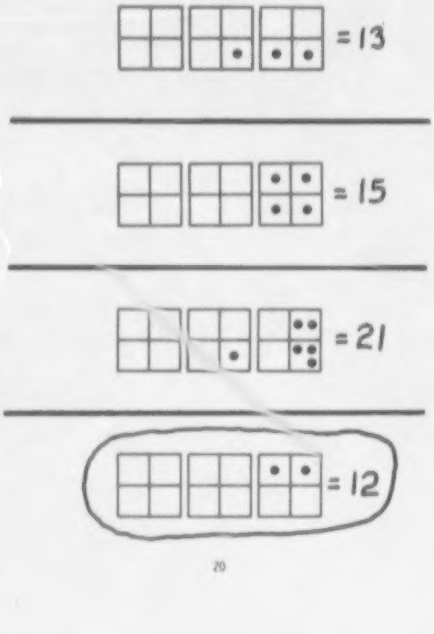
19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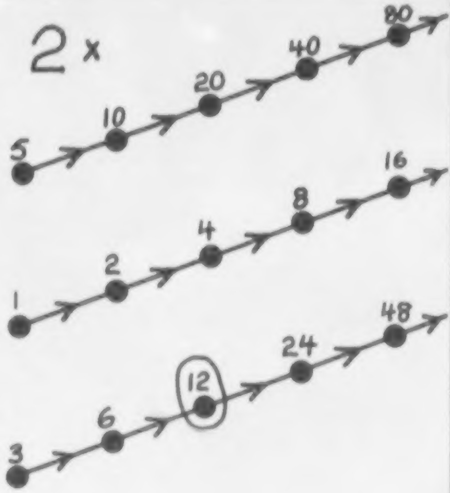
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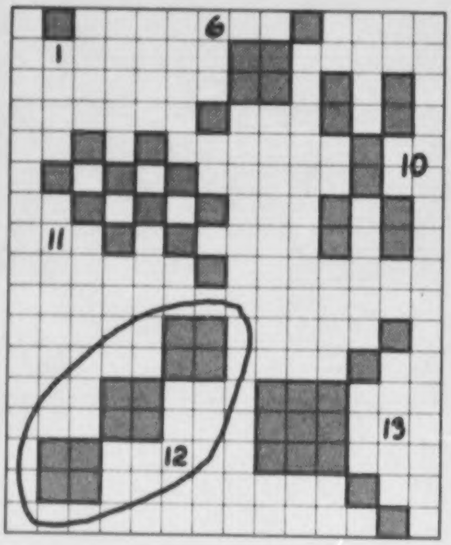

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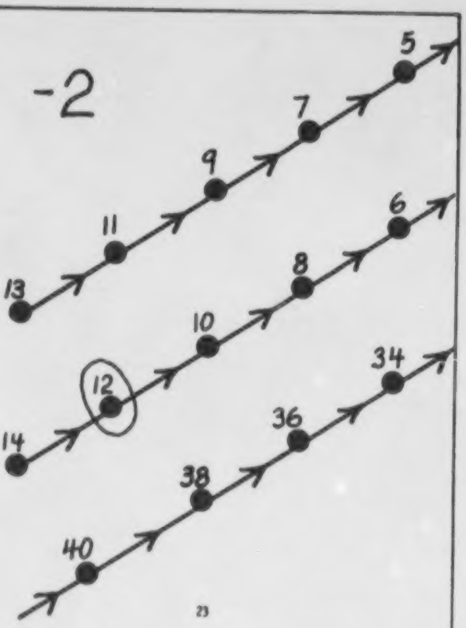
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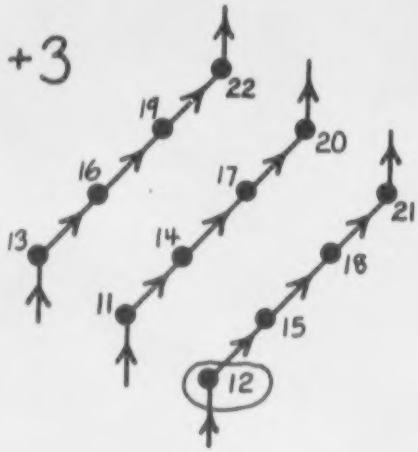
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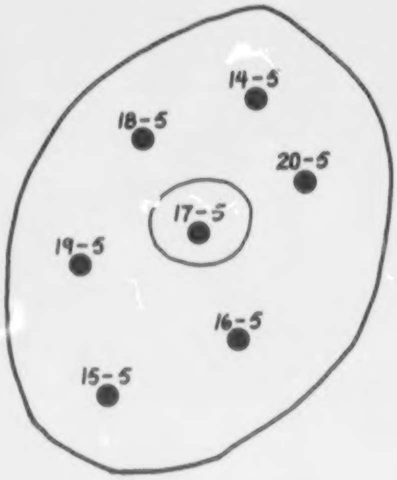
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19-5
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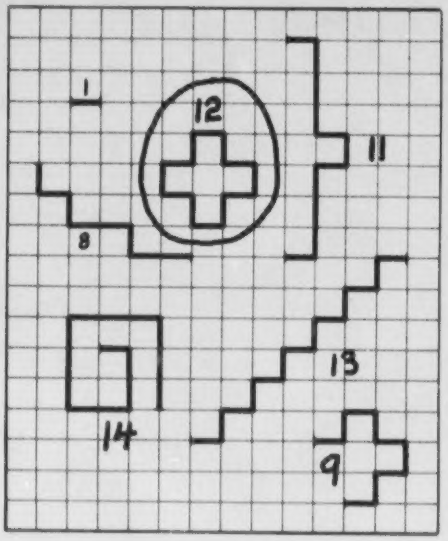
15-5

25



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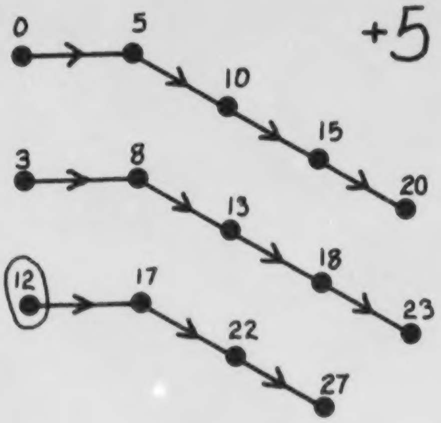
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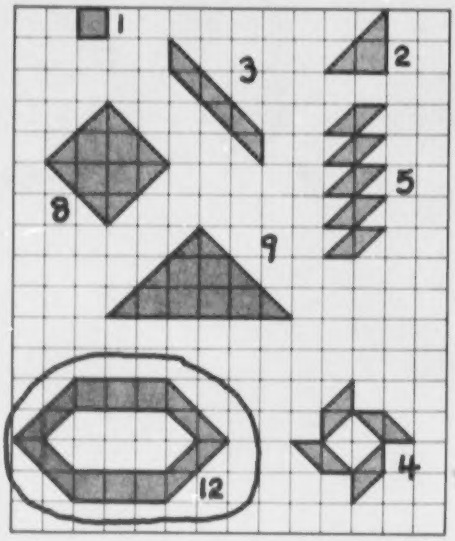
27



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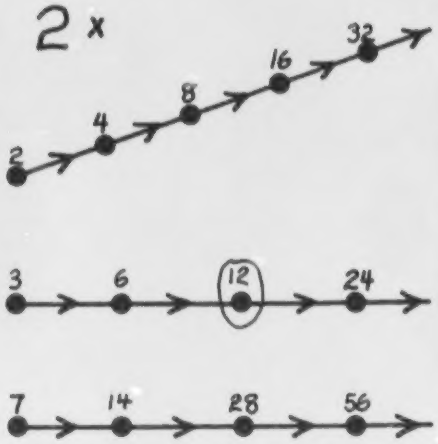
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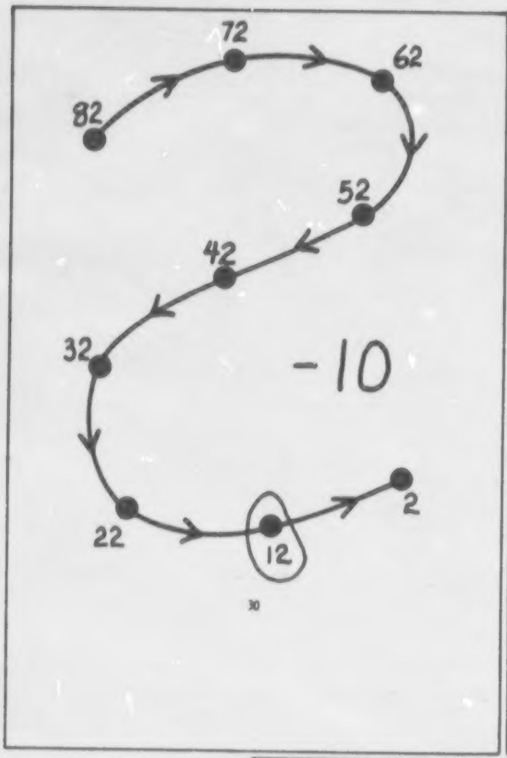
82
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62
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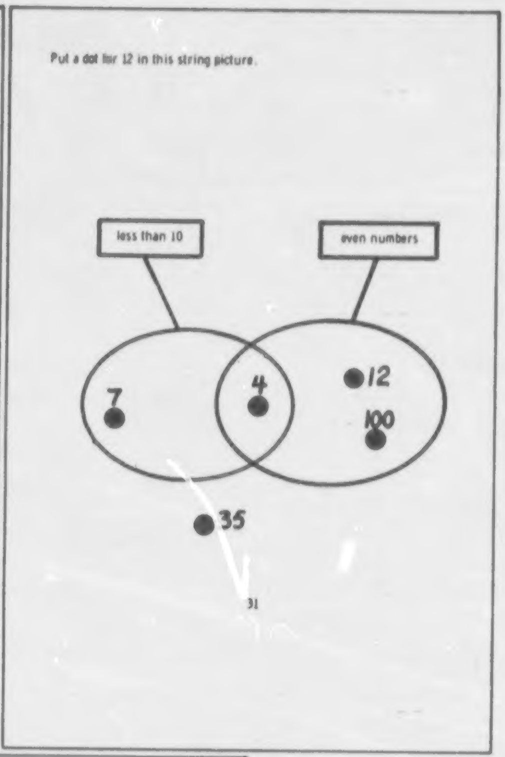
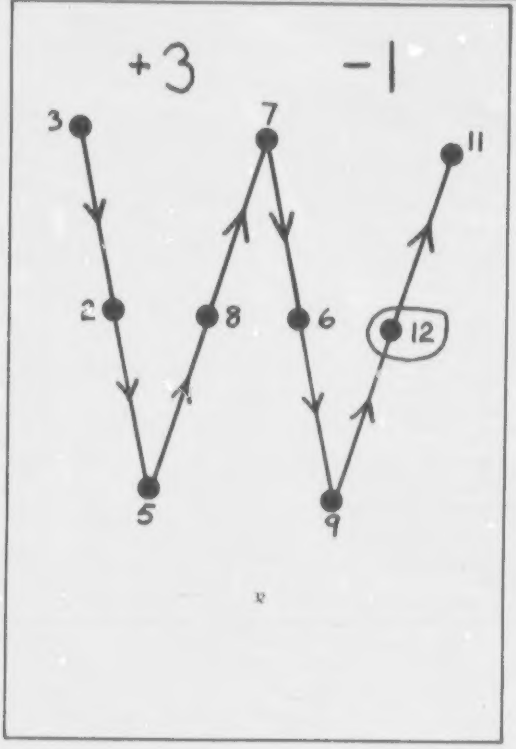
30

Put a dot for 12 in this string picture.


less than 10
even numbers
7
4
12
100
35

31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32

W5 Eli the Elephant / Detective Story #2

CAPSULE LESSON SUMMARY

Introduce negative checkers to represent the number of magic peanuts in Eli's bag. Solve a detective story with clues involving the Minicomputer, an arrow picture, and a string diagram.

MATERIALS

Teacher: Minicomputer set; colored chalk
Student: Paper; pencil; Worksheet W5

DESCRIPTION OF LESSON

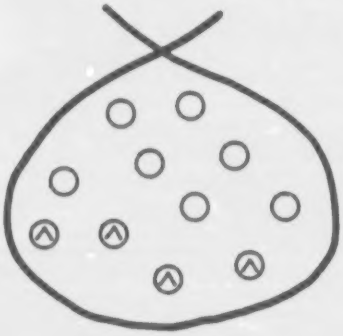
Exercise 1: Eli the Elephant

T: Who remembers Eli the Elephant?

Allow students to recall whatever they remember. If everyone has forgotten, mention that Eli collects peanuts; he sometimes finds regular peanuts and sometimes magic peanuts. Review what happens when a magic peanut and a regular peanut meet.

T: Today Eli went walking in the park and found seven regular peanuts and four magic peanuts. He put all the peanuts in his bag. He plans to eat the peanuts tonight while he watches television. How many peanuts will he find when he opens his bag?

Draw a picture of the bag with regular and magic peanuts in it.



W 5

Write a number story for this situation on the board.

$$7 + \widehat{4} = ?$$

T: Seven regular peanuts plus four magic peanuts is . . . ?

S: Three regular peanuts.

Complete the number story. Display two Minicomputer boards.

T: Let's show this story about Eli's peanuts on the Minicomputer. How many regular peanuts did Eli find?

S: Seven.

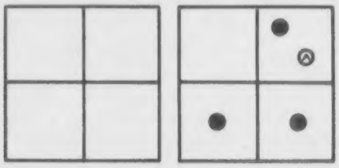
Invite someone to put 7 on the Minicomputer. Show the class the negative checkers " ⊙ ".

T: We can use these negative, or magic, checkers to show how many magic peanuts Eli found in the park. How many magic peanuts did he find?

S: Four.

T: Who can show this on the Minicomputer using negative checkers?

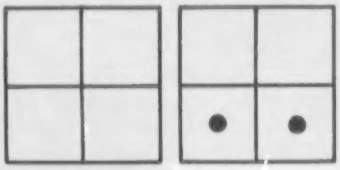
Do not allow the students to remove any checkers from the Minicomputer.



T: What will happen?

S: The regular checker and the magic checker on the purple square will disappear.

Explain that four magic peanuts and four regular peanuts disappear and ask a volunteer to show the class what happens on the Minicomputer. The student should remove both checkers from the purple square.



T: Seven regular peanuts and four magic peanuts is the same as . . . ?

S: Three regular peanuts.

Erase the board and remove the checkers from the Minicomputer.

T: Last week Eli found a lot of peanuts. One day he found 39 magic peanuts and 25 regular peanuts. When Eli opened his bag that night, what kind of peanuts did he find?

S: Magic peanuts.

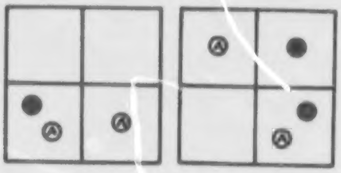
Write a number story for this situation on the board.

$$\widehat{39} + 25 = ?$$

If several students know the answer, ask them to tell you the number with whispers.

W 5

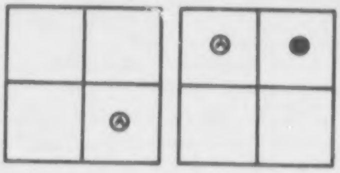
T: Who can put $\widehat{39}$ on the Minicomputer with negative checkers?
Who can put 25 on the Minicomputer with regular checkers?



T: What happens?

S: The checkers on the 1-square disappear, and the checkers on the 20-square disappear.

Invite students, one at a time, to remove the checkers from the 1-square and then from the 20-square. Encourage students to explain that one regular and one magic peanut disappear and that twenty regular and twenty magic peanuts disappear.



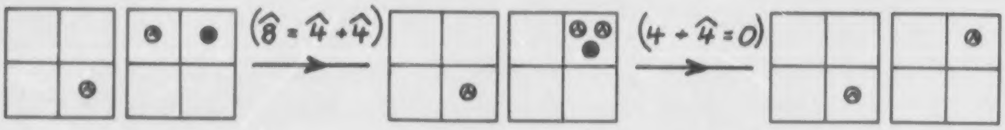
T: What could we do to make the number easier to read?

S: Make a backward play.

Do not expect the verbalization of this play to be exact. If a student says "8 = 4 + 4," accept it, but rephrase it by saying that eight magic peanuts is the same as four magic peanuts plus four magic peanuts.

If a student suggests that checkers not on the same square will disappear, for example $\widehat{8}$ and 4, remind the class that those checkers are for eight magic peanuts and four regular peanuts.

If no one suggests a backward play, suggest it yourself.



Complete the number story on the board.

$$\widehat{39} + 25 = \widehat{14}$$

Repeat this exercise with "$\widehat{46}$ + 22 = ?". Before continuing, erase the board and remove all the checkers from the Minicomputer.

Exercise 2: Detective Story

Do a short review of even and odd numbers before beginning the detective story. Write a large "12" on a slip of paper and fold the paper so that the "12" is hidden.

T: The name of a secret number is written on this paper. Listen carefully to this story and you can discover my secret number.

Display one Minicomputer board.

T: The secret number is on the Minicomputer using exactly two checkers. You cannot see the checkers because they are invisible. What are some numbers it could be?

Each time a number is suggested, ask a student to show how it can be put on the Minicomputer with two checkers. Record each number on the board as it is suggested, leaving space so that they can be recorded in order. Accept three or four suggestions from the students before continuing.

W 5

T: What is the smallest number it could be? (2) What is the largest number it could be? (16) Let's finish our list of numbers that could be the secret number.

Continue until your list has all ten possible numbers.

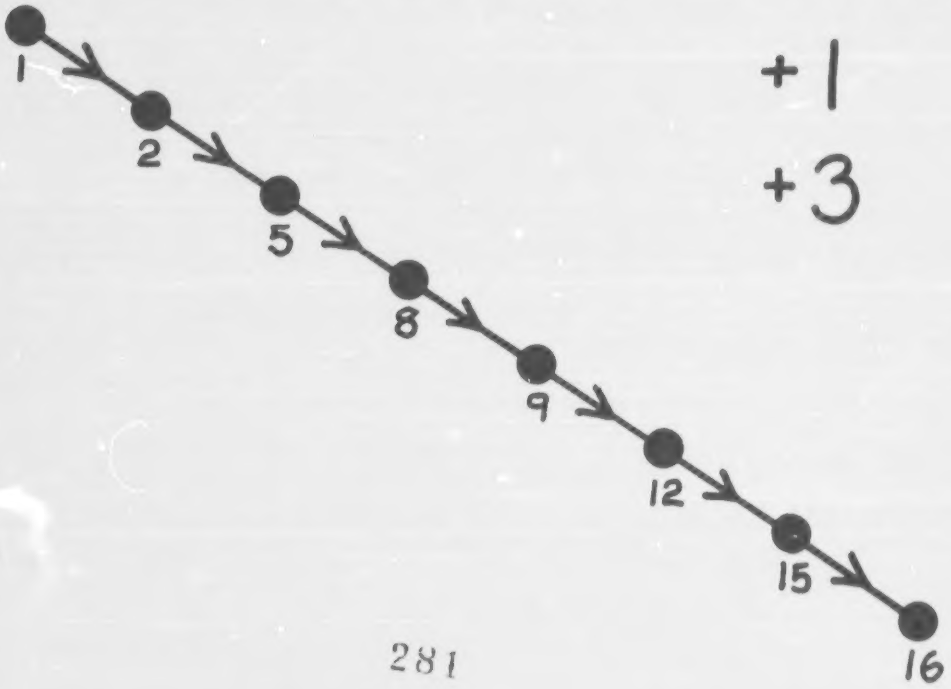
2, 3, 4, 5, 6, 8, 9, 10, 12, 16

T: Now we know that the secret number is one of these numbers, but we do not know which one. We need another clue.

Distribute copies of Worksheet W5 to the students.

T: The secret number is one of the numbers in this arrow picture. Which number could it be?

Allow the students to work on the arrow picture individually. While they are working, draw the arrow picture on the board. When many of the students have completed the worksheet, complete the arrow picture on the board collectively.



Point to the list of numbers on the board from the first clue.

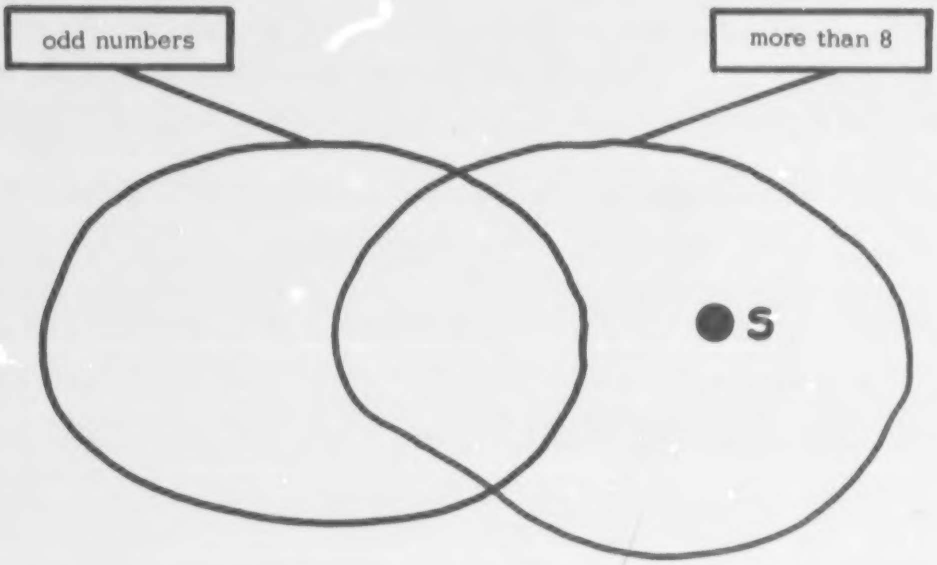
T: Which of these numbers cannot be the secret number? Help me cross them off our list.

Students should tell you to cross off 3, 4, 6, and 10 because these numbers are not in the arrow picture.

2, ~~3~~, ~~4~~, 5, ~~6~~, 8, 9, ~~10~~, 12, 16

T: Now we know that the secret number could be 2, 5, 8, 9, 12, or 16. Here is another clue.

Draw this string picture on the board.



T: Our secret number is in this string picture (point to "s"). Is the secret number an odd number? (No) Is it more than 8? (Yes) Which of these numbers could be the secret number?

W 5

When a number is suggested, use it to label the dot and then ask the class if that number could be there. Suppose a student suggests that the secret number could be 8.

T: Could this be 8? (Disagreement)
Is 8 an odd number? (No)
Then 8 must be outside the red string.
Is 8 more than 8? (No)
Then 8 must be outside the blue string.
Where should 8 be in this string picture?

Let a volunteer point to the correct region.

T: 8 cannot be the secret number.

Erase "8" from the string picture and cross it off the list on the board.

For a correct suggestion, elicit class agreement that it could be the secret number and circle that number on your list. Continue until all of the numbers in the original list are either circled or crossed off. Erase those which have been crossed off.

12, 16

T: The secret number is either 12 or 16. I will give you one more clue.
The secret number can be put on the Minicomputer with three checkers, all on the same square.

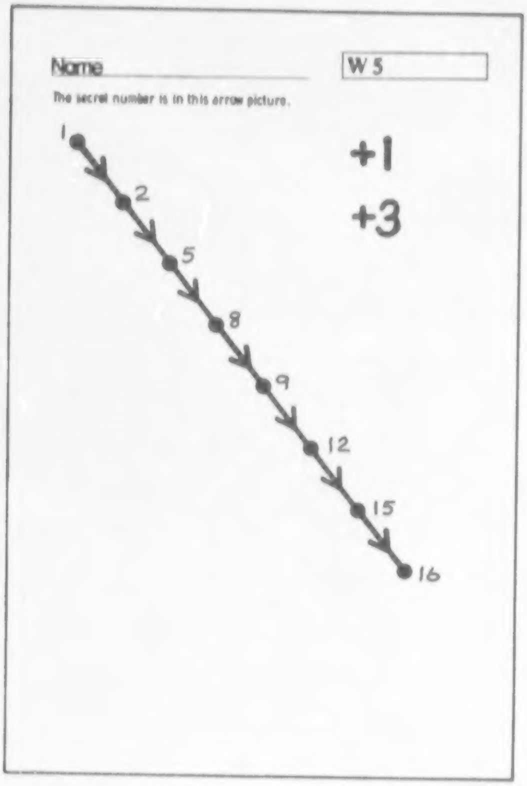
Ask the students to write the secret number on a piece of paper and check a few of their answers.

T: Which is our secret number?

S: 12.

Invite someone to put 12 on the Minicomputer with three checkers, all on the same square. Reveal that "12" is written on your paper.

281



Name ____________________ W5

The secret number is in this arrow picture.

+1
+3
1
2
5
8
9
12
15
16

W6 A Catalog of Problems #1 (Lesson 1)

CAPSULE LESSON SUMMARY

Extend the experiences in various content strands in the Workbook "A Catalog of Problems #1".

MATERIALS

Teacher:	(If needed) Minicomputer set; colored chalk
Student:	Workbook "A Catalog of Problems #1"; pencil; colored pencils

DESCRIPTION OF LESSON

Distribute copies of the Workbook "A Catalog of Problems #1" and let the students begin working independently. If the questions you are receiving indicate that many of your students are having difficulty with a particular page, hold a collective discussion about that page.

Allow about 30 minutes of the lesson for the workbook and then collect them for use in Lesson W7. Spend the remaining time playing the Number Line Game.

W 7

W7 A Catalog of Problems #1 (Lesson 2)

CAPSULE LESSON SUMMARY

Decide who has more rocks in his collection - Derrick or Jeremy - after various additions are made. Continue working in "A Catalog of Problems #1". (This is the second of two lessons using this workbook.)

MATERIALS

Teacher: None

Student: Workbook "A Catalog of Problems #1"; pencil; colored pencils

DESCRIPTION OF LESSON

T: One day Derrick and Jeremy each found three special rocks to bring to school for a science lesson.

Record this information on the board.

Derrick	Jeremy
3	3

T: The next day Derrick found two more special rocks and Jeremy found six more.

Add this information to what is on the board.

Derrick	Jeremy
3 + 2	3 + 6

T: Who found the most rocks during those two days?

S: Jeremy.

Complete this number story on the board as you read it "three plus two is less than three plus six".

Derrick Jeremy

$$3+2 < 3+6$$

T: Derrick and Jeremy each displayed his rocks on a piece of cardboard to bring to class. How many rocks did Derrick put in his display?

S: Five.

Replace "3 + 2" with "5".

Derrick Jeremy

$$5 < 3+6$$

T: How many rocks did Jeremy put in his display?

S: Nine.

Derrick Jeremy

$$5 < 9$$

T: The teacher was pleased with the work these boys had done and gave each of them another special rock, a piece of quartz, for their collections. How can I show this in our number story (point to "5 < 9")?

S: Add 1 to each side.

W 7

Ask a student to write this on the board.

Derrick Jeremy

$5 + 1 < 9 + 1$

T: Do we need to change this (point to " < ")? Is 5 + 1 less than 9 + 1?

S: Yes.

T: 5 + 1 = . . . ? (6)

9 + 1 = . . . ? (10)

Replace the expressions on the board with their sums.

Derrick Jeremy

$6 < 10$

T: Derrick's older brother gave Derrick ten new rocks for his collection. How can we show this in our number story?

S: Add 10 on Derrick's side.

Ask a student to write this on the board.

Derrick Jeremy

$6 + 10 < 10$

T: Is this number story true?

S: No.

T: Who can fix it up?

Invite someone to correct the statement on the board and then to read the number story.

Derrick Jeremy
$6 + 10 > 10$

S: Six plus ten is more than ten.

T: Now who has the most rocks?

S: Derrick; he has sixteen.

Replace "6 + 10" with "16".

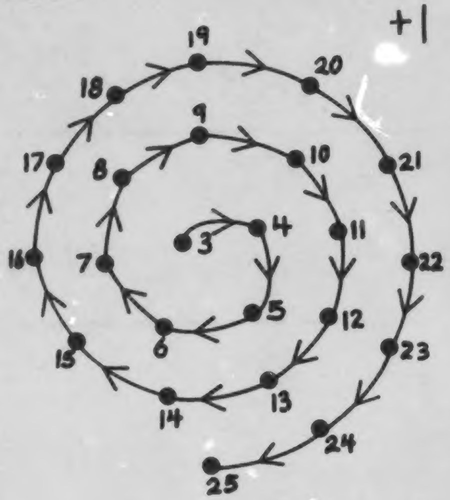
Derrick Jeremy
$16 > 10$

Distribute copies of the Workbook "A Catalog of Problems #1" and let the students work independently. At the end of the lesson, collect the workbooks for your review.

A
CATALOG
OF
PROBLEMS
I

A Catalog of Problems 1

Label the dots.


+1
3
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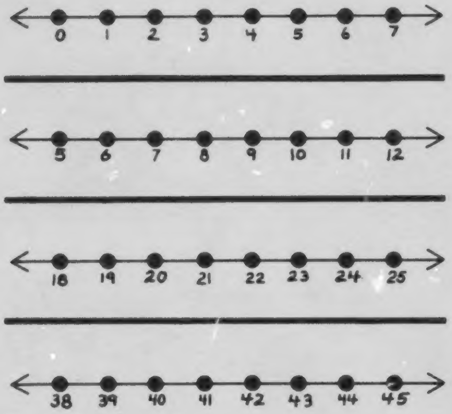
25

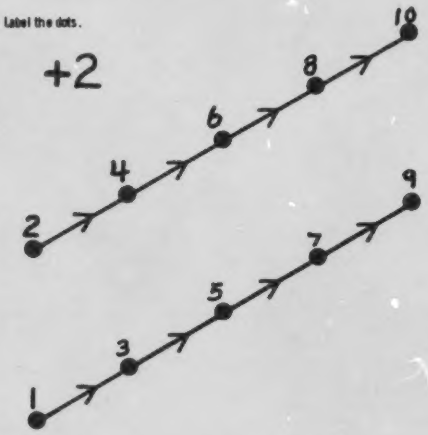
2

Label the dots on these number lines.


0 1 2 3 4 5 6 7
5 6 7 8 9 10 11 12
18 19 20 21 22 23 24 25
38 39 40 41 42 43 44 45

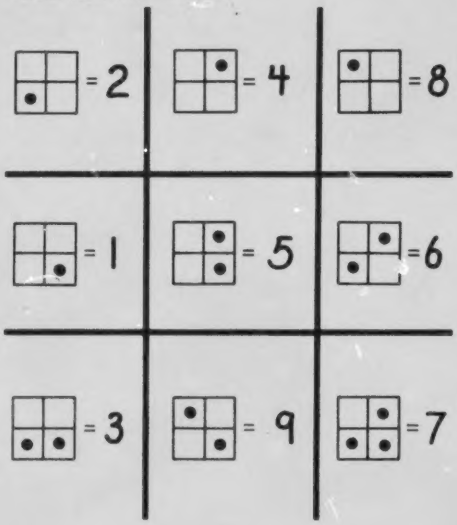
3

Label the dots.


+2
2
4
6
8
10
1
3
5
7
9

4

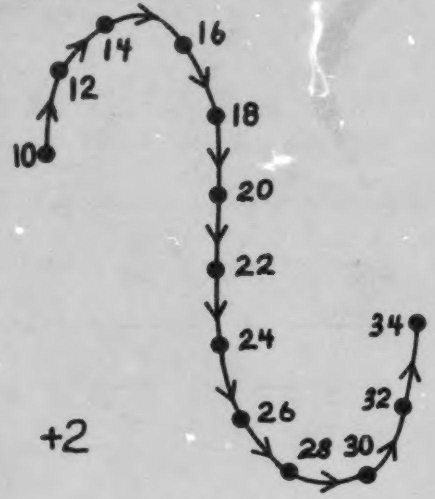
Put each number on the Minicomputer.



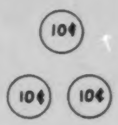
5

Standard configuration is given here for each number; however, many other solutions are possible.

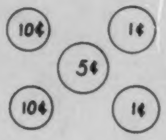
Label the dots.


10
12
14
16
18
20
22
24
26
28
30
32
34
+2

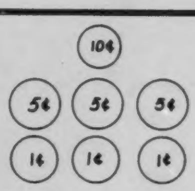
How much money?



30 ¢

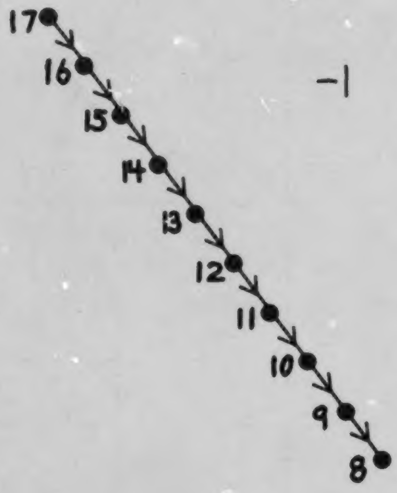


27 ¢



28 ¢

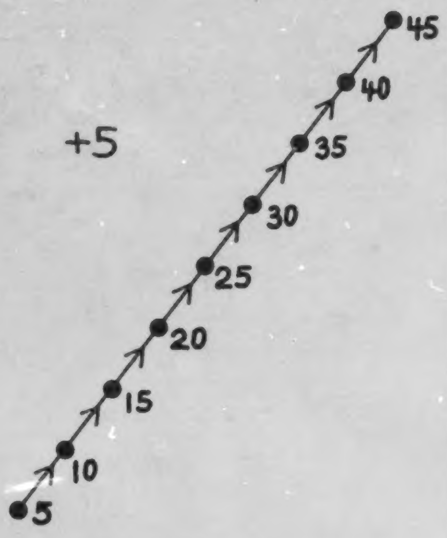
Label the dots.


17
16
15
14
13
12
11
10
9
8
−1

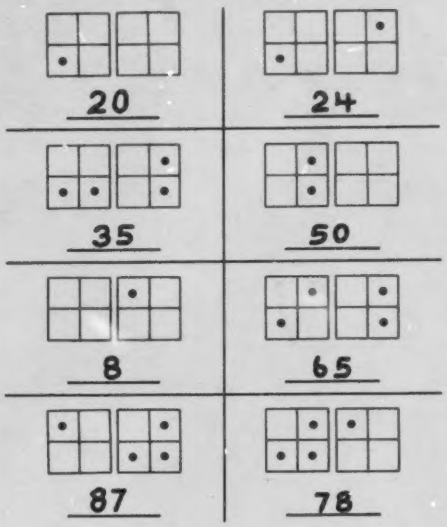
Write "<", ">", or "=" to make true number stories. One is done for you.

3 + 2	<	6
9	=	5+4
3 + 3	>	5
6 + 5	<	6 +6
10 + 2	=	9+3
3 + 8	<	4+8
12 + 3	<	16
11 + 5	>	12+2

Label the dots.


+5
5
10
15
20
25
30
35
40
45

What number is on the Minicomputer?



20

24

35

50

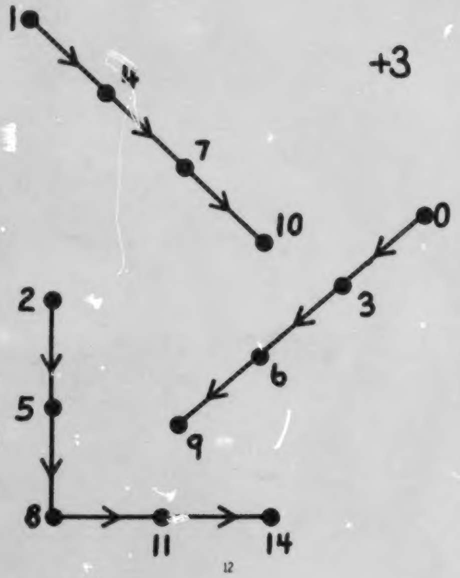
8

65

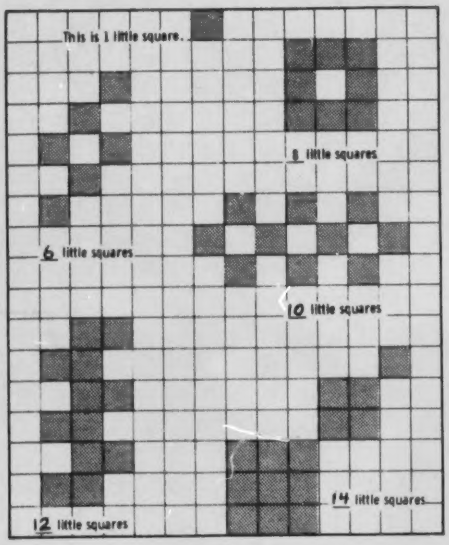
87

78

Label the dots.


1
4
7
10
+3
0
3
6
9
2
5
8
11
14

What is the area of each shape? One problem is done for you.



This is 1 little square.

8 little squares

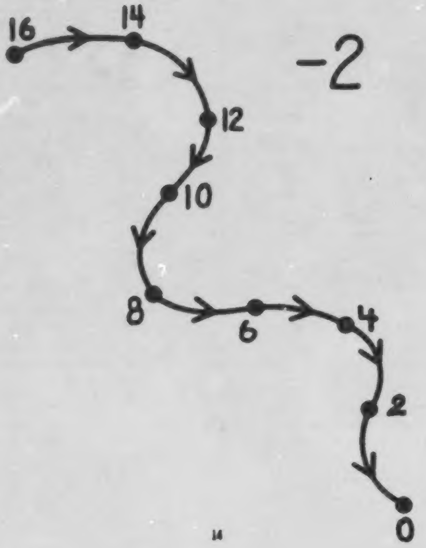
6 little squares

10 little squares

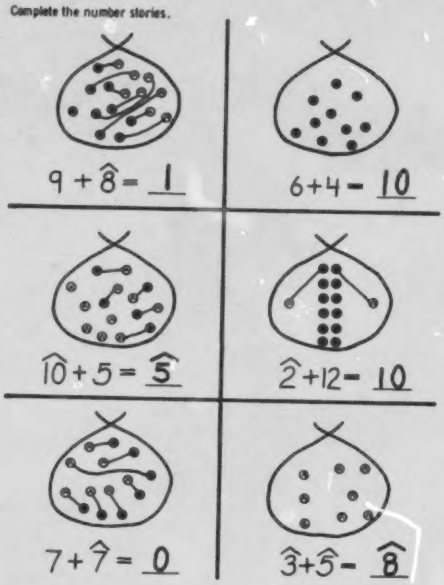
12 little squares

14 little squares

Label the dots.



Complete the number stories.



$9+\hat{8}=\underline{1}$

$6+4-\underline{10}$

$\hat{10}+5=\underline{\hat{5}}$

$\hat{2}+12-\underline{10}$

$7+\hat{7}=\underline{0}$

$\hat{3}+\hat{5}-\underline{\hat{8}}$

CODE

A - 1
B - 2
C - 3
D - 4
E - 5
F - 6
G - 7
H - 8
I - 9
J - 10
K - 11
L - 12
M - 13
N - 14
O - 15
P - 16
Q - 17
R - 18
S - 19
T - 20
U - 21
V - 22
W - 23
X - 24
Y - 25
Z - 26

Decode:

W	H	A	T
23	4+4	1+0	10+10

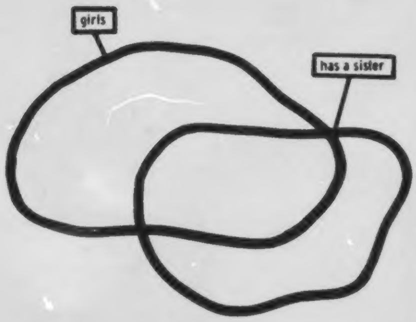
I	S
6+3	19

Y	O	U	R
20+5	15	22-1	18

N	A	M	E ?
10+4	1-0	13	7-2

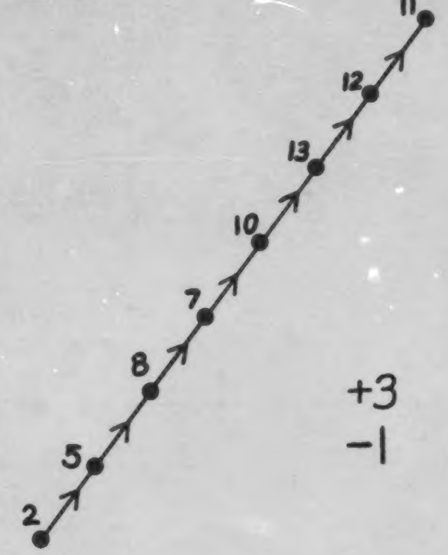
Answer: Student's Name

Draw a dot for yourself.



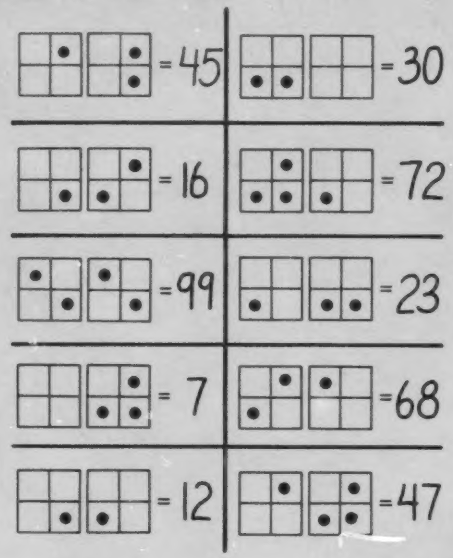
Solution depends on the student.

Label the dots.


11
12
13
10
7
8
5
2
+3
−1

18

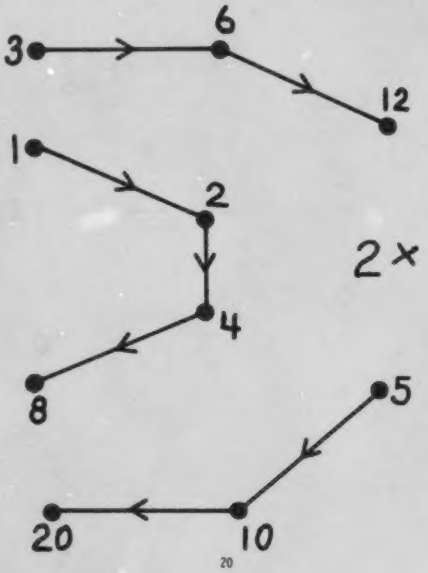
Put each number on the Minicomputer.


= 45
= 30
= 16
= 72
= 99
= 23
= 7
= 68
= 12
= 47

19

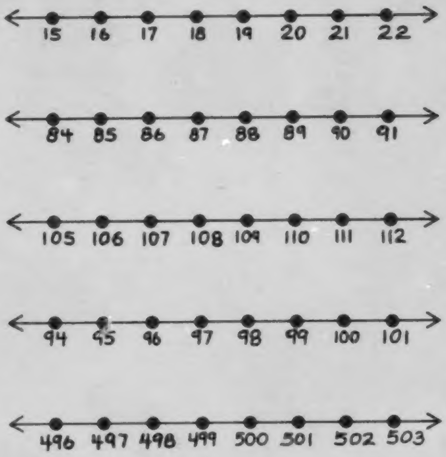
Standard configuration is given here for each number; however, many other solutions are possible.

Label the dots.


3
6
12
1
2
4
8
2x
5
10
20

20

Label the dots on these number lines.


15 16 17 18 19 20 21 22
84 85 86 87 88 89 90 91
105 106 107 108 109 110 111 112
94 95 96 97 98 99 100 101
496 497 498 499 500 501 502 503

21

Complete the number stories.

$10 + 10 = \boxed{20}$

$9 + \boxed{11} = 20$

$\boxed{8} + 12 = 20$

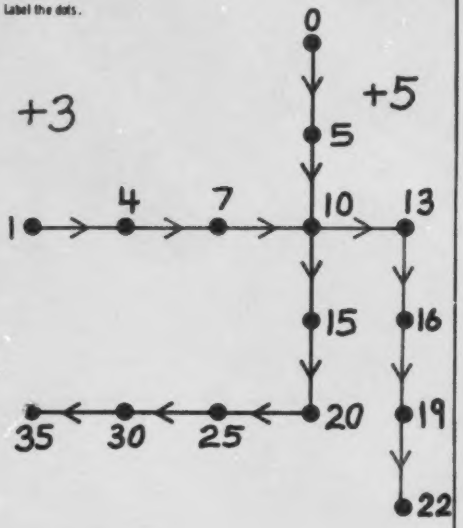
$\boxed{7} + 13 = 20$

$6 + \boxed{14} = 20$

$5 + \boxed{15} = 20$

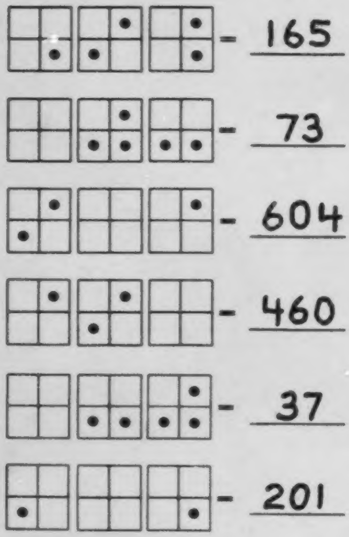
22

Label the dots.


0
+3
+5
5
1
4
7
10
13
15
16
35
30
25
20
19
22

23

What number is on the Minicomputer?



- 165

- 73

- 604

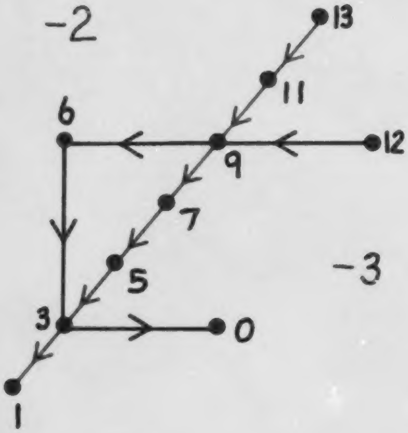
- 460

- 37

- 201

24

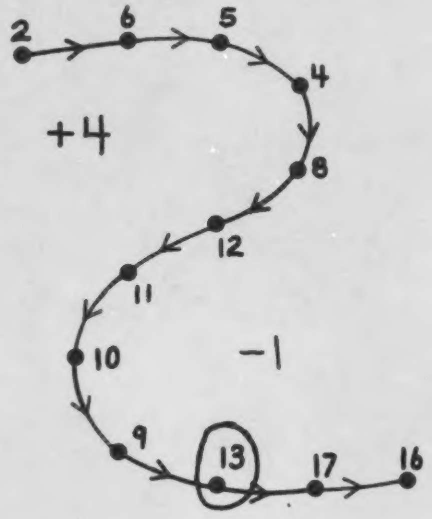
Label the dots.


-2
13
11
6
9
12
7
5
-3
3
0
1

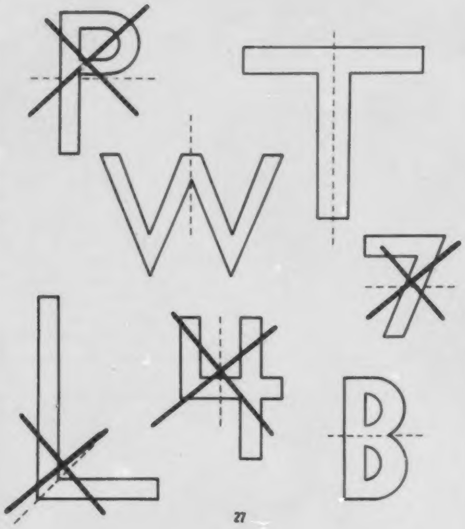
25

297

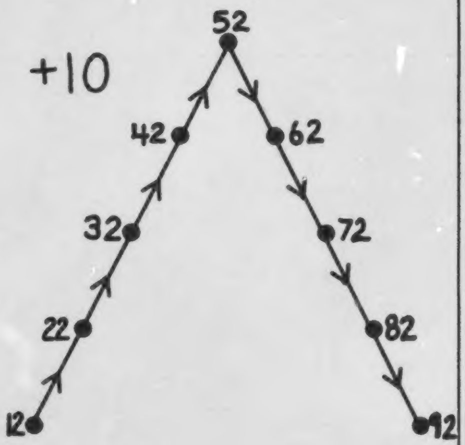
Find 13 in this picture and circle its dot.


2
6
5
4
+4
8
12
11
10
-1
9
13
17
16

Cross out the shapes which are not cut fairly in half.



Label the dots.


+10
52
42
62
32
72
22
82
12
92

CODE

A - 1
B - 2
C - 3
D - 4
E - 5
F - 6
G - 7
H - 8
I - 9
J - 10
K - 11
L - 12
M - 13
N - 14
O - 15
P - 16
Q - 17
R - 18
S - 19
T - 20
U - 21
V - 22
W - 23
X - 24
Y - 25
Z - 26

Decode:

D	O	Y	O	U
2+2	9+6	20+5	15+0	19+2

H	A	V	E	A
2×4	1+0	2×11	3+2	1-0

B	I	K	E	?
10-8	3×3	10+1	6-1	

Answer: Yes (or No)

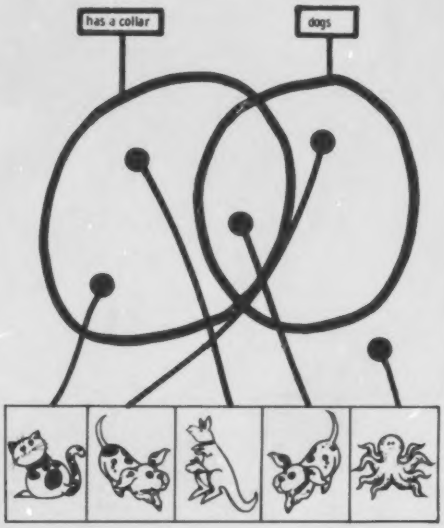
Show ways to make 16¢. One way is given.

Dimes	Nickels	Pennies
	5¢ 5¢ 5¢	1¢
10¢	5¢	1¢
10¢		1¢ 1¢ 1¢ 1¢ 1¢ 1¢
	5¢ 5¢	1¢ 1¢ 1¢ 1¢ 1¢ 1¢
	5¢	Eleven pennies#
		Sixteen pennies#

30

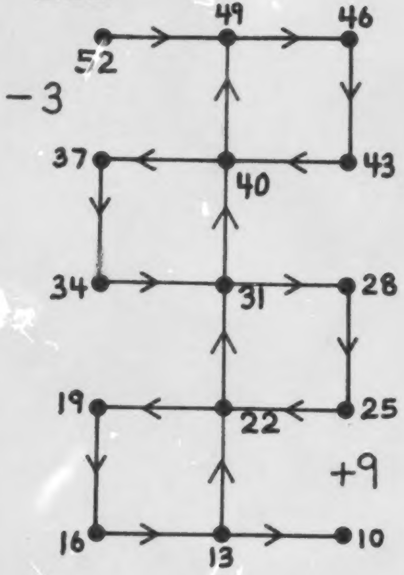
Students will draw the coins, but space prohibits our doing so in this answer key.

Match each dot with an animal. One is done for you.


has a collar
dogs

31

Label the dots.


49
46
52
-3
37
40
43
34
31
28
19
22
25
+9
16
13
10

32

W8 Detective Story #3 / Eli the Elephant Stories

CAPSULE LESSON SUMMARY

Solve a detective story with clues involving an arrow picture, a string picture, and the Minicomputer. Review the story of Eli the Elephant and continue solving addition problems involving positive and negative numbers.

MATERIALS

Teacher: Colored chalk; Minicomputer set

Student: Pencil; Worksheets W8*, **, ***, and ****

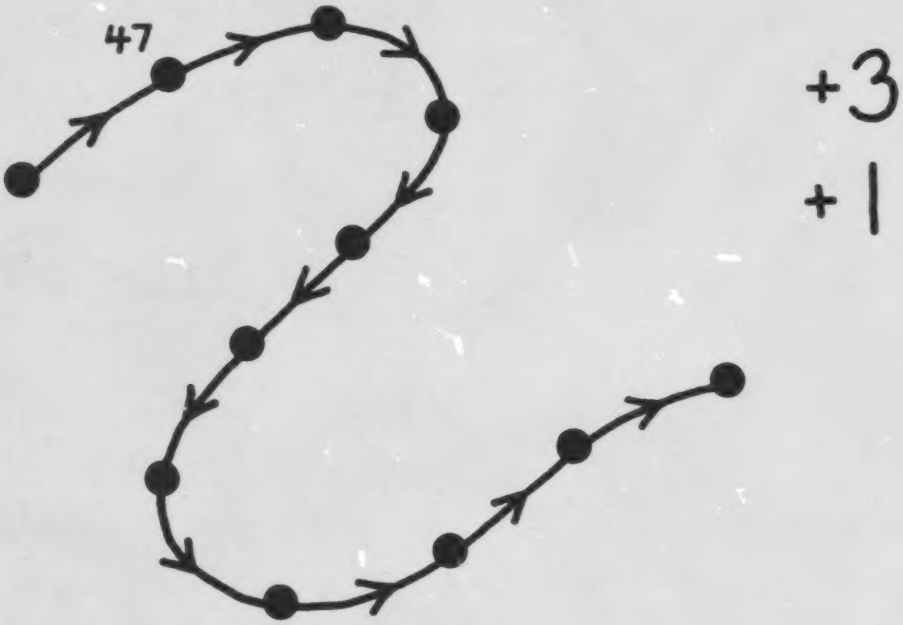
DESCRIPTION OF LESSON

Exercise 1: Detective Story

Write a large "60" on a slip of paper and fold it so that the "60" is hidden.

T: The name of a secret number is written on this paper. Listen carefully to my clues and you can discover it.

Draw this arrow picture on the board.



W 8

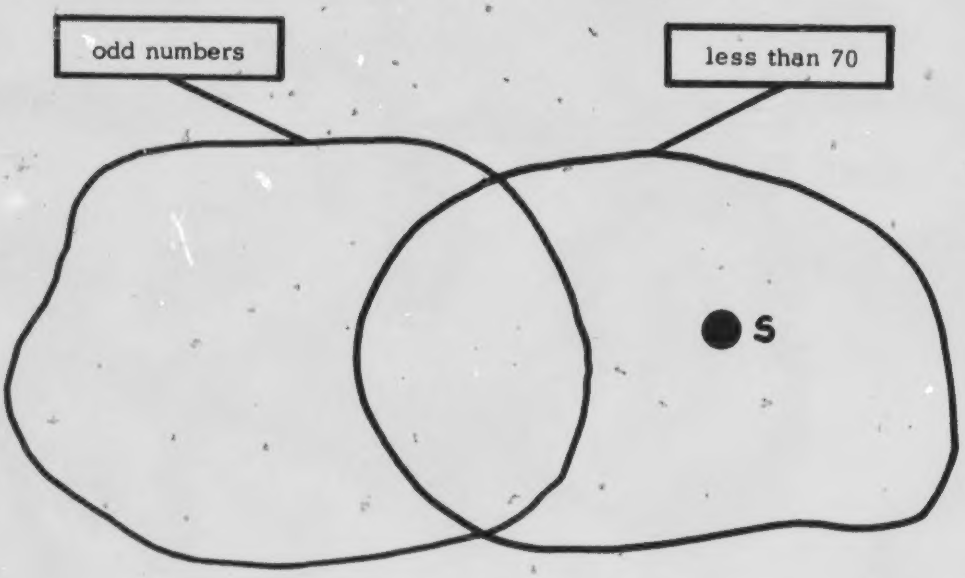
T: The secret number is one of the numbers in this arrow picture. Which numbers could it be?

When a number is suggested, ask the student to label the appropriate dot. After all the dots have been labeled, make a list of the numbers that could be the secret number.

44, 47, 50, 53, 56, 59, 60, 61, 64, 67, 70

T: We know that the secret number is one of these numbers but we don't know which. We need another clue.

Erase the arrow picture and then draw this string picture on the board.



T (pointing to "s"): Here is the secret number. Is it an odd number? (No) Is it less than 70? (Yes) Which of these numbers could be the secret number?

When a number is suggested, use it to label the dot and then ask the class if that number could be there. Suppose a student suggests that the secret number could be 67.

T: Could this be 67? (Disagreement)
Is 67 less than 70? (Yes)
Then 67 must be inside the red string.
Is 67 odd? (Yes)
Then 67 must be inside the blue string.
Where should 67 be in this picture?

Let a student point to the correct region.

T: 67 cannot be the secret number.

Erase "67" from the string picture and cross it off the list on the board.

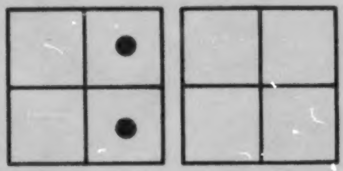
For a correct suggestion, elicit class agreement that it could be the secret number and circle it on your list. Continue until many of the numbers in the original list are either circled or crossed off. Point to each unchecked number and guide your class in deciding if it could be the secret number. When all the numbers in the list have been checked, erase those which have been crossed off.

㊹ ㊿ (56) (60) (64)

T: The secret number is one of these numbers. I will give you one more clue.

W 8

Display two Minicomputer boards with this configuration.



T: There are two checkers on the Minicomputer. We need only one more checker to show the secret number.

Ask students to write the secret number on a piece of paper and check a few of their answers.

T: Which is the secret number?

S: 60.

Invite a volunteer to place one more checker on the Minicomputer to show 60. Reveal that "60" is written on your paper.

Exercise 2: Eli the Elephant

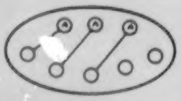
Talk briefly with the students about Eli the Elephant until everyone remembers what happens when a regular and a magic peanut meet. Distribute in turn Worksheets W8*, **, ***, and ****. Emphasize that the pictures are there to help them think about the number stories, but that they do not have to finish the picture.

If some students finish these worksheets very quickly, ask them to write their own Eli the Elephant stories.

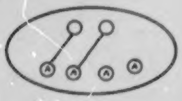
Name ____________ W 8 *

Finish the number stories.

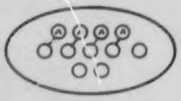
$\hat{3}+5=$ 2



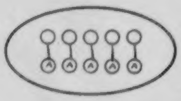
$2+\hat{4}=$ $\hat{2}$



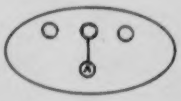
$\hat{4}+7=$ 3



$5+\hat{5}=$ 0



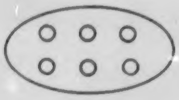
$3+\hat{1}=$ 2



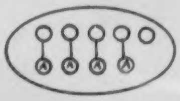
Name ____________ W 8 **

Finish the number stories.

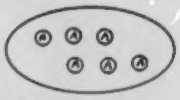
$2+4=$ 6



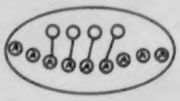
$5+\hat{4}=$ 1



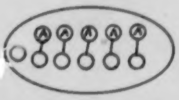
$\hat{3}+\hat{3}=$ $\hat{6}$



$4+\hat{9}=$ $\hat{5}$



$\hat{5}+6=$ 1



Name ____________ W 8 ***

Complete these number stories.

$\hat{2}+3=$ 1

$8+\hat{8}=$ 0

$10+\hat{1}=$ 9

$4+3=$ 7

$\hat{9}+\hat{3}=$ $\hat{12}$

Name ____________ W 8 ****

Complete these number stories.

$22+\hat{2}=$ 20

$\hat{18}+18=$ 0

$20+\hat{10}=$ 10

$5+\hat{15}=$ $\hat{10}$

$\hat{1}+100=$ 99

$\widehat{1{,}000}+1{,}000=$ 0

W 9

W9 8? - 15? (Lesson 1)

CAPSULE LESSON SUMMARY

Pose the problem: Is the number 10 on a +3 road which contains the number 6 or on a +3 road which contains the number 16? Once decided, locate the dot for 10 and label it. Do a similar activity finding the number 10 by extending part of a +2 road. Do pages 2 and 3 of the Workbook "8? - 15?" collectively. In the first section of this workbook, find the number 8 on each page or extend the picture to include 8. In the second section, find the number 15 or extend the picture to include 15.

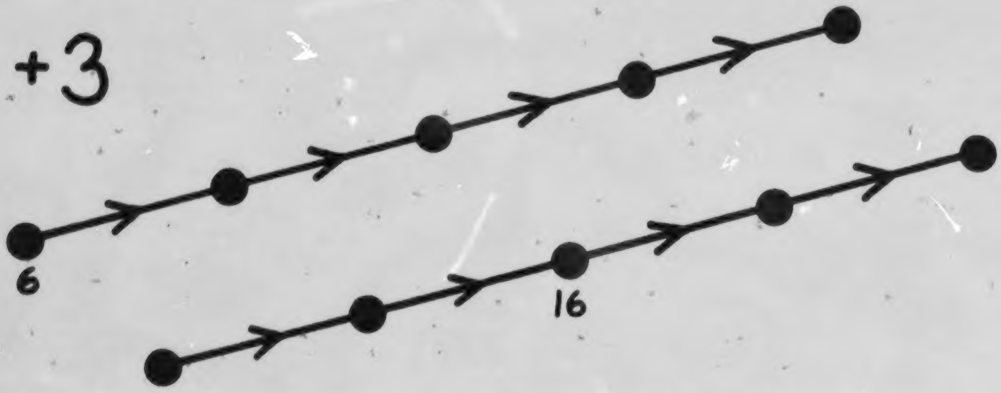
MATERIALS

Teacher:	Colored chalk
Student:	Workbook "8? - 15?"; pencil

DESCRIPTION OF LESSON

Exercise 1

Draw these arrow roads on the board.



T: Do you think the number 10 is in this arrow picture? If you think it is, which road is it on?

Ask a few students to explain their choice of either the top or the bottom road. If many students believe that 10 is in the top road, collectively label the dots of this road and discover that 10 is skipped.

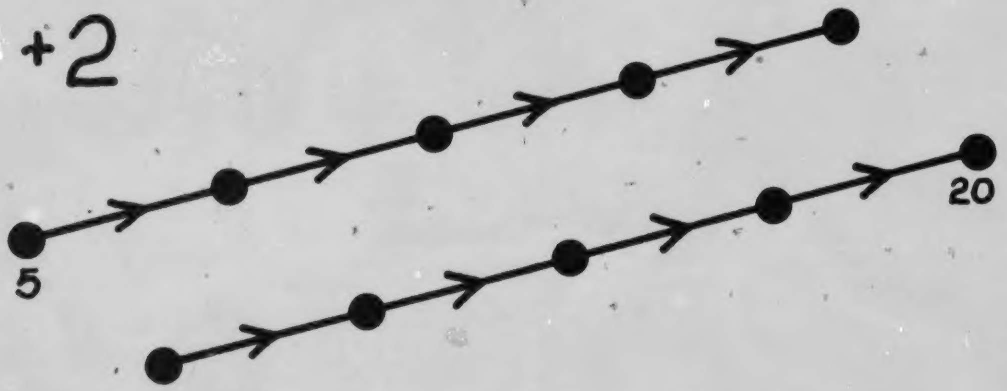
When your class has decided that 10 cannot be in the top road, direct their attention to the bottom road. Ask whether 10 could be to the right or to the left of "16" and let students explain their answers. Label the dots until 10 has been found.

It is possible that your class will need to label only two dots to find 10, but they may wish to label the other dots. Make it clear that it is alright to label the other dots but not necessary because your objective was only to find 10.

Erase the numerals and the key, but leave the arrow roads on the board.

Exercise 2

Indicate that now the red arrows will be for +2. Label the first dot of the top road "5" and the last dot of the bottom road "20".



T: We are still looking for the number 10. Could 10 be in the top road? How do you know?

Someone may comment that 10 is an even number and that the numbers in the top picture will be odd. If someone thinks 10 could be in the top picture, label the dots with the class and observe that 10 is skipped.

W 9

T: Is 10 in the bottom road?

S: No, because there aren't enough dots there.

Ask whether more red arrows could be drawn in the bottom road so that 10 would be in the picture. Let one of the students extend the road. (If a student just draws a dot and labels it "10", ask if there are any new arrows that can be drawn. Of course, since 12 is already in the picture, an arrow can be drawn from 10 to 12.)

Distribute the workbooks. Look at several pages with your class and very briefly go over the directions. Stress that in the first section they are to find 8 in every picture or to extend the picture to include 8; in the second section they are to find 15 in every picture or to extend the picture to include 15. Do pages 2 and 3 collectively with your class. Ask the students to work on those pages while you copy the pictures on the board. In both cases, they will need to extend their pictures to include 8.

At the end of the lesson, collect the workbooks for use in Lesson W10.

W10 8? - 15? (Lesson 2)

CAPSULE LESSON SUMMARY

Pose the problem: Is the number 13 on a +3 road which contains the number 6 or on a +3 road which contains the number 28? Decide that 13 cannot be on the first road. Extend the part of the second road which has been drawn on the board to include 13. Continue working in the Workbook "8? - 15?".

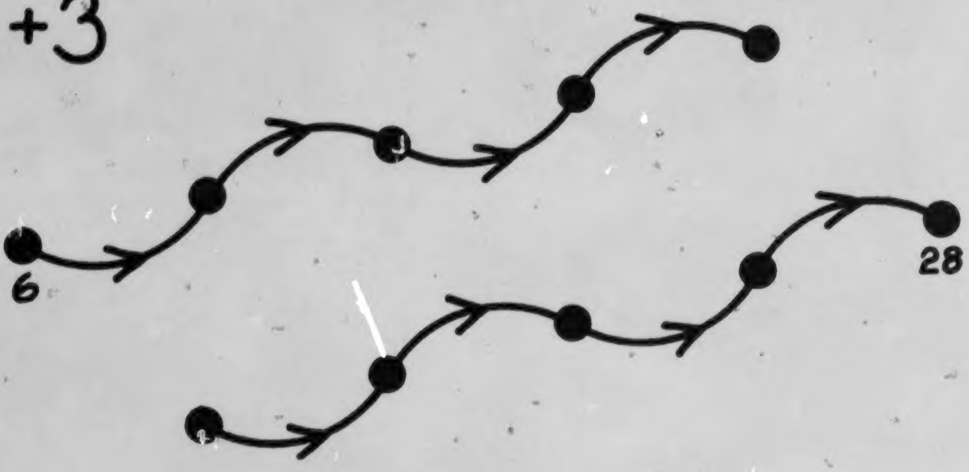
MATERIALS

Teacher: Colored chalk

Student: Workbook "8? - 15?", pencil

DESCRIPTION OF LESSON

Draw these arrow roads on the board.



T: We are looking for the number 13. Could 13 be in this road (trace the road starting at 6)?

If necessary, collectively label the dots of this road and discover that 13 is skipped.

W 9

T (pointing to the dots in the road ending at 28): Is one of these dots for 13?

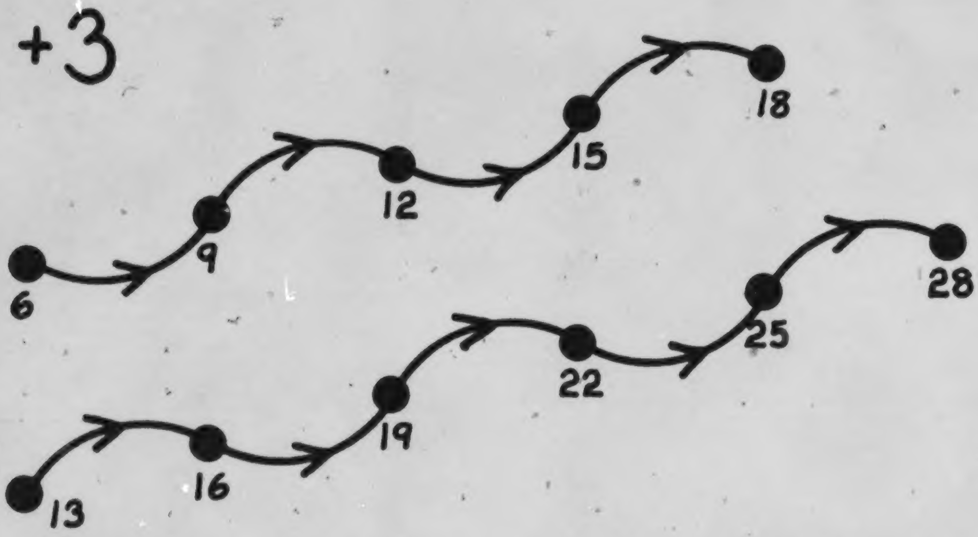
Label dots until it is clear that 13 is not one of the dots.

T: Did we skip over 13?

S: No.

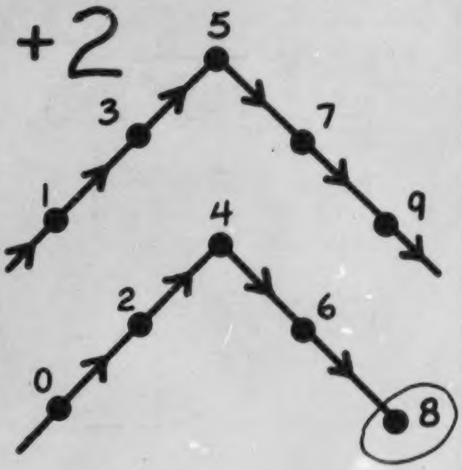
T: Can we extend the road by drawing more red arrows so that we would meet the number 13?

Let a student draw the appropriate arrow and label the dot for 13.



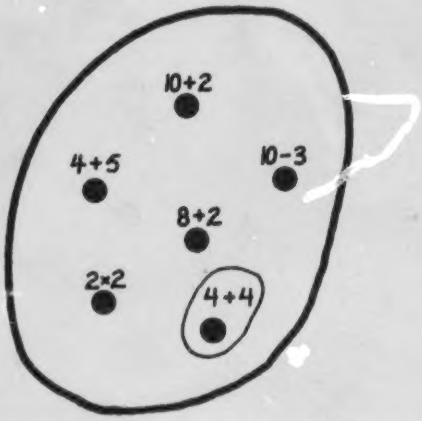
Distribute Workbook "8? - 15?". Ask the class to continue working in this booklet. If some students finish the workbook early, encourage them to make up their own problems for a similar workbook.

8? - 15?


+2
5
3
7
1
9
4
2
6
0
8

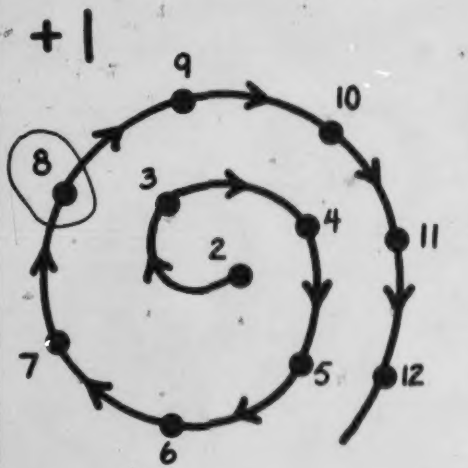
2

Extend the picture to include 8.

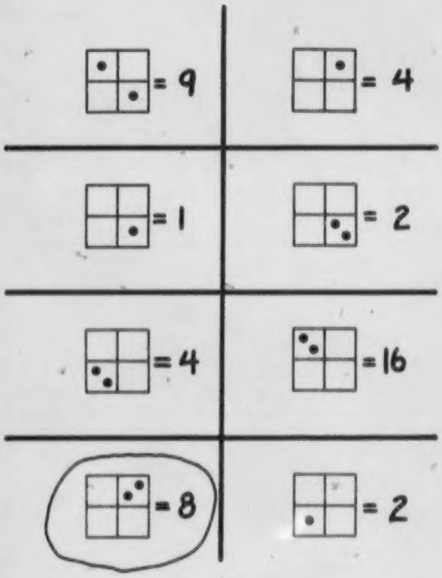

10+2
4+5
10-3
8+2
2×2
4+4

3

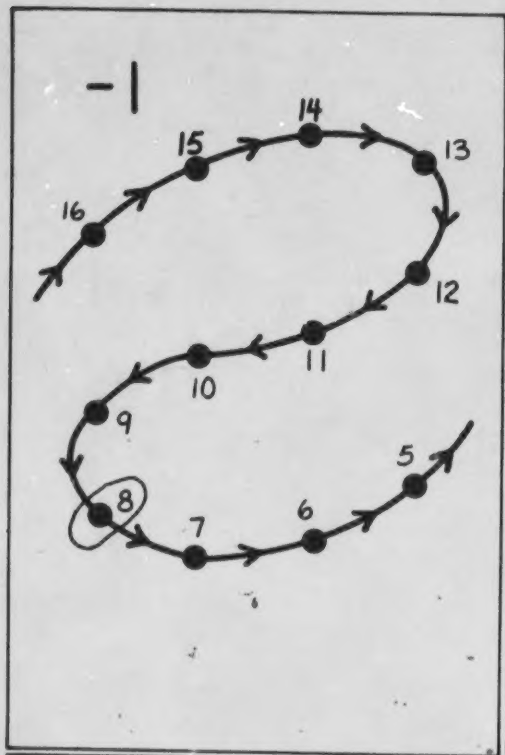
8 was not in the string;
extend the picture to include 8.
There are many ways to label the dot for 8.

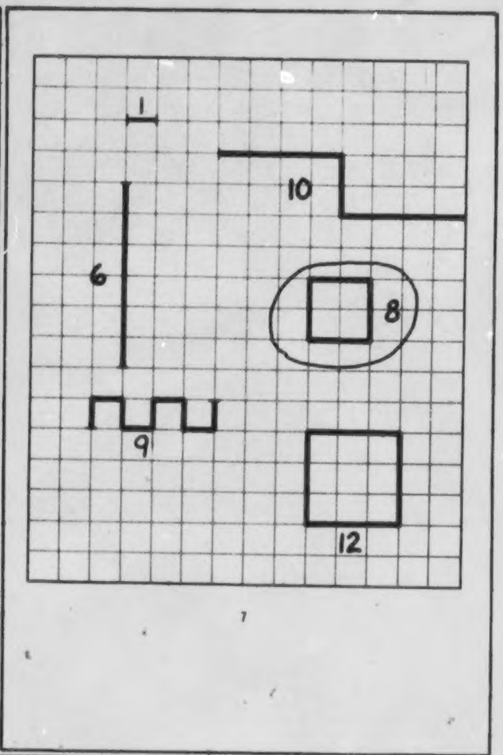

+1
9
10
8
3
4
2
11
7
5
12
6

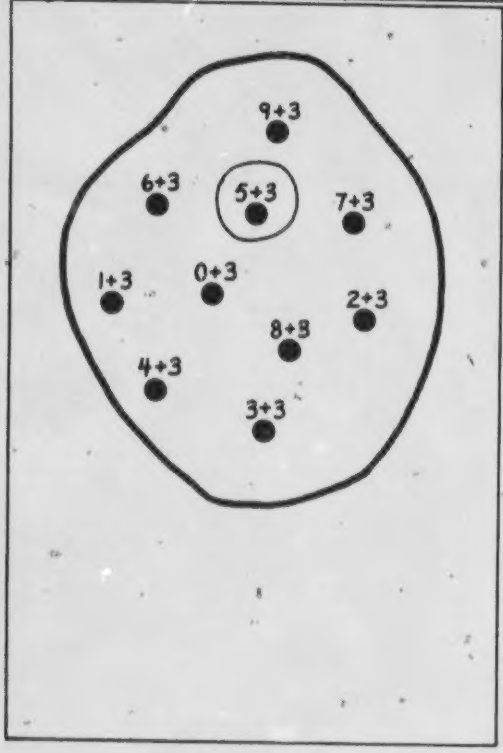
4

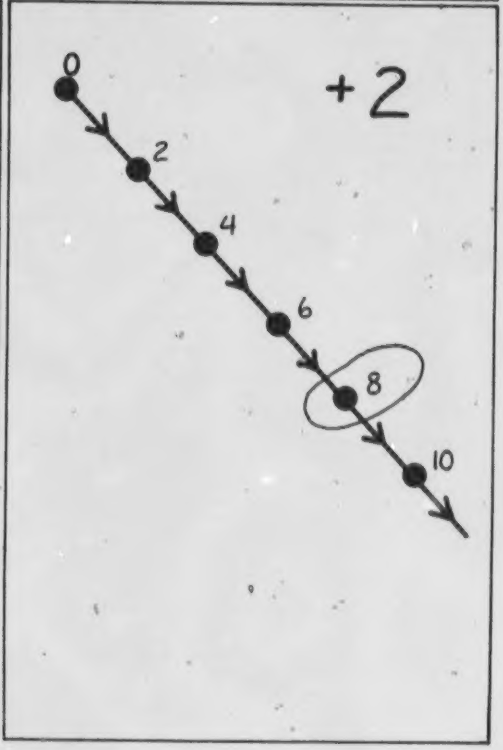


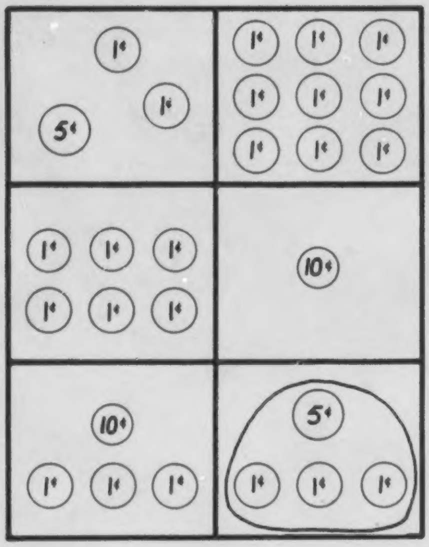
5


-1

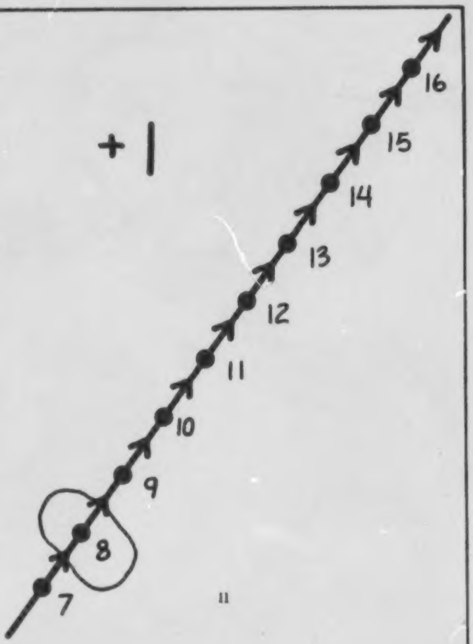



9+3
6+3
5+3
7+3
1+3
0+3
2+3
8+3
4+3
3+3
8

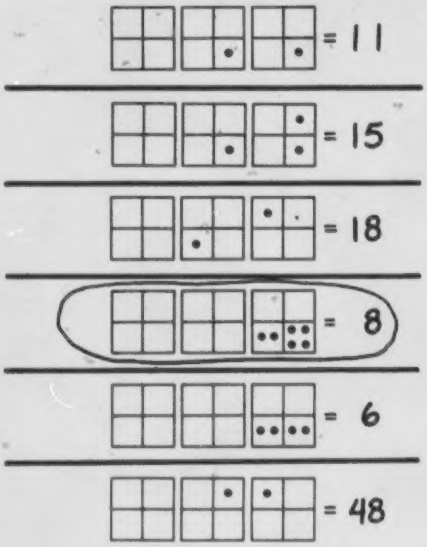

+2



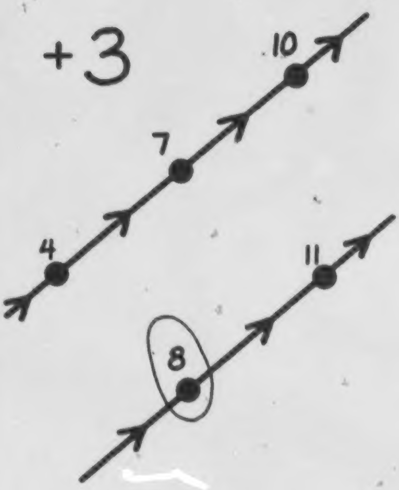
10


+1
7
8
9
10
11
12
13
14
15
16

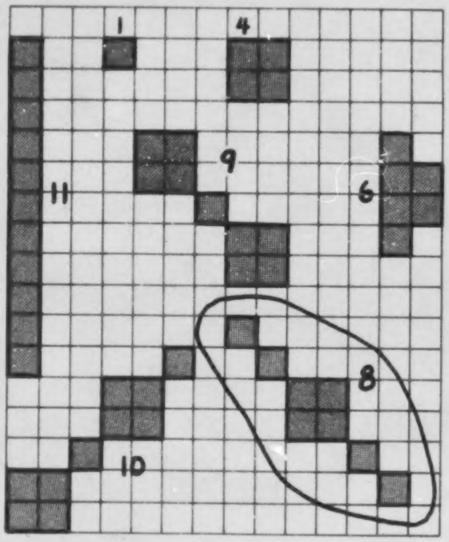
11


= 11
= 15
= 18
= 8
= 6
= 48

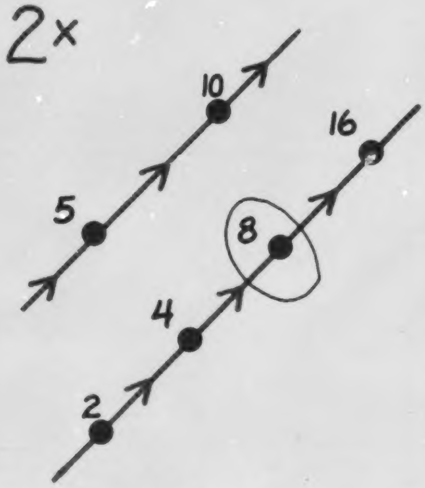
12



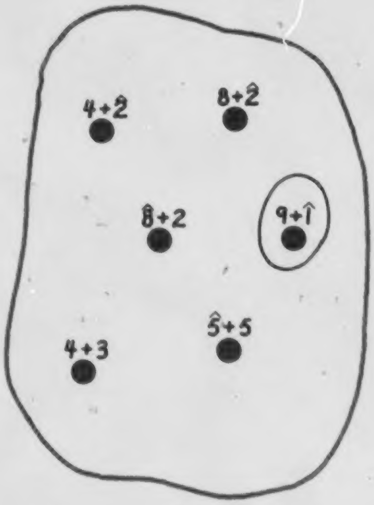
13



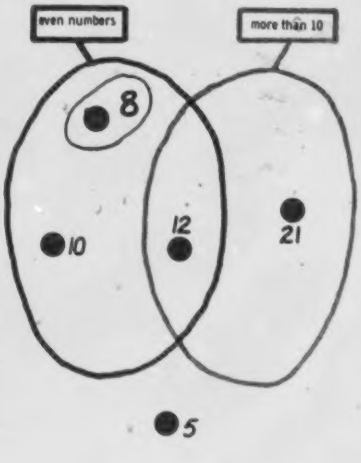
14


2x
10
16
5
8
4
2

15

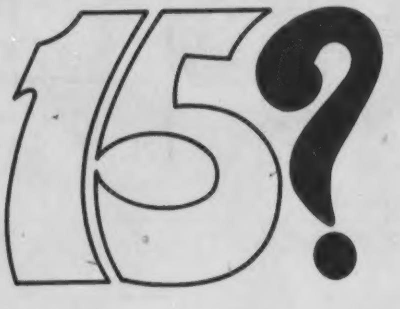

4+2̂
8+2̂
8̂+2
9+1̂
4+3
5̂+5

16


even numbers
more than 10
8
10
12
21
5

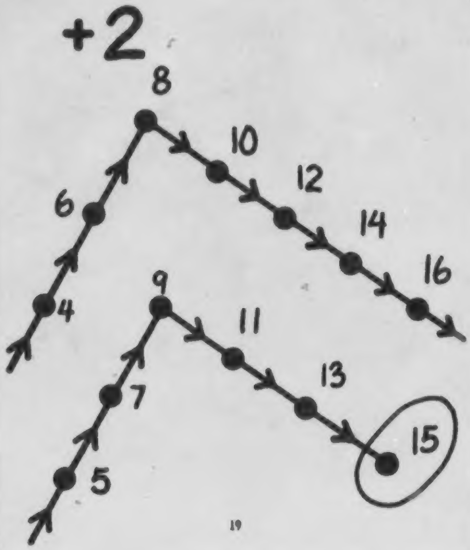
17

8 was not in the picture; include a dot for 8.



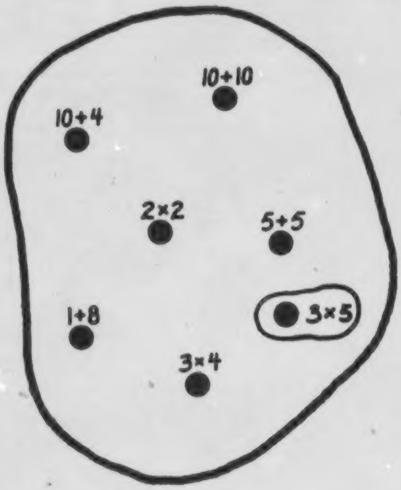
Find 15 on each page in this section of the workbook. If 15 is not on a page, extend the picture to include 15.

18


+2
8
10
12
14
16
6
4
9
11
13
15
7
5

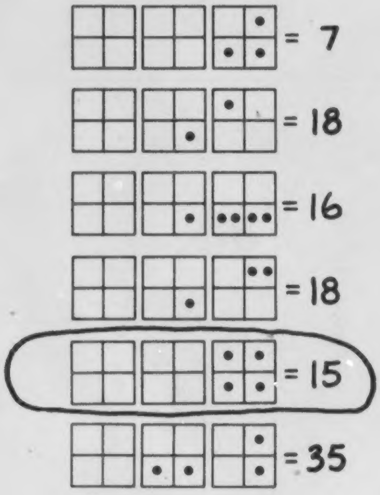
19

Extend the picture to include 15.

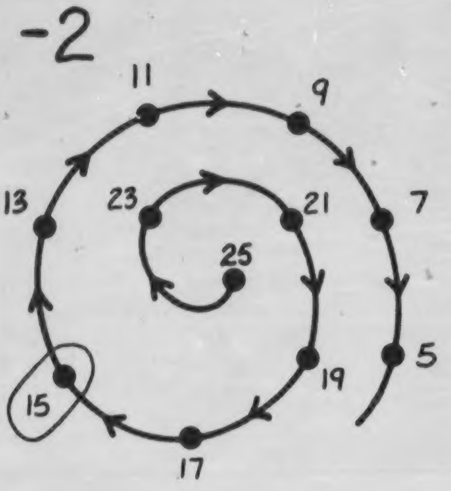

10+10
10+4
2×2
5+5
1+8
3×5
3×4

20

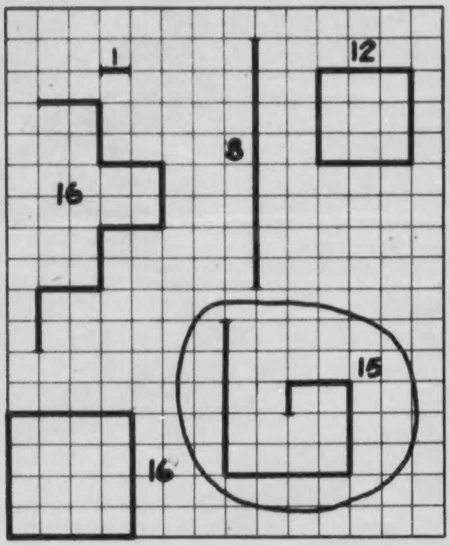
15 was not in the string; extend the picture to include 15. There are many ways to label the dot for 15.


= 7
= 18
= 16
= 18
= 15
= 35

21


-2
11
9
13
23
21
7
25
5
15
19
17

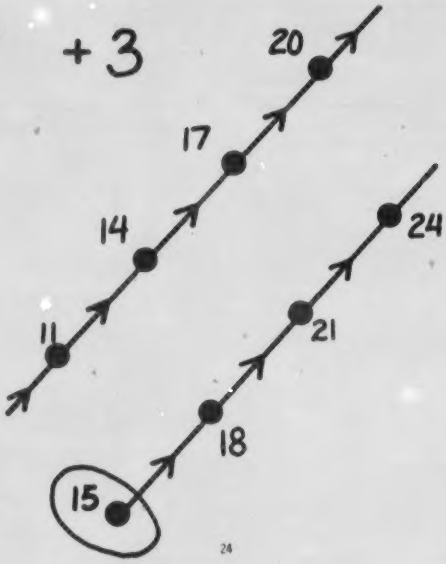
22


1
12
8
16
15
16

23

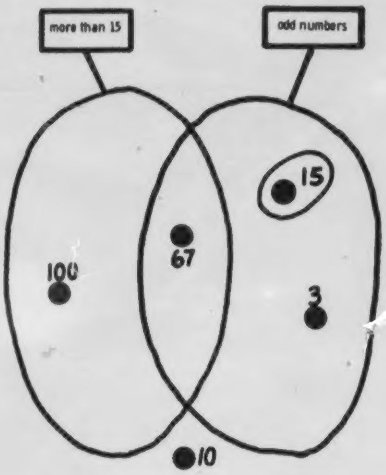
Extend the picture to include 15.

There are many ways to do this.


+3
20
17
14
24
11
21
18
15

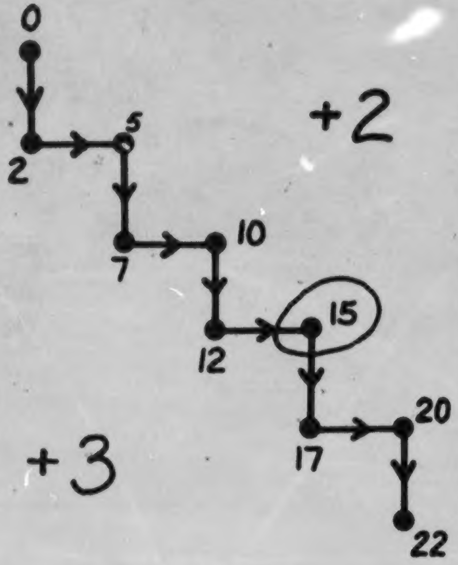
24

Extend the picture to include 15.

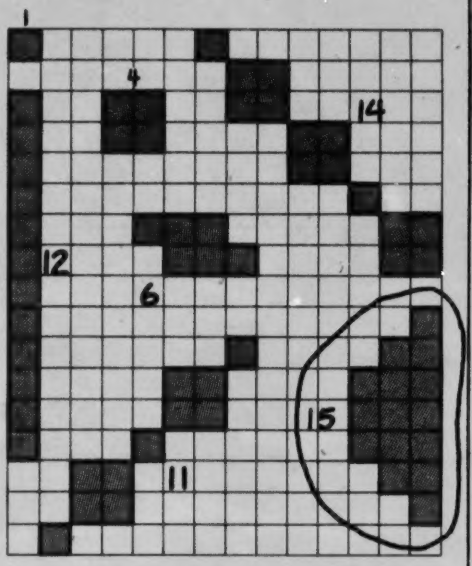

more than 15
odd numbers
15
67
100
3
10

25

15 was not in the picture; include a dot for 15.

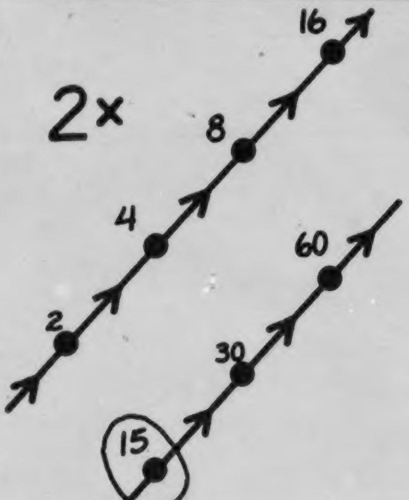

0
2
5
+2
7
10
12
15
17
20
22
+3

26

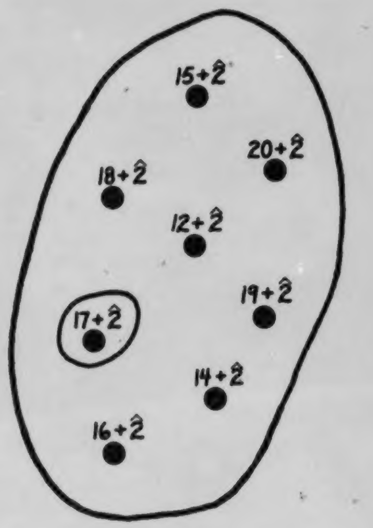

1
4
14
12
6
15
11

27

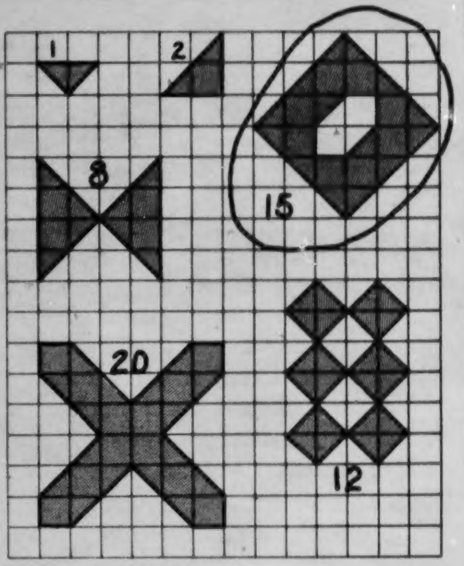
Extend the picture to include 15.
There are many ways to do this.


2x
16
8
4
2
60
30
15

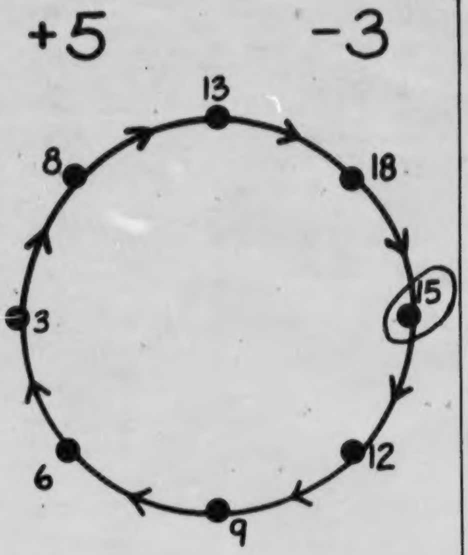
28


15+2̂
20+2̂
18+2̂
12+2̂
19+2̂
17+2̂
14+2̂
16+2̂

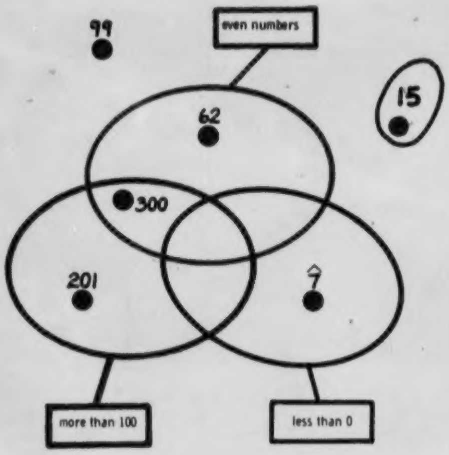
29



30


+5
-3
13
8
18
15
3
12
6
9

31


99
even numbers
15
62
300
201
7
more than 100
less than 0

32

15 was not in the picture; include a dot for 15.

W11 Detective Story #4 / Eli's Peanut Collection

CAPSULE LESSON SUMMARY

Solve a detective story with clues involving the Minicomputer and two arrow pictures. Pose a problem in which Eli picks peanuts from three different bushes. What could be in his bag after the peanuts meet? Display the possibilities on the Minicomputer.

MATERIALS

Teacher: Colored chalk; Minicomputer set

Student: None

DESCRIPTION OF LESSON

Exercise 1: Detective Story

Write a large "14" on a slip of paper and fold it so that the "14" is hidden.

T: The name of a secret number is written on this paper. Listen carefully to my clues and you can discover it.

Display one Minicomputer board.

T: The secret number is on the Minicomputer using exactly three (regular) checkers. You cannot see the checkers because they are invisible.

Accept three or four suggestions from the students and check each suggestion on the Minicomputer. As they are suggested, make a list of numbers that could be the secret number, spacing them so that they will be recorded in order.

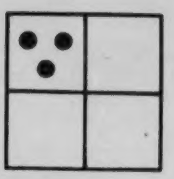
T: What is the smallest number it could be? (3)
What is the largest number it could be? (24)
What other numbers could be the secret number?

W 11

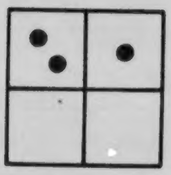
Continue until all the possible numbers have been found.

3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,20,24

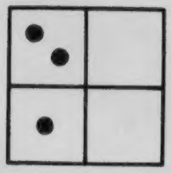
NOTE: The largest number which can be put on the ones' board of the Minicomputer with three checkers is 24, all three checkers on the brown square.



The next largest number is 20 with two checkers on the brown square and one checker on the purple square.



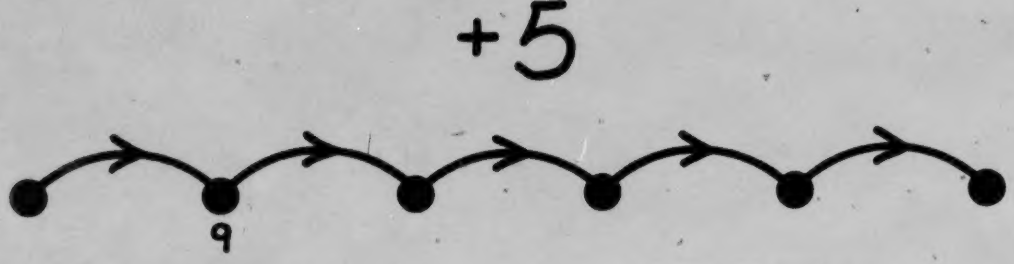
The next number would be 18.



Therefore, it is impossible to put 19, 21, 22, and 23 on the ones' board of the Minicomputer with only three checkers. 15 is also impossible because it requires at least four checkers, one checker on each of the four squares of the ones' board.

T: Now we know that the secret number is one of these numbers, but we don't know which. We need another clue.

Draw this arrow picture on the board.



T: The secret number is one of the numbers in this arrow picture. Which numbers could it be?

Invite students to label the dots.

Remind the class of the first clue by pointing to the list of numbers on the board.

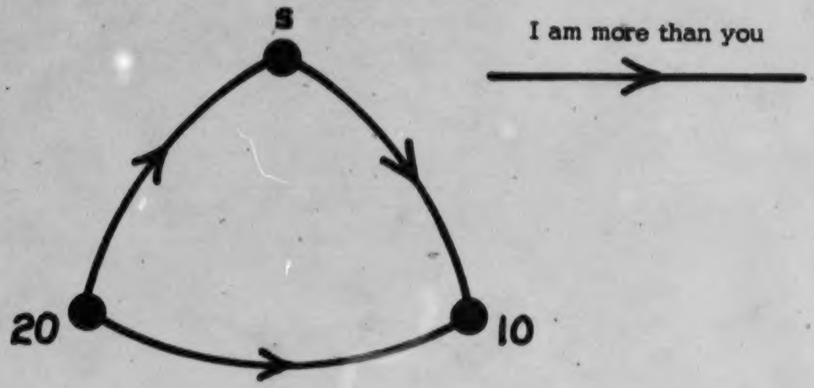
T: Which of these numbers cannot be the secret number? Tell me which ones to cross off our list.

~~3~~, 4, ~~5~~, ~~6~~, ~~7~~, ~~8~~, 9, ~~10~~, ~~11~~, ~~12~~, ~~13~~, 14, ~~16~~, ~~17~~, ~~18~~, ~~20~~, 24

T: Now we know that the secret number could be 4, 9, 14, or 24. Here is my last clue.

W 11

Erase the + 5 arrow picture before drawing this one.



T: Our secret number is here (point to "s") in this arrow picture. Tell me with whispers which number is the secret number.

Accept whispers from many students and then ask a student to give the answer aloud.

S: 14.

Check each of the remaining numbers on the list and then reveal "14" is writwritten on the paper.

Exercise 2: Eli the Elephant

T: This morning Eli the Elephant went for a walk. Along the way he pulled up three peanut bushes. He put four peanuts from the first bush, one peanut from the second bush, and two peanuts from the third bush in his bag. How many peanuts do you think Eli has in his bag?

Allow the class to discuss this problem. Perhaps a student will suggest that some, or all, of the peanuts may have been magic peanuts; if necessary suggest this yourself.

Display one Minicomputer board.

T: How many peanuts from the first bush did Eli put in his bag?

S: 4.

T: Were they regular or magic peanuts?

S: We don't know.

NOTE: It is possible that a student will suggest that regular and magic peanuts came from the same bush. This is an interesting idea, but the lesson will become very complicated if you accept this possibility. Assure the students that only magic peanuts grow on magic peanut plants and only regular peanuts grow on regular peanut plants.

T: We really don't know for sure what kind of peanuts they were, but let's suppose they were four regular peanuts.

Put a regular checker on the 4-square of the Minicomputer.

T: How many peanuts from the second bush did Eli put in his bag?

S: 1.

T: Was it a regular or a magic peanut?

S: It could be either kind.

T: Let's suppose it was a regular peanut.

Put a regular checker on the 1-square of the Minicomputer.

W 11

T: How many peanuts from the third bush did Eli put in his bag?

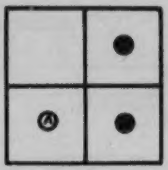
S: 2.

T: What kind of peanuts were they?

S: We don't know.

T: Let's suppose they were two magic peanuts.

Put a negative checker on the 2-square of the Minicomputer.



T: How many regular peanuts did Eli put in his bag altogether?

S: 5.

T: How many magic peanuts did Eli put in his bag altogether?

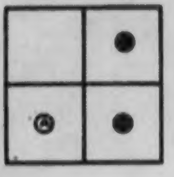
S: 2.

Write "$5 + \widehat{2}$" on the board to the right of the Minicomputer.

T: If Eli picked five regular peanuts and two magic peanuts, what will there be in Eli's bag?

S: Three regular peanuts.

Complete the number story on the board.

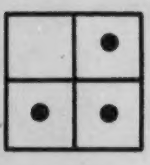


$= 5 + \widehat{2} = 3$

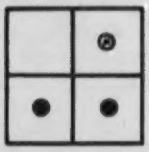
If your class is not certain that the number on the Minicomputer is 3, draw a picture of a bag with five regular and two magic peanuts. Ask a student to show which peanuts would disappear.

T: Eli could have three regular peanuts in his bag. Do you think Eli could have a different number of peanuts in his bag?

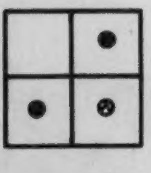
Let the students make suggestions about the kind of peanuts Eli could put in his bag. Represent each new possibility on the Minicomputer and in a number story. There are eight different possibilities (see the next illustration), but do not expect your students to find all of them.



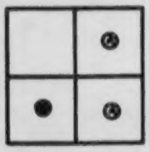
$= 7$



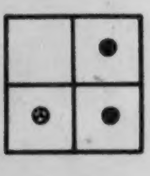
$= \widehat{4} + 3 = \widehat{1}$



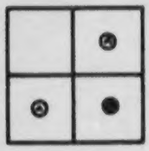
$= 6 + \widehat{1} = 5$



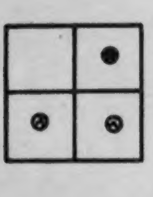
$= \widehat{5} + 2 = \widehat{3}$



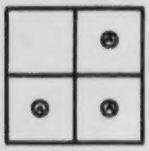
$= 5 + \widehat{2} = 3$



$= \widehat{6} + 1 = \widehat{5}$



$= 4 + \widehat{3} = 1$



$= \widehat{7}$

W 11

Continue this activity as long as there is student interest. Emphasize that we are not sure which kind of peanuts grow on the three bushes so there are several possible answers to the question, "How many peanuts does Eli have in his bag?"

W12 A Catalog of Problems #2 (Lesson 1)

CAPSULE LESSON SUMMARY

Extend the experiences in various strands in the Workbook "A Catalog of Problems #2".

MATERIALS

Teacher: (If needed) Minicomputer set; colored chalk; A-Blocks

Student: Workbook "A Catalog of Problems #2"; pencil; colored pencils

DESCRIPTION OF LESSON

Distribute copies of the Workbook "A Catalog of Problems #2" and let the students begin working independently. If the questions you are receiving indicate that many of your students are having difficulty with a particular page, hold a collective discussion about that page.

Allow about 30 minutes of the lesson for the workbook and then collect them for use in Lesson W13. Spend the remaining time playing the Number Line Game.

W 13

W13 A Catalog of Problems #2 (Lesson 2)

CAPSULE LESSON SUMMARY

To build a road between 20 and 32 using +4 and +1 arrows, decide the road must start from 20 and then proceed to build it. Continue working in the Workbook "A Catalog of Problems #2". (This is the second of two lessons using this workbook.)

MATERIALS

Teacher: Colored chalk

Student: Workbook "A Catalog of Problems #2"; colored pencils

DESCRIPTION OF LESSON

On the board draw two dots, one dot to the right of the other with plenty of space between them.

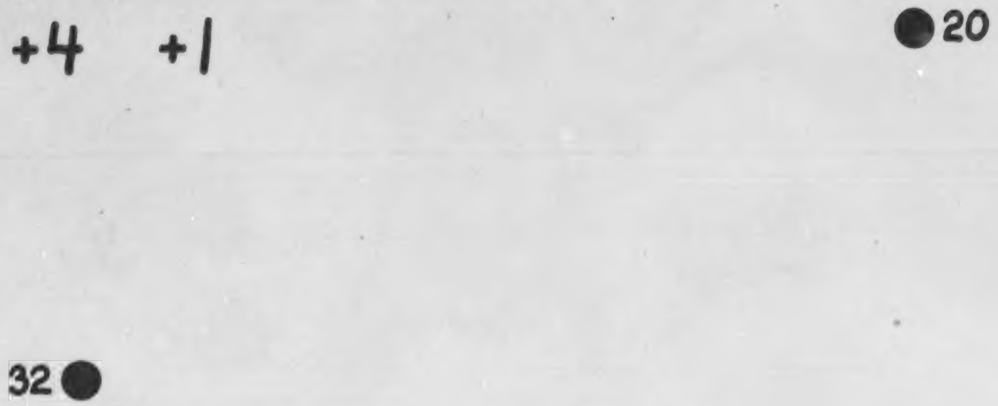
T: These dots are for numbers. Which of these numbers is larger?

S: We don't know which is larger; the larger number could be in either place.

Label the dot on the right "20" and the dot on the left "32".

T: We are going to build a road between these two numbers with red and blue arrows. The red arrows are for +4 and the blue arrows are for +1.

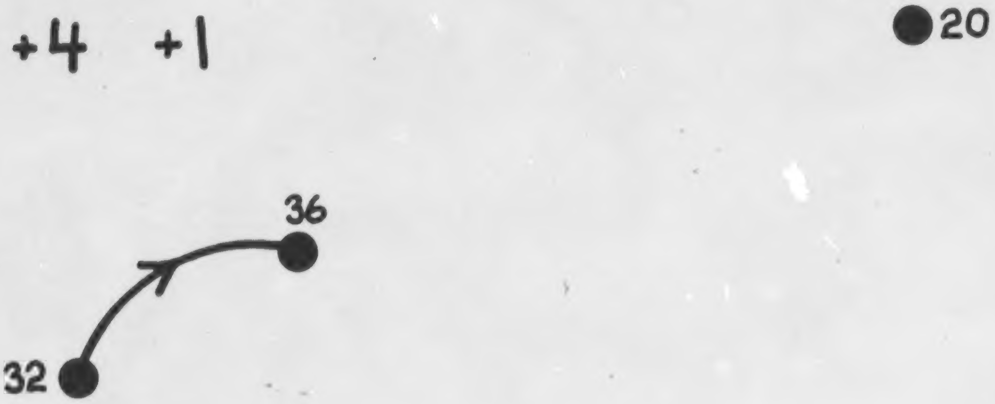
Write the key information "+4" and "+1" on the board.



T: Who would like to draw the first arrow?

This exercise may be too simple for your better students; try to involve other students at this time and call on the better students when the situation becomes more challenging. Allow any student to comment on a suggested arrow.

Perhaps a student will suggest a +4 arrow starting at 32.



Trace the red arrow.

T: Is 32 + 4 = 36?

S: Yes.

W 13

T: Does this arrow help us build a road between 20 and 32?

S: No.

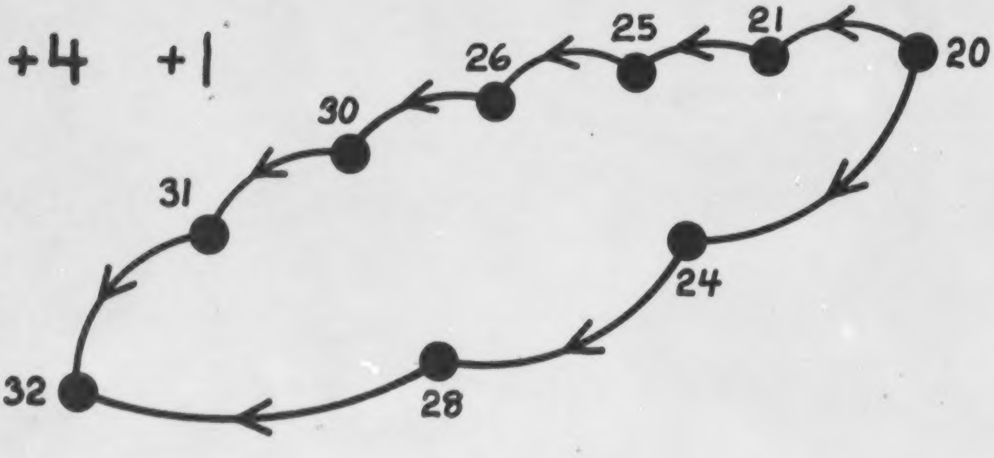
T: Why not?

S: 20 is less than 32, but if you start at 32 and can only use +4 and +1 arrows, the number will get larger.

Do not expect students to be able to express this idea precisely. Erase the arrow from 32 to 36 and the dot for 36 and ask for another way to start the road. Continue asking students to draw arrows until a road is built between 20 and 32.

T: How many +1 arrows are in the road we built? (0, 4, 8, or 12) Could we build a shorter road?

If the road built contains any +1 arrows, some students should observe that they can replace every four +1 arrows with one +4 arrow. In any case, the students should conclude the shortest road between 20 and 32 contains three +4 arrows. It is possible your arrow picture will look like this one.



330

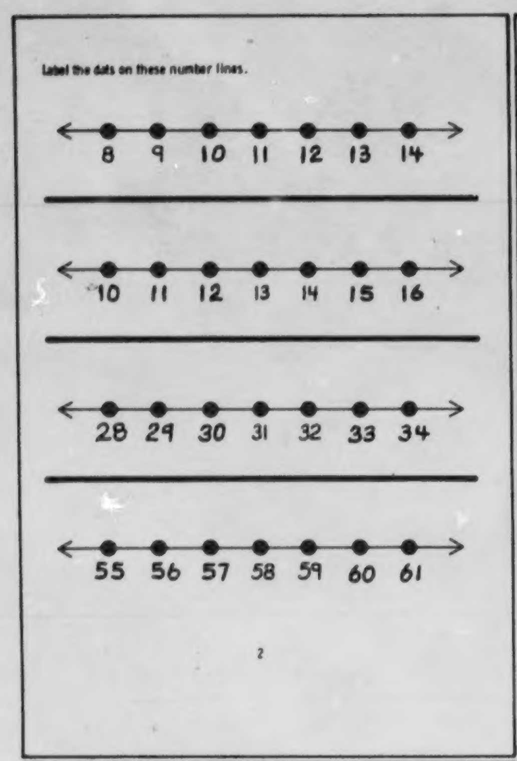
T: What does the longest road between 20 and 32 look like?

S: Twelve + 1 arrows.

Distribute students' copies of the Workbook "A Catalog of Problems #2". Allow about 30 minutes of independent work and then collect the workbooks for your review.

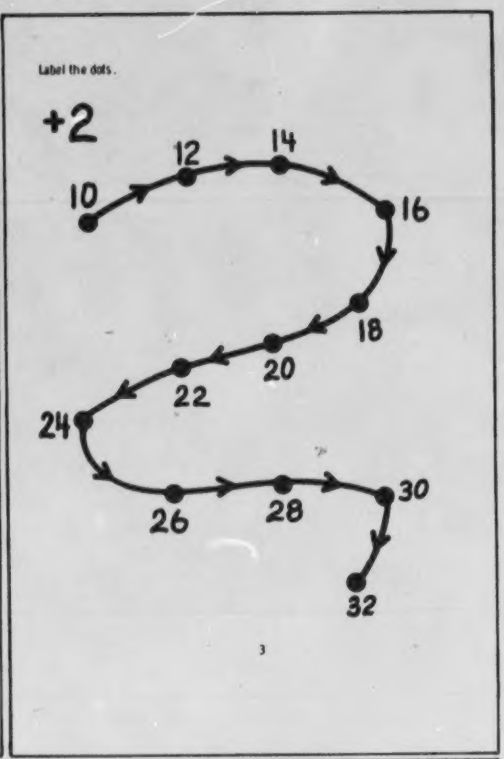
A
CATALOG
OF
PROBLEMS
2

A Catalog of Problems 2



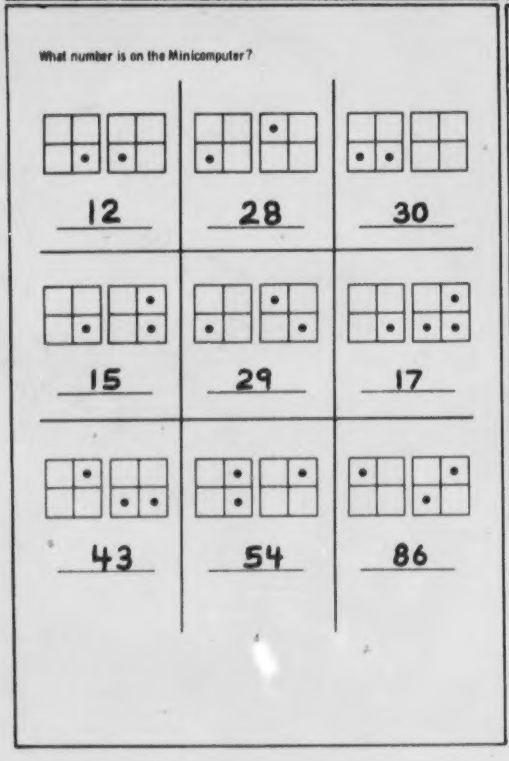
Label the dots on these number lines.

8 9 10 11 12 13 14
10 11 12 13 14 15 16
28 29 30 31 32 33 34
55 56 57 58 59 60 61
2



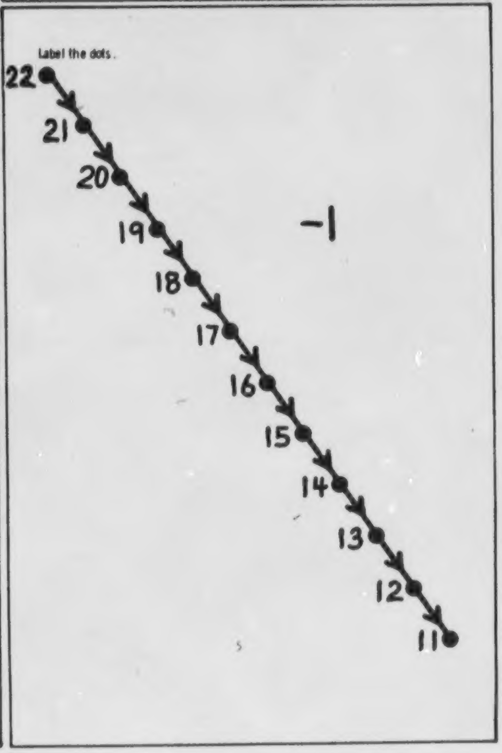
Label the dots.

+2
10 12 14 16 18 20 22 24 26 28 30 32
3



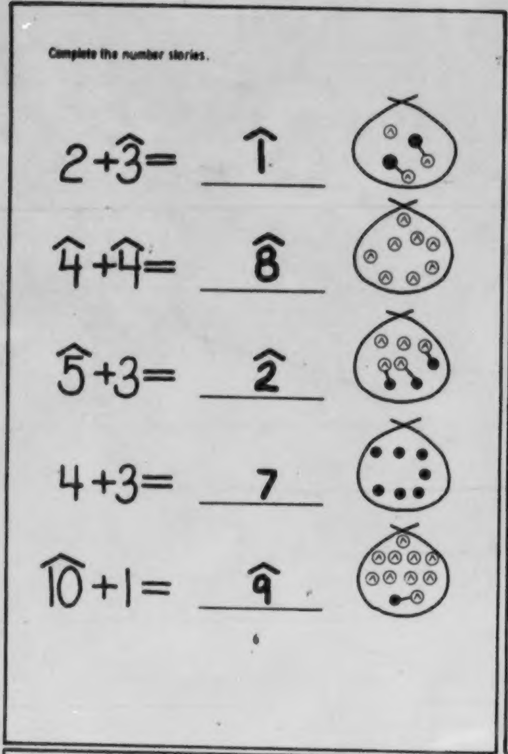
What number is on the Minicomputer?

12
28
30
15
29
17
43
54
86
4



Label the dots.

-1
22 21 20 19 18 17 16 15 14 13 12 11
5



Complete the number stories.

$2+\hat{3}=\hat{1}$

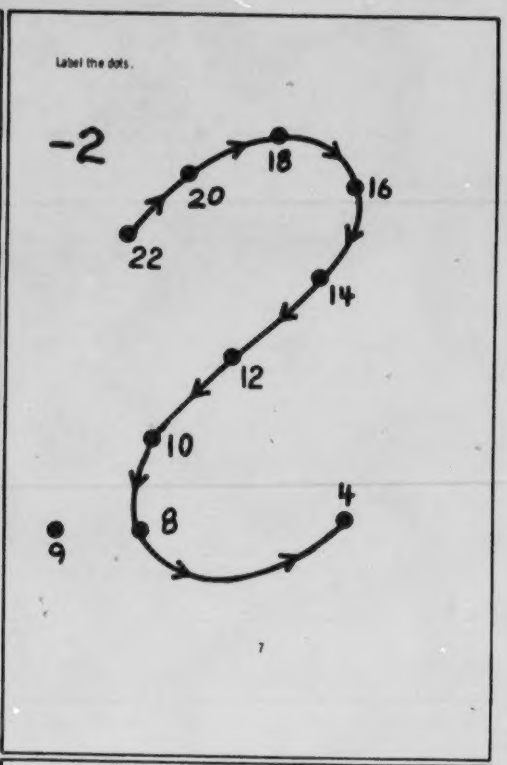
$\hat{4}+\hat{4}=\hat{8}$

$\hat{5}+3=\hat{2}$

$4+3=7$

$\hat{10}+1=\hat{9}$

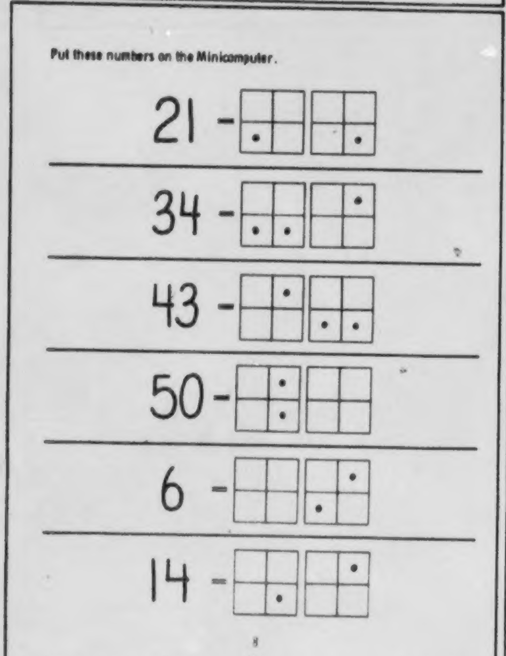
6



Label the dots.

-2
22
20
18
16
14
12
10
8
4
9

7

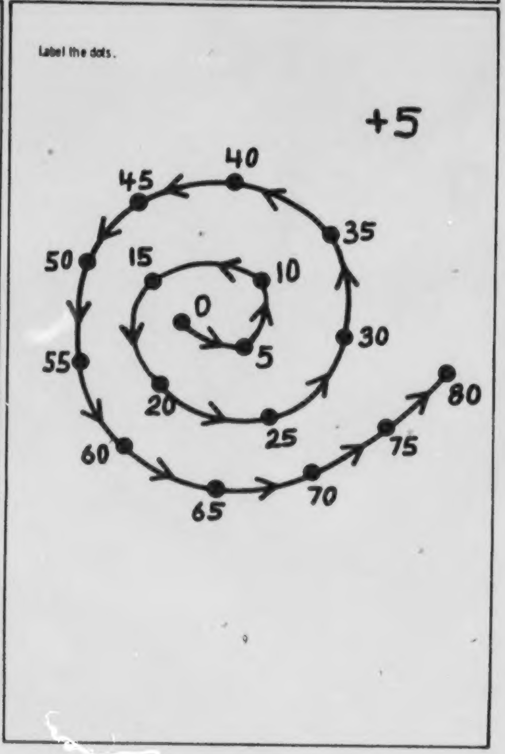


Put these numbers on the Minicomputer.

21 -
34 -
43 -
50 -
6 -
14 -

8

Standard configuration is given here for each number; however, many other solutions are possible.

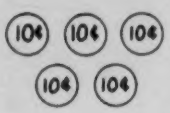


Label the dots.

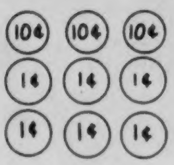
+5
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80

9

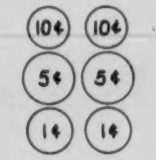
How much money?


10¢
10¢
10¢
10¢
10¢

50 ¢

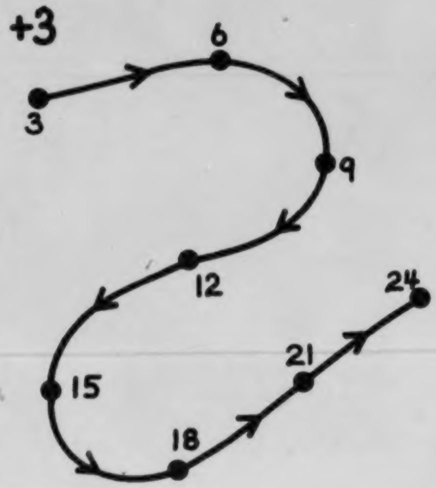

10¢
10¢
10¢
1¢
1¢
1¢
1¢
1¢
1¢

36 ¢



32 ¢

Label the dots.


+3
3
6
9
12
15
18
21
24

Complete these number stories.

$4 + 3 = 7$

$4 + \boxed{4} = 8$

$4 + \boxed{5} = 9$

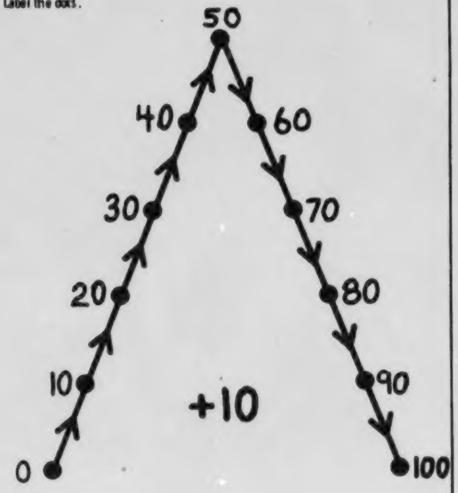
$4 + \boxed{6} = 10$

$4 + 7 = \boxed{11}$

$4 + 8 = \boxed{12}$

$4 + 9 = \boxed{13}$

Label the dots.


50
40
60
30
70
20
80
10
90
+10
0
100

CODE

A - 1
B - 2
C - 3
D - 4
E - 5
F - 6
G - 7
H - 8
I - 9
J - 10
K - 11
L - 12
M - 13
N - 14
O - 15
P - 16
Q - 17
R - 18
S - 19
T - 20
U - 21
V - 22
W - 23
X - 24
Y - 25
Z - 26

Decode:

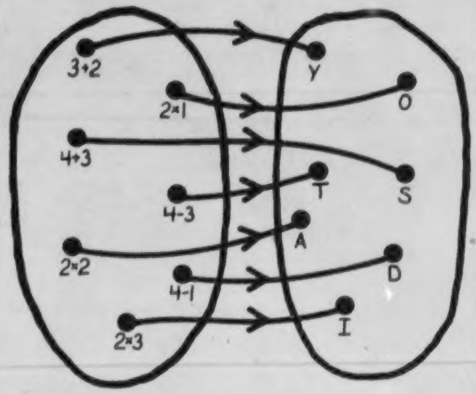
H	O	W
5+3	13+2	20+3
O	L	D
10+5	6+6	2×2
A	R	E
6−5	9+9	10−5
Y	O	U ?
21+4	9+6	22−1

Answer: 7 (or 8)

14

Other answers are possible depending on the student.

What is the secret message?

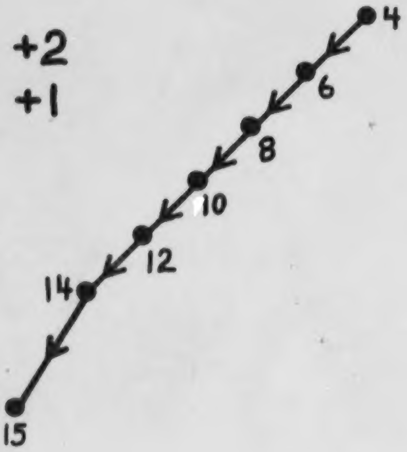

3+2
2×1
4+3
4-3
2×2
4-1
2×3
Y
O
S
T
A
D
I

TODAY IS *________________

15

*Answer depends on which day of the week today is.

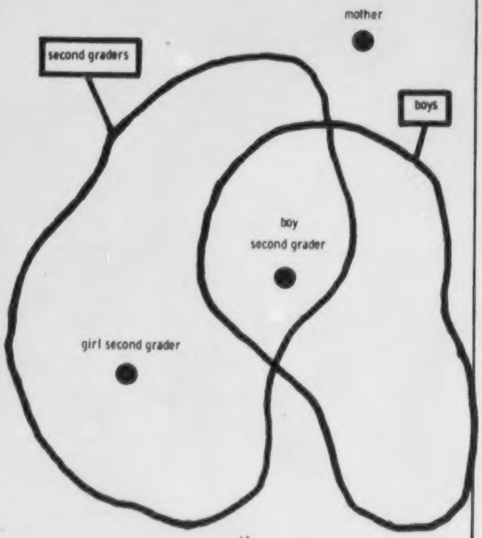
Build a red-blue road from 4 to 15. Use only +2 and +1 arrows.


+2
+1
4
6
8
10
12
14
15

16

Many solutions are possible.

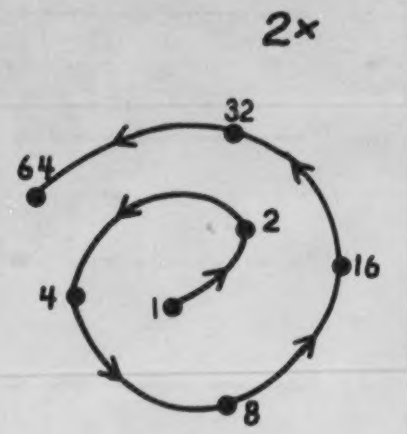
Put a dot for yourself. Put a dot for your mother.


mother
second graders
boys
boy second grader
girl second grader

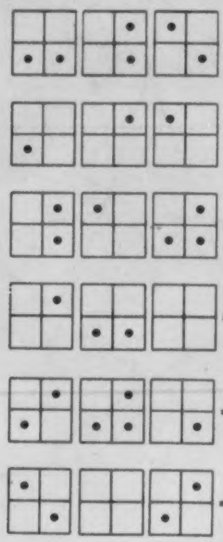
17

The dot for a girl (boy) who is not a second grader is outside both strings (outside the blue string and inside the red string).

Label the dots.


2x
1
2
4
8
16
32
64

What number is on the Minicomputer?



– 359

– 248

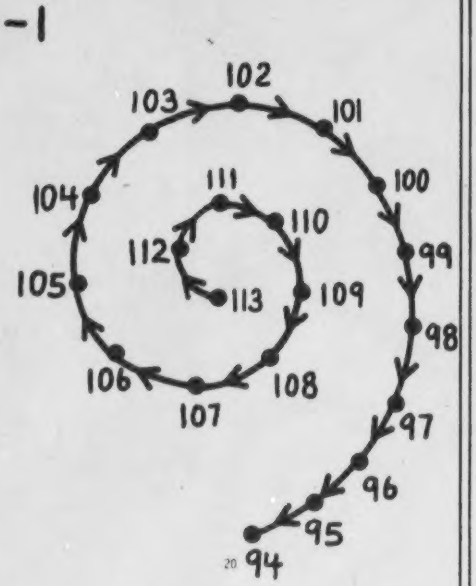
– 5[illegible]

– 430

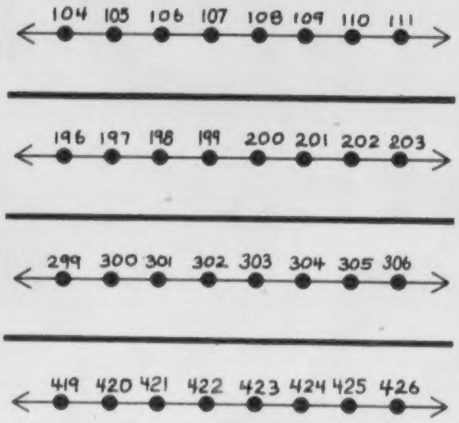
– 671

– 906

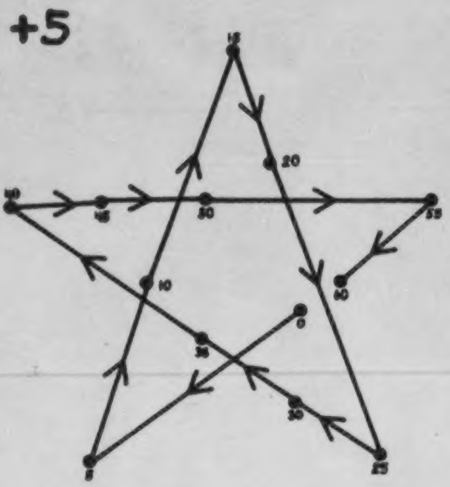
Label the dots.


-1
113
112
111
110
109
108
107
106
105
104
103
102
101
100
99
98
97
96
95
94

Label the dots on these number lines.

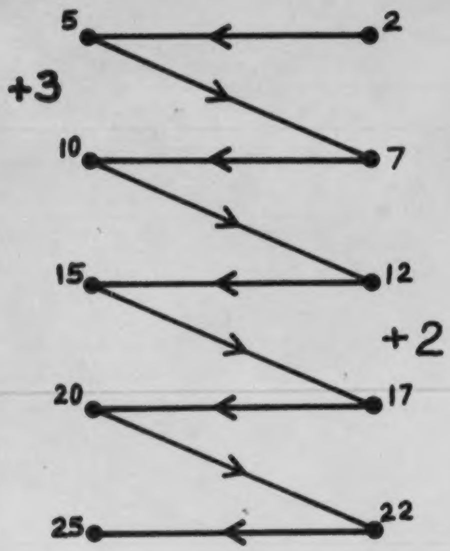

104 105 106 107 108 109 110 111
196 197 198 199 200 201 202 203
299 300 301 302 303 304 305 306
419 420 421 422 423 424 425 426

Draw all the red arrows.


+5
15
20
40
45
50
55
60
10
0
35
30
5
25

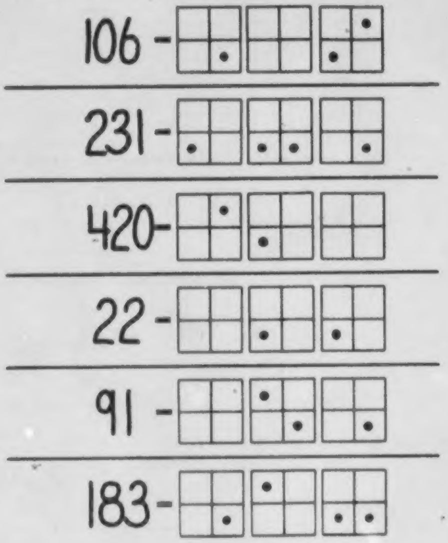
22

Label the dots.


5
2
+3
10
7
15
12
+2
20
17
25
22

23

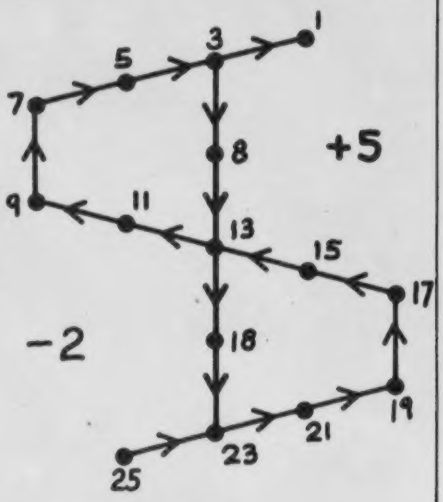
Put the numbers on the Minicomputer.


106 -
231 -
420 -
22 -
91 -
183 -

24

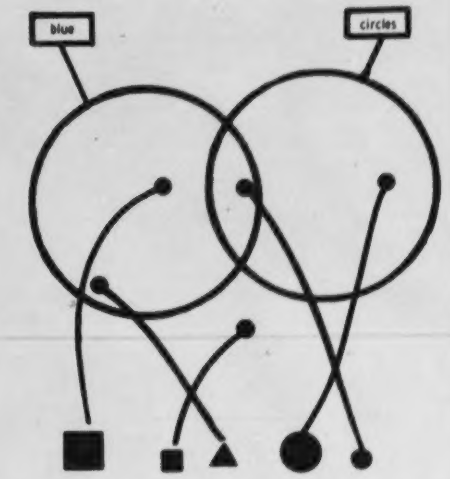
Standard configuration is given here for each number; however, many other solutions are possible.

Label the dots.


1
3
5
7
+5
8
9
11
13
15
17
-2
18
19
21
23
25

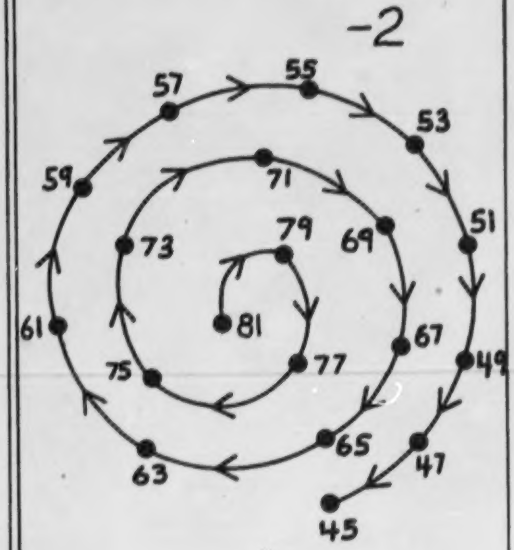
25

Match each dot with an A-block. One is done for you.


blue
circles

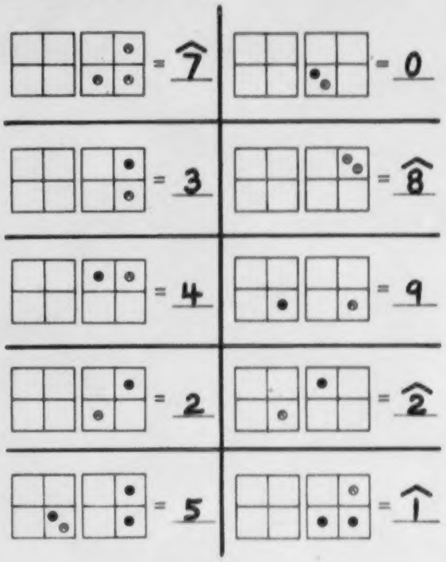
26

Label the dots.


-2
57
55
53
59
71
51
73
79
69
61
81
67
49
75
77
65
63
47
45

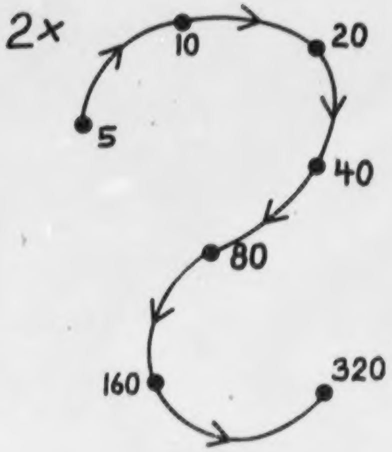
27

What number is on the Minicomputer?

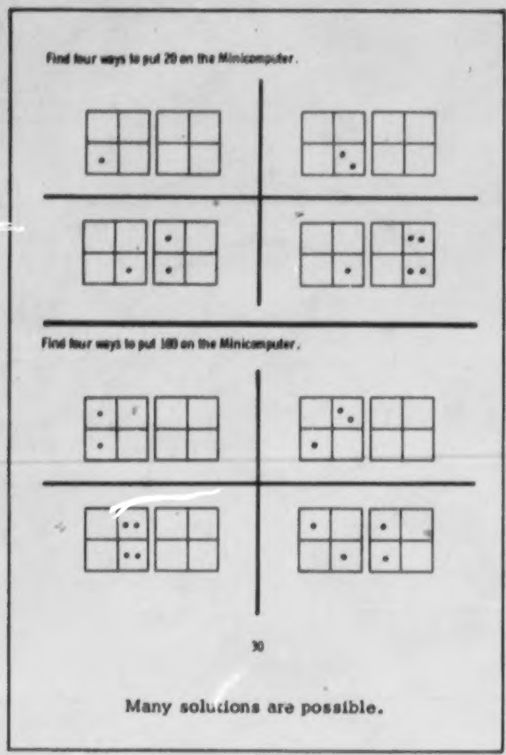

= 7̂
= 0
= 3
= 8̂
= 4
= 9
= 2
= 2̂
= 5
= 1̂

28

Label the dots.


2x
10
20
5
40
80
160
320

29

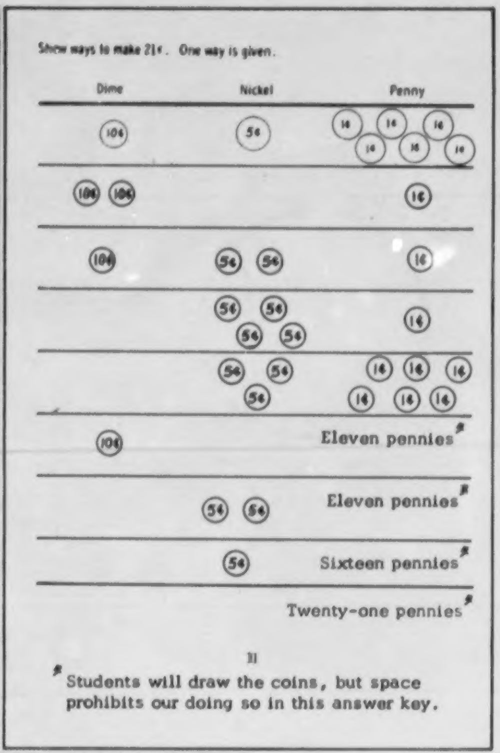


Find four ways to put 20 on the Minicomputer.

Find four ways to put 100 on the Minicomputer.

30

Many solutions are possible.

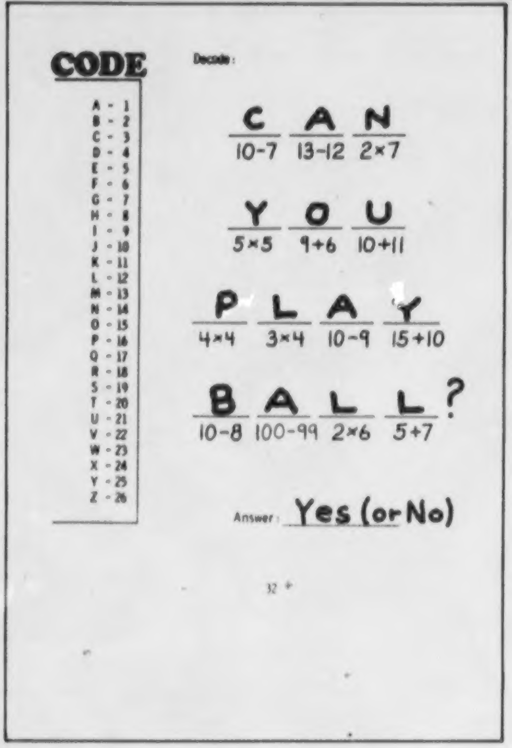


Show ways to make 21¢. One way is given.

Dime	Nickel	Penny
10¢	5¢	1¢ 1¢ 1¢ 1¢ 1¢ 1¢
10¢ 10¢		1¢
10¢	5¢ 5¢	1¢
	5¢ 5¢ 5¢ 5¢	1¢
	5¢ 5¢ 5¢	1¢ 1¢ 1¢ 1¢ 1¢ 1¢
10¢		Eleven pennies*
	5¢ 5¢	Eleven pennies*
	5¢	Sixteen pennies*
		Twenty-one pennies*

31

* Students will draw the coins, but space prohibits our doing so in this answer key.



CODE

A - 1
B - 2
C - 3
D - 4
E - 5
F - 6
G - 7
H - 8
I - 9
J - 10
K - 11
L - 12
M - 13
N - 14
O - 15
P - 16
Q - 17
R - 18
S - 19
T - 20
U - 21
V - 22
W - 23
X - 24
Y - 25
Z - 26

Decode:

C	A	N
10-7	13-12	2×7

Y	O	U
5×5	9+6	10+11

P	L	A	Y
4×4	3×4	10-9	15+10

B	A	L	L?
10-8	100-99	2×6	5+7

Answer: Yes (or No)

32

W14 Detective Story #5 / Eli Visits His Grandmother

CAPSULE LESSON SUMMARY

Solve a detective story with clues involving the hand-calculator, a string picture, and the Minicomputer. Present an Eli the Elephant episode. After picking many peanuts, Eli arrives at his grandmother's to find only one peanut in his bag. What could have happened? Discuss the possibilities.

MATERIALS

Teacher:	Hand-calculator* (two, or more if possible); colored chalk; Minicomputer set
Student:	None

DESCRIPTION OF LESSON

Exercise 1: Detective Story

Since this is the first lesson using hand-calculators, you may want to give calculators to students who have had some experience with them before, either at home or at school. Slowly and clearly announce each step on the calculator for the entire class. The students with calculators and those close enough to observe the display of the calculator can verify what number is on the display.

Write a large "32" on a slip of paper and fold it so that the "32" is hidden.

T: Today you must discover a secret number. These calculators will help us with the first clue. We are going to press some buttons; I will tell you which ones. One of the numbers which will appear on the calculator's display will be the secret number.

Ask students with calculators to turn them "on" and remind these students to do exactly what you say. Give the following instructions.

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* See NOTE ON THE USE OF HAND-CALCULATORS in the General Introduction.

W 14

T: Start with 0 on the display. Press $\boxed{2}$. What number is on the display?

S: 2.

Record "2" on the board.

T: Press $\boxed{+}\boxed{5}\boxed{=}$.
What number is on the display?

S: 7.

T: What did we do?

S: We added 5; 2 + 5 = 7.

Record "7" on the board.

T: Press $\boxed{=}$ again. What number is on the display?

S: 12.

T: What did the calculator do?

S: Added 5 again; 7 + 5 = 12.

Record "12" on the board.

T: We can add 5 again by pressing $\boxed{=}$ again.
Press $\boxed{=}$.
What number is on the display?

S: 17.

Continue your list on the board.

2, 7, 12, 17

T: Suppose we continue pressing $\boxed{=}$. What are the next three numbers we will get on the calculator?

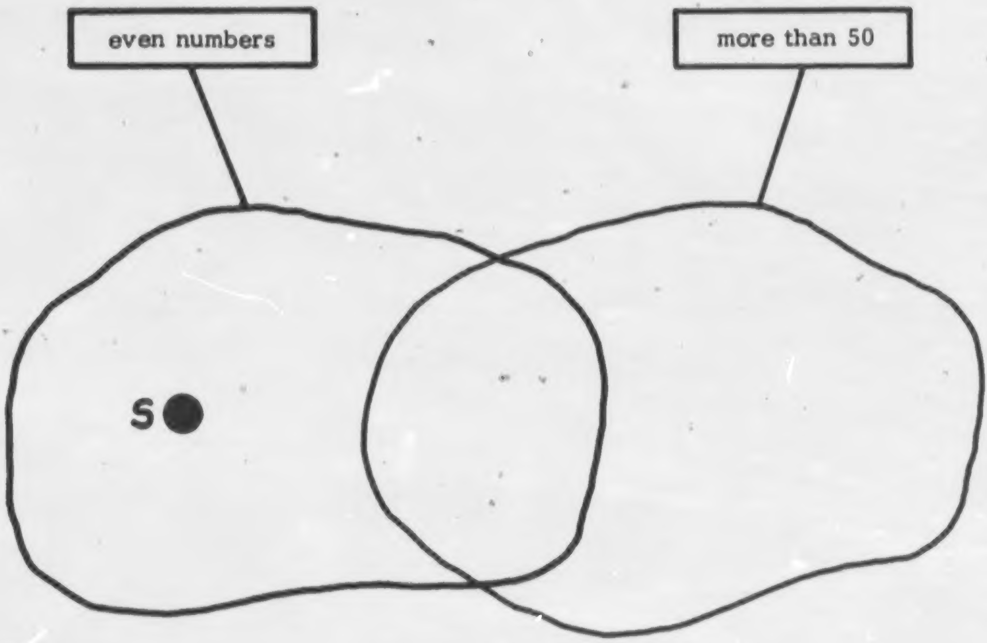
Allow the class to predict these numbers. Then record them on the board as the students press $\boxed{=}$ three more times and note them on their displays. As soon as most of your students are able to predict the sequence quickly you can abandon the calculator. Continue counting and recording until the class realizes this sequence goes on and on. Your list on the board should have several numbers greater than 50.

2,7,12,17,22,27,32,37,42,47,52,57,62,67,72,...

Explain that " . . . " means that the list goes on forever.

T: Our secret number is one of these numbers.
The second clue is a string picture.

Draw this picture on the board.



W 14

T: Here is the secret number (point to "s").

Is it an even number? (Yes) Is it more than 50? (No)

Which of these numbers could be the secret number?

Each time a number is suggested, label the dot with that number and ask the class if the number is placed correctly. Suppose, for example, a student suggests 72.

T: Could this be 72? (Disagreement)

Is 72 even? (Yes)

Then it must be inside the red string.

Is 72 more than 50? (Yes)

Then it must be inside the blue string.

Where should 72 be?

Ask a student to point to the correct region.

T: 72 cannot be the secret number.

Erase "72" from the string picture and cross "72" off the list on the board.

If a correct number is suggested, elicit class agreement and circle it on your list. Continue until many of the numbers in the original list are either circled or crossed off.

Point to the three dots.

T: Could the secret number be any of the numbers we didn't write on the board? (No) How do you know?

S: Because the secret number is not more than 50.

Cross off the three dots.

Point to each unchecked number and guide your class in deciding if it could be the secret number. When all the numbers have been checked, erase those which have been crossed off.

T: Now we know the secret number is one of these (point to the list of remaining numbers).

2, 12, 22, 32, 42

Display two Minicomputer boards.

T: This will be the final clue. When you know which of these numbers is the secret number, tell me with whispers.

At least three (regular) checkers are needed to put the secret number on the Minicomputer. We cannot use less than three checkers to put it on the Minicomputer. Who is the secret number?

Accept several whispers and after most of the students realize that 32 is the secret number, ask a student to say the secret number aloud. Check the other numbers on the Minicomputer by letting students demonstrate that each of them can be put on the Minicomputer with less than three checkers. Reveal that "32" is written on your paper.

Exercise 2: Eli the Elephant

T: Today Eli the Elephant went to visit his grandmother. On the way to her house, he took a detour through a field where he had seen many peanut plants. Eli's grandmother loves peanuts so Eli planned to bring her a bag full of peanuts as a surprise.

Eli spent several hours gathering peanuts. He was thinking about how pleased his grandmother would be when he gave her the bag of peanuts.

Eli's grandmother met him at the gate and gave Eli a big hug. Eli held out the bag and said: "Grandmother, I've brought you a bag of peanuts as a present." Eli's grandmother was very happy. She gave Eli a big elephant kiss and went to bring a bowl for the peanuts. She shook the bag and out fell - one peanut! She searched inside the bag and then Eli searched inside the bag, but there weren't any more peanuts. Eli felt very confused.

What do you think happened to the peanuts Eli picked?

Allow the students to comment. Very likely someone will suggest that the other peanuts disappeared. If a student suggests that Eli ate the other peanuts, or that there was a hole in the bag, or any other reasonable answer, accept it as a possibility.

T: Why would the peanuts disappear?

Guide this discussion until it is clear that the same number of regular and magic peanuts have disappeared.

T: Do we know how many peanuts Eli put in his bag?

S: No.

If students begin to give you suggestions of how many regular and magic peanuts Eli picked, record them on the board. If no one suggests any specific numbers, ask:

T: How many regular and magic peanuts could Eli have picked?

Each suggestion should be considered to see if all but one peanut would disappear. Write the corresponding number story for each suggestion. Some possible correct answers are on the next page.

$$10 + \widehat{9} = 1$$ (one regular peanut left in Eli's bag)

$$\widehat{10} + 9 = \widehat{1}$$ (one magic peanut left in Eli's bag)

$$58 + \widehat{59} = \widehat{1}$$

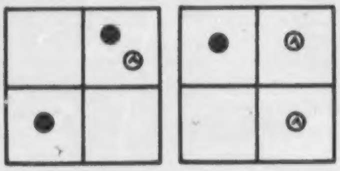
$$\widehat{1,000,000} + 1,000,001 = 1$$

T: Poor Eli, he still doesn't know about magic peanuts. We will never know exactly how many peanuts Eli gathered that day for his grandmother. Another day Eli put 68 regular peanuts and 45 magic peanuts in his bag. When Eli sat down to eat the peanuts, would there be regular or magic peanuts in his bag?

S: Regular.

Write "68 + $\widehat{45}$" on the board. If some students know how many peanuts are in Eli's bag, let them tell you with whispers.

T: Who can put 68 on the Minicomputer with regular checkers? Who can put $\widehat{45}$ on the Minicomputer with negative checkers?



T: What will happen?

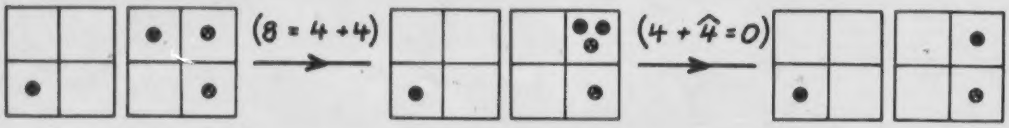
S: The checkers on the 40-square will disappear.

Ask a student to take these checkers off the Minicomputer. If someone suggests that checkers not on the same square will disappear, for example 8 and $\widehat{4}$, remind the class that those checkers are for eight regular peanuts and four magic peanuts.

T: What could we do to make the number easier to read?

S: Make some plays.

This is one possible sequence of plays.



Interrupt the activity to ask these questions.

T: What number do we have with these checkers (point to the regular checkers)?

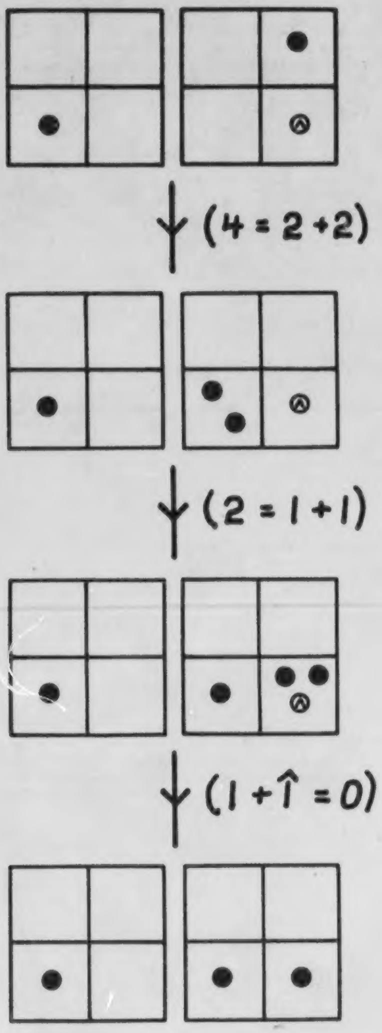
S: 24.

T: What number do we have with this checker (point to the negative checker)?

S: $\hat{1}$.

T: We have 24 + $\hat{1}$ on the Minicomputer. Now some of you might know the number of peanuts Eli has, but let's continue to make some more plays.

Continue until all the negative checkers have been removed from the board; you may have to ask some leading questions or suggest the backward play "4 = 2 + 2" yourself.


(4 = 2 + 2)
(2 = 1 + 1)
(1 + $\hat{1}$ = 0)

T: How many peanuts does Eli have in his bag?

S: 23 regular peanuts.

Complete the number story on the board.

$$68 + \widehat{45}' = 23$$

W 15

W15 Fishing for Numbers (Lesson 1)

CAPSULE LESSON SUMMARY

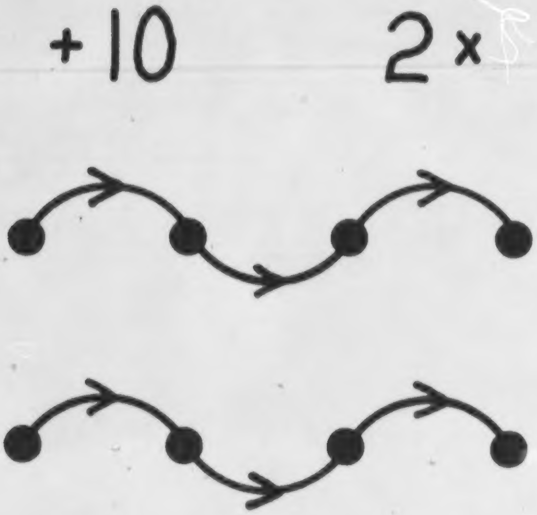
Put the numbers in a string picture in their proper places in an arrow picture. Begin the Workbook "Fishing for Numbers".

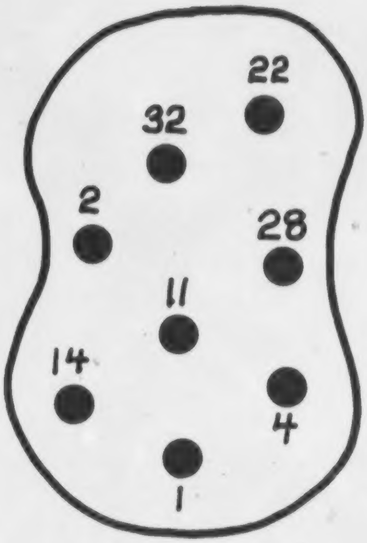
MATERIALS

Teacher: Colored chalk
Student: Workbook "Fishing for Numbers"; pencil

DESCRIPTION OF LESSON

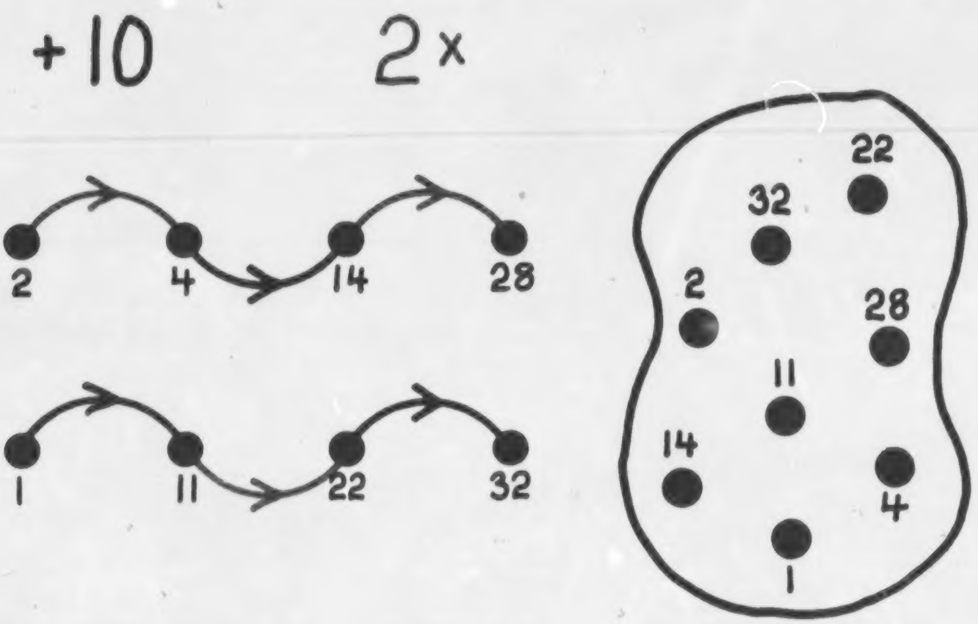
Draw these pictures on the board.





Tell the class that the problem is to put all the numbers shown in the string picture in their proper places in the arrow picture. Ask someone to suggest a number for the first dot of one of the roads. Then pursue that choice and see where it leads. If it leads to an incorrect situation, let the students discover this and discuss why it is wrong and what they should do. A student may suggest that the smallest number must go at the beginning of one of the roads; ask the student to convince the rest of the class. The students might suggest crossing out the numbers inside the string after they have been placed in the arrow picture. This procedure is questionable since in a trial and error method many mistakes may be made and new starts may be necessary. In the course of solving this problem collectively, the opportunity to discuss various strategies should arise, but allow ample opportunity for students to discover some good strategies themselves.

The completed arrow picture is given below.



Distribute copies of the Workbook "Fishing for Numbers". At the end of the lesson collect the workbooks for use in Lesson W16.

W 16

W16 Fishing for Numbers (Lesson 2)

CAPSULE LESSON SUMMARY

Put all the numbers in a string picture in their proper places in an arrow picture. Continue working on "Fishing for Numbers". (This is the second of two lessons using this workbook.)

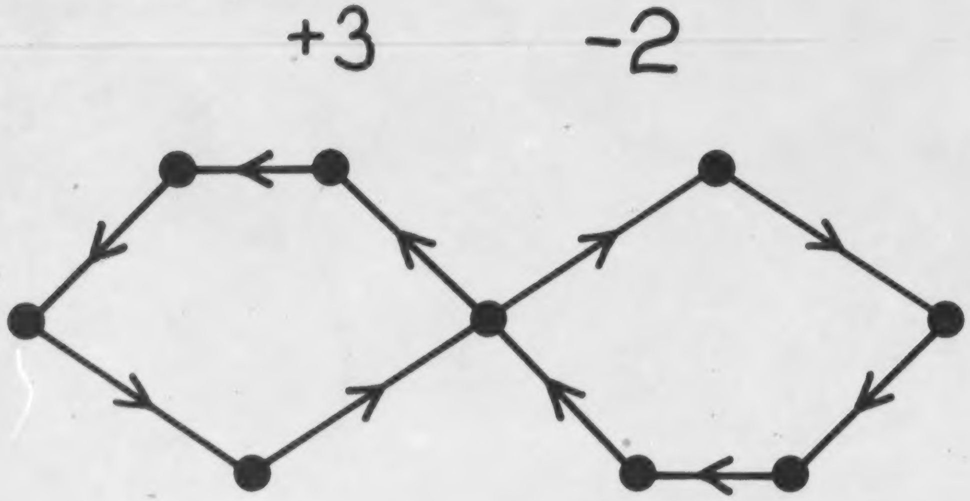
MATERIALS

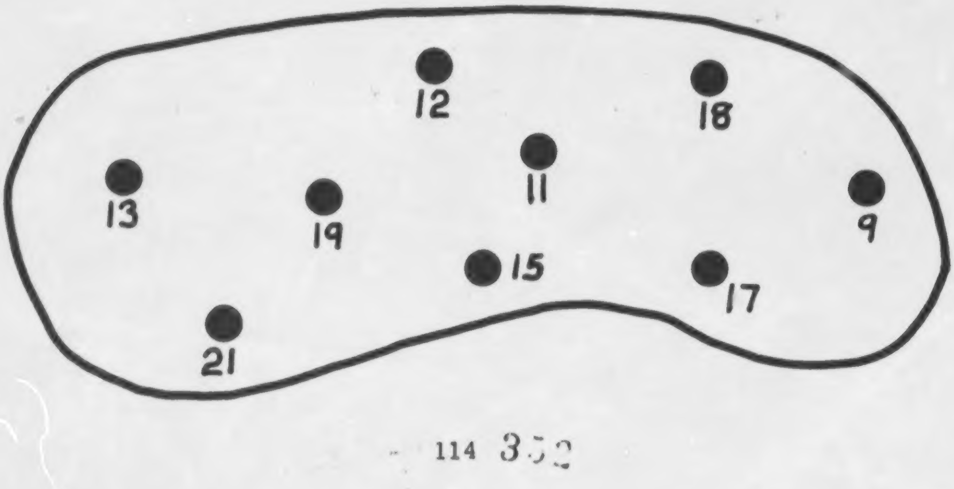
Teacher: Colored chalk

Student: Workbook "Fishing for Numbers"; pencil

DESCRIPTION OF LESSON

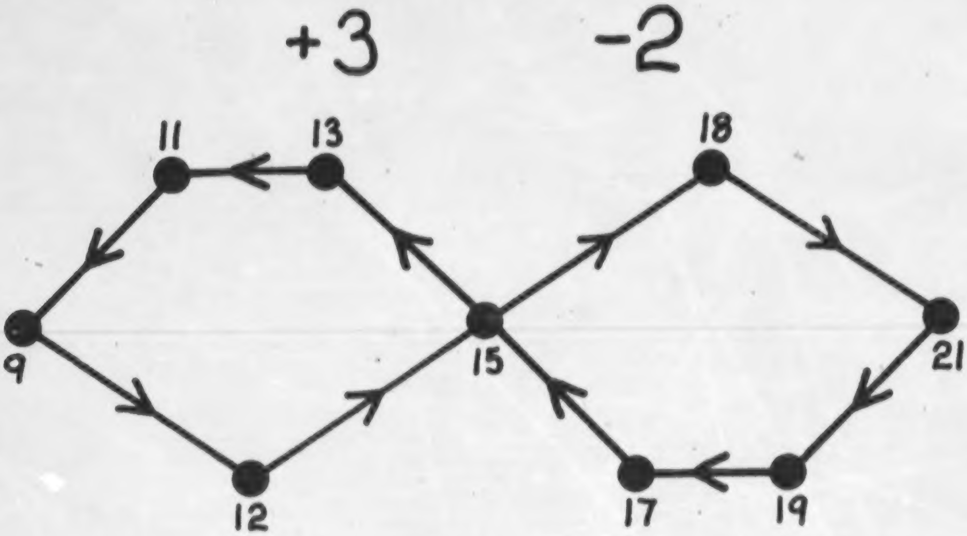
Draw these pictures on the board.

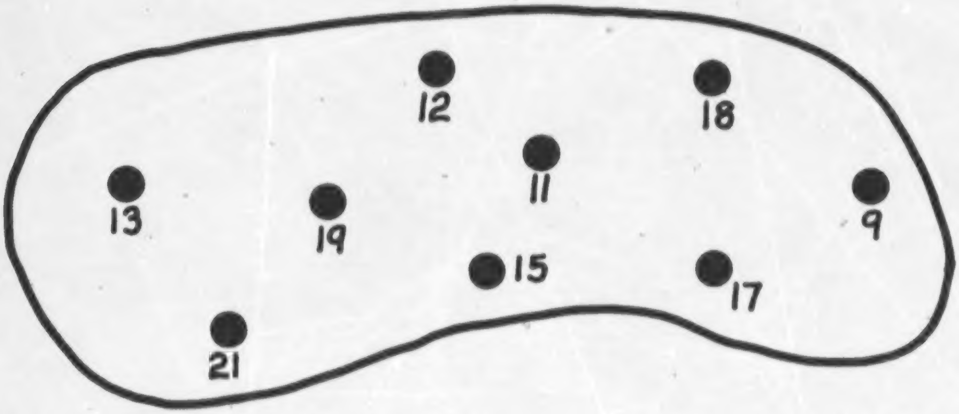




Tell the class that the problem is to put all the numbers in the string picture in their proper places in the arrow picture. Let students label dots by trial and error. Encourage the strategy of finding where the smallest number is (at the far left) and where the largest number is (at the far right).

Continue until all the dots in the arrow picture are labeled.



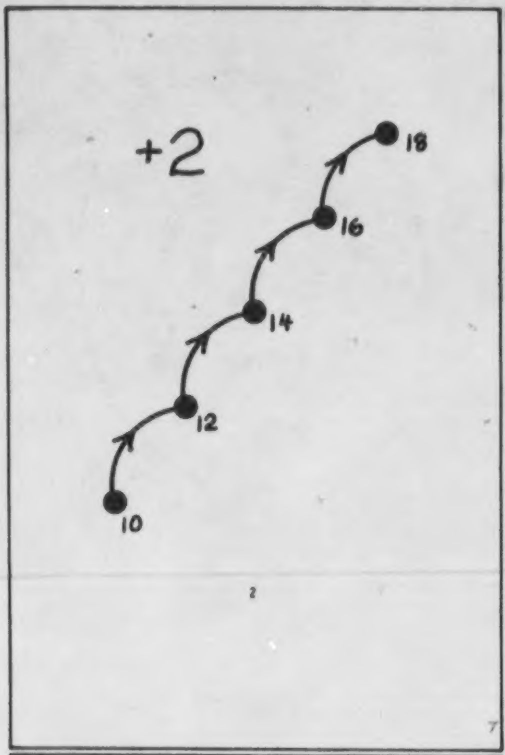


Distribute students' copies of the Workbook "Fishing for Numbers". At the end of the lesson, collect the workbooks for your review.

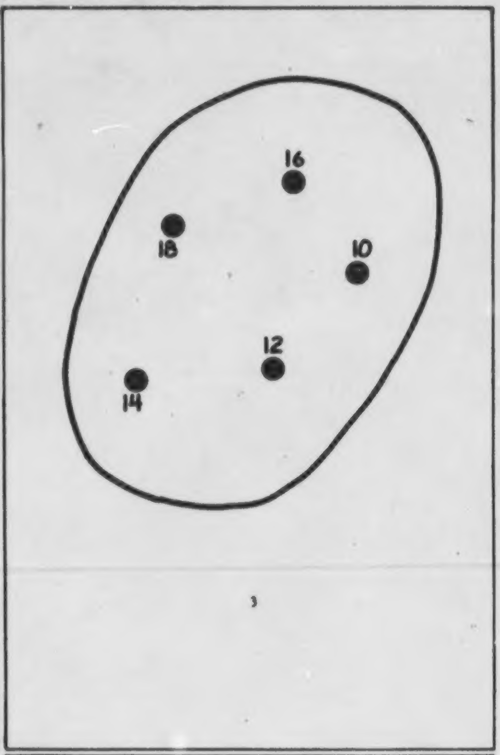
FISHING FOR NUMBERS

(Part I)

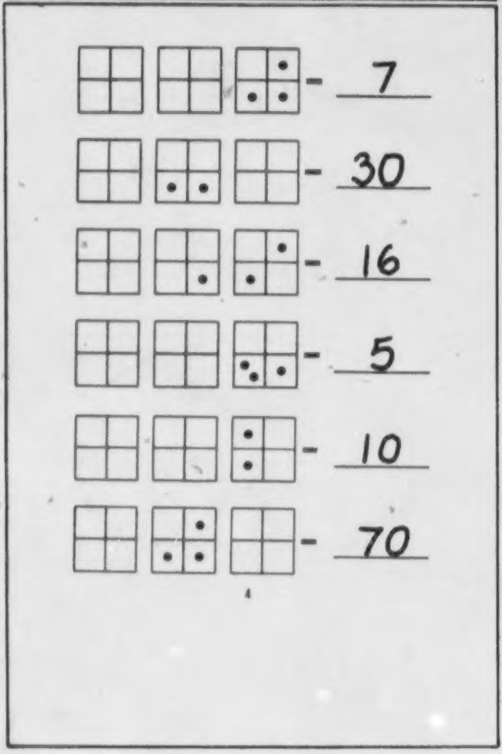
Fishing for Numbers (Part I)


+2
10
12
14
16
18

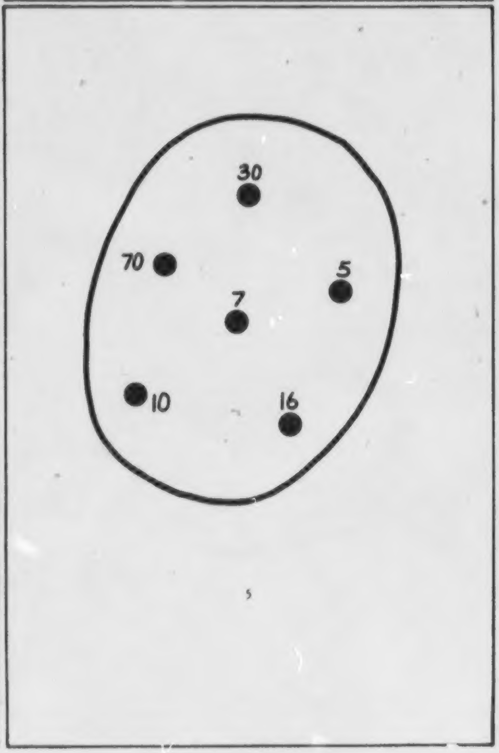
2


16
18
10
12
14

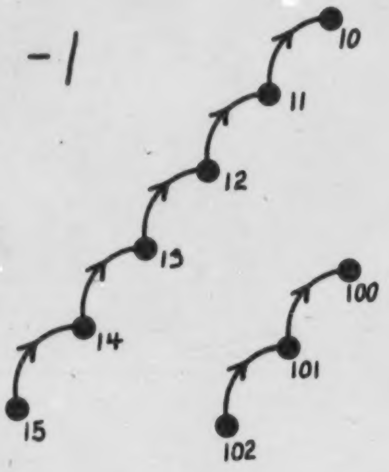
3


7
30
16
5
10
70

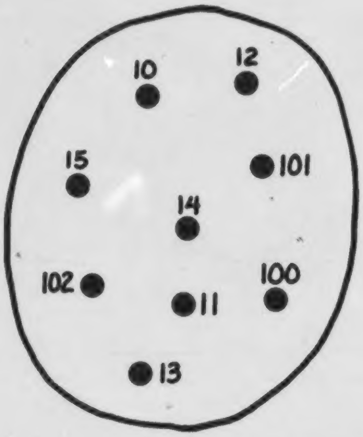
4


30
70
5
7
10
16

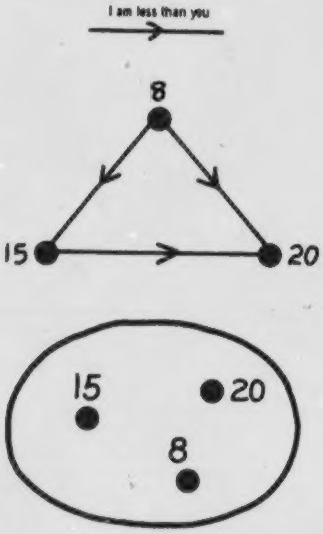
5


-1
10
11
12
13
14
15
100
101
102

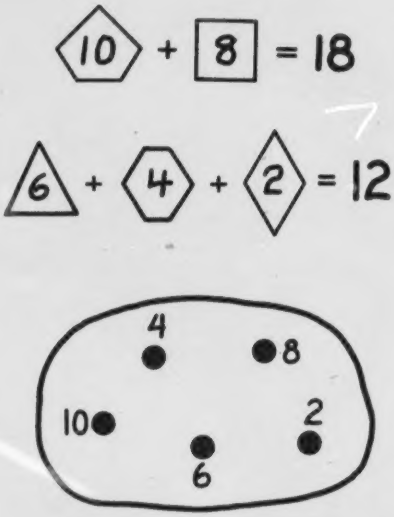
6


10
12
15
101
14
102
100
11
13

7

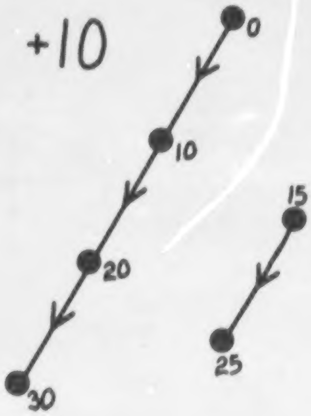

I am less than you
8
15
20
15
20
8

8

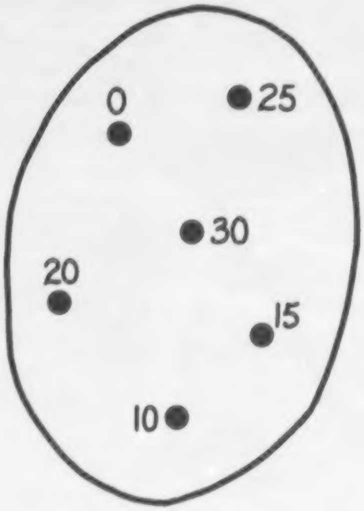

10 + 8 = 18
6 + 4 + 2 = 12
4
8
10
2
6

9

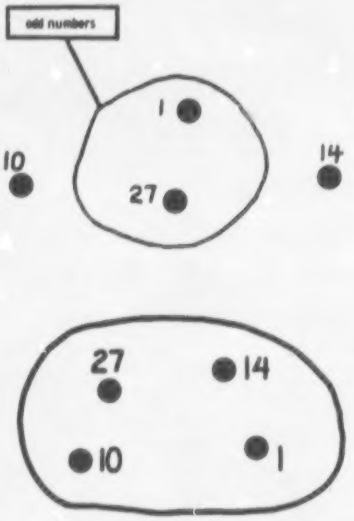
Rearranging the order of the addends gives other solutions.


+10
0
10
20
30
15
25

10

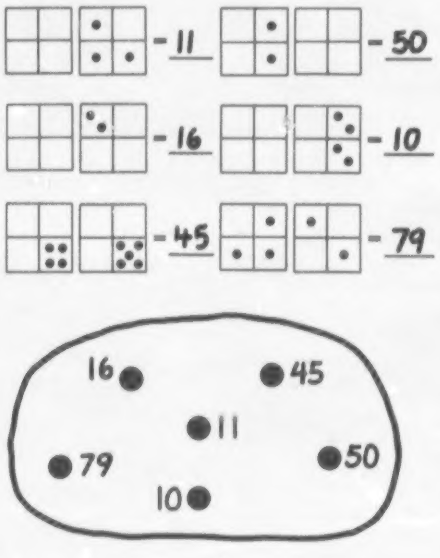

0
25
30
20
15
10

11

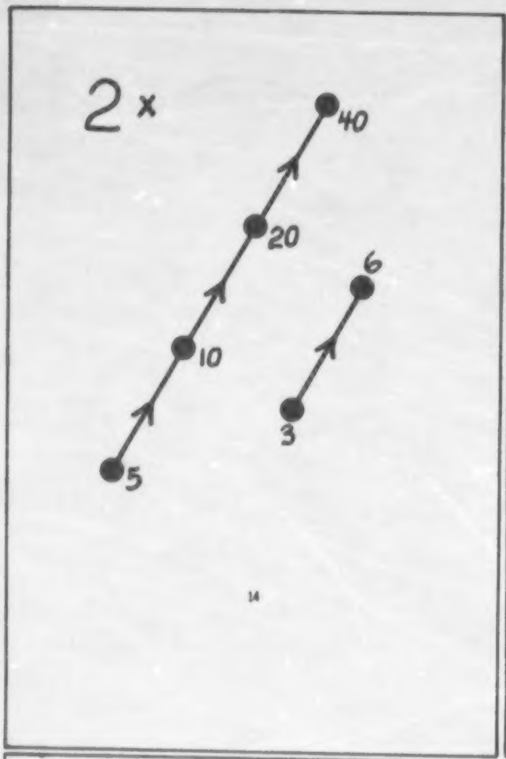

odd numbers
1
10
14
27
27
14
10
1

12

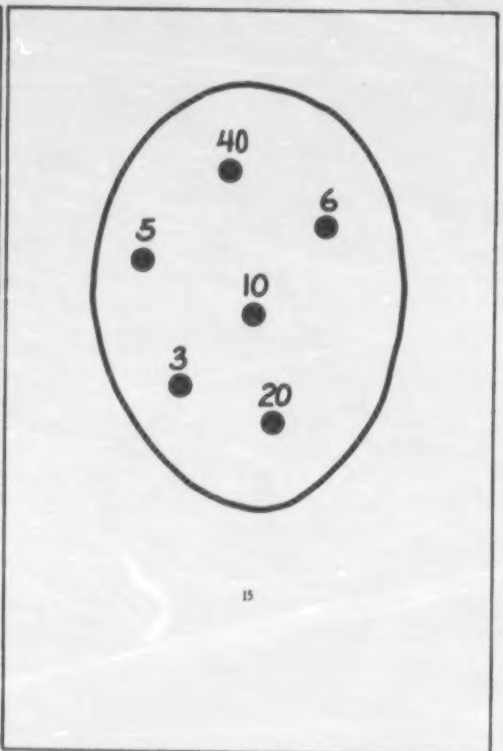
The labels for dots in the same region may be interchanged to obtain other solutions.


11
50
16
10
45
79
16
45
11
79
50
10

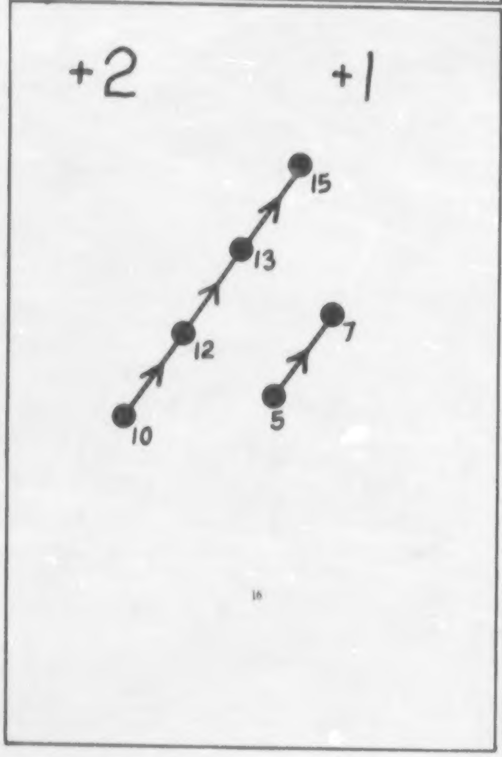
13


2 ×
40
20
10
5
6
3

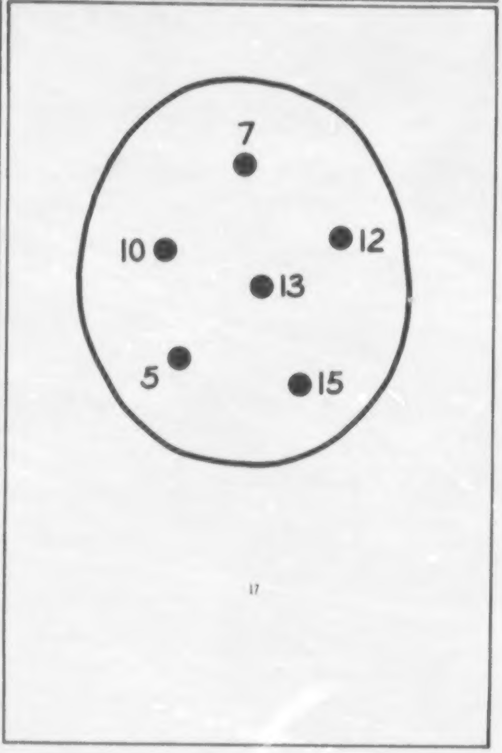
14



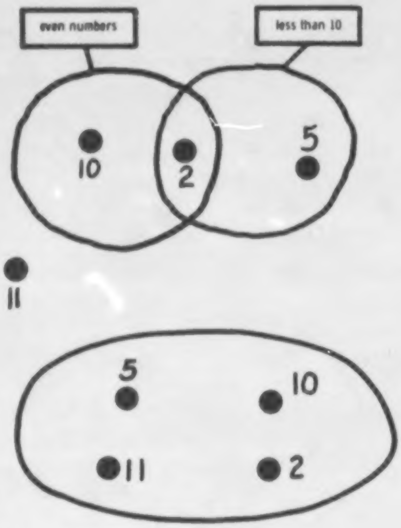
15


+2
+1
15
13
12
10
7
5

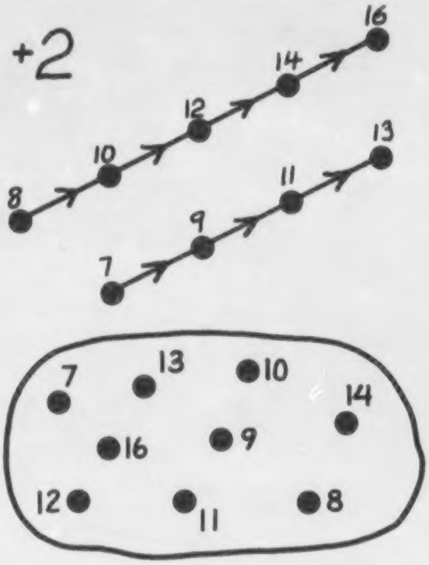
16


7
10
12
13
5
15

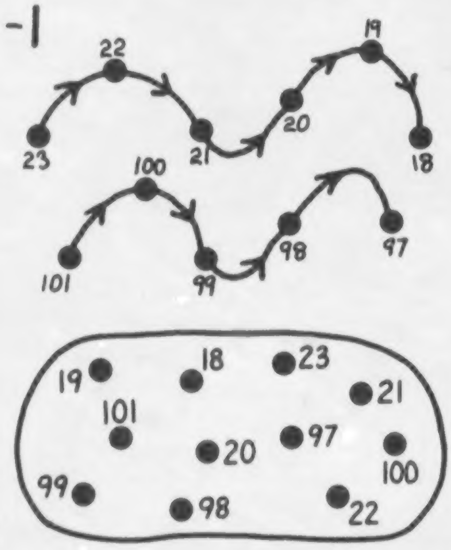
17


even numbers
less than 10

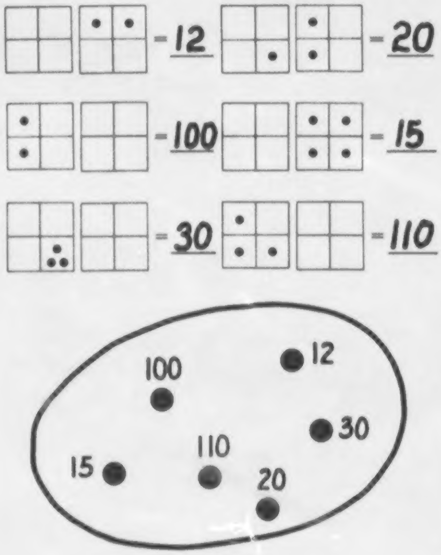
18


+2

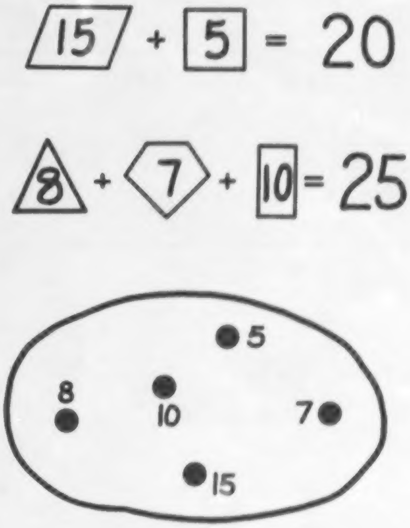
19


-1

20

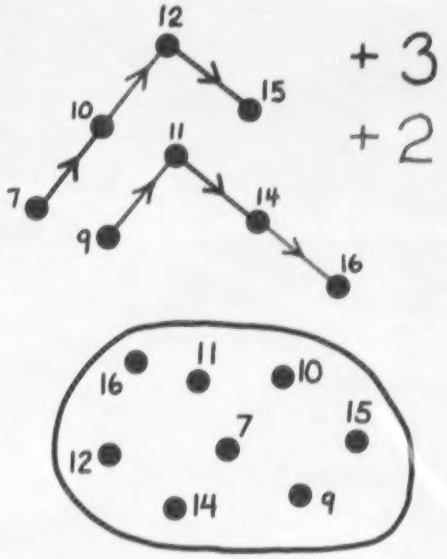

-12
-20
-100
-15
-30
-110
100
12
30
110
15
20

21

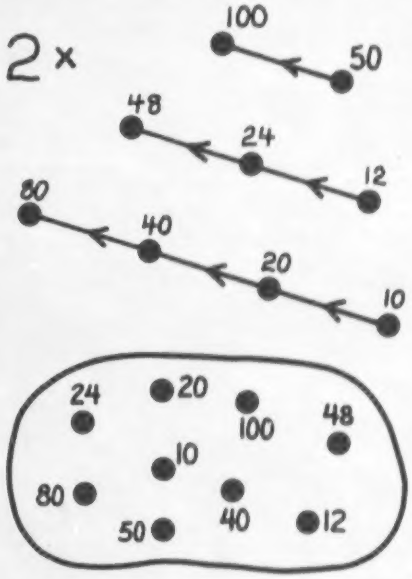

15 + 5 = 20
8 + 7 + 10 = 25
5
8
10
7
15

22

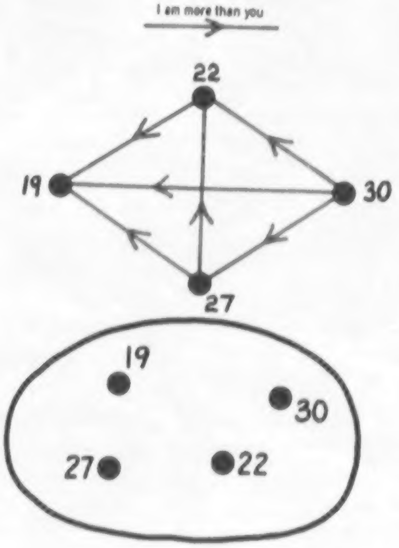
Rearranging the order of the addends gives other solutions.


+ 3
+ 2
12
10
15
7
11
9
14
16
16
11
10
15
12
7
14
9

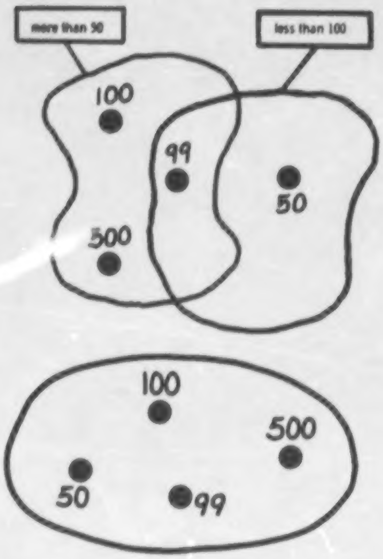
23


2 ×
100
50
48
24
12
80
40
20
10
24
20
100
48
10
80
40
50
12

24

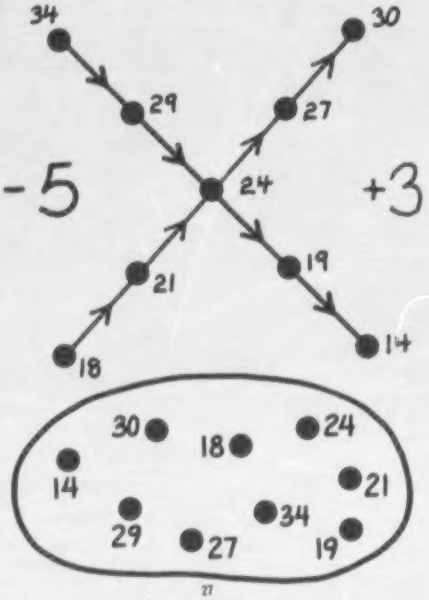

I am more than you
22
19
30
27
19
30
27
22

25

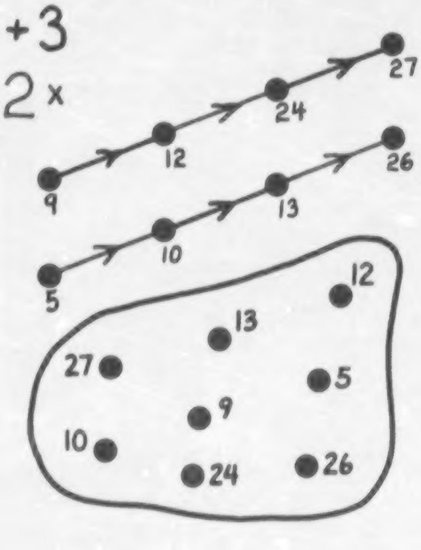

more than 90
less than 100
100
99
50
500
100
500
50
99

26

"100" and "500" may be interchanged.


34
30
29
27
-5
24
+3
21
19
18
14
30
18
24
14
21
29
34
27
19

27


+3
2x
27
24
12
26
9
13
10
5
12
13
27
5
9
10
24
26

28

6×3 = 18

20−9 = 11

41−5 = 36

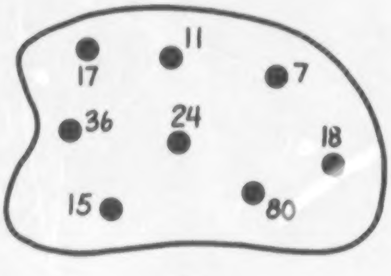
3×8 = 24

½×30 = 15

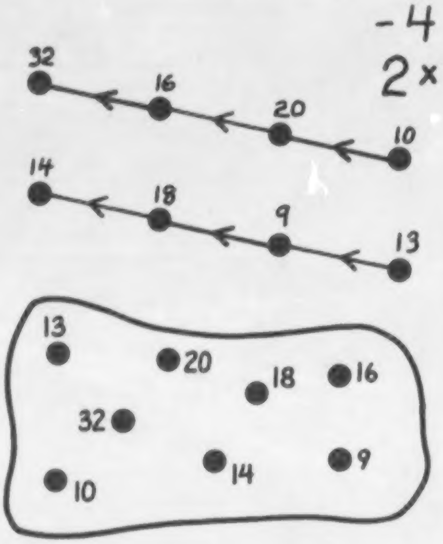
15−8 = 7

48+32 = 80

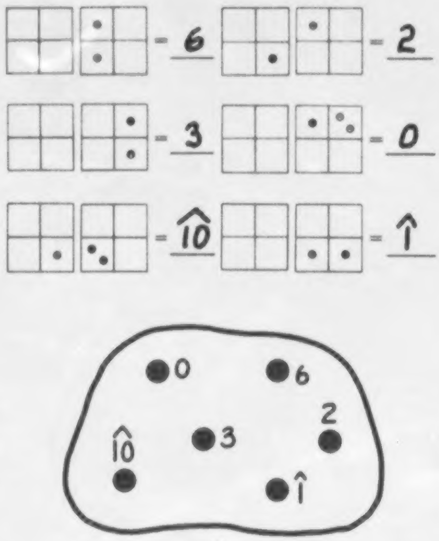
½×34 = 17


17
11
7
36
24
18
15
80

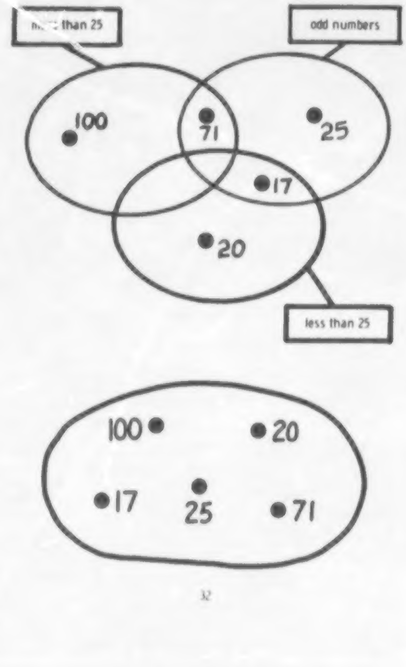
29


-4
2×

30


= 6
= 2
= 3
= 0
= $\hat{10}$
= $\hat{1}$
0
6
2
$\hat{10}$
3
$\hat{1}$

31


more than 25
odd numbers
less than 25

32

W17 Detective Story #6 / Eli Tricked by Clarence the Crocodile

CAPSULE LESSON SUMMARY

With an arrow picture, limit the possibilities for a secret number. Give a clue that this secret number is an odd number, and then locate it in a string picture. Present Clarence the Crafty Crocodile who plays a trick on Eli the Elephant. Eli discovers some of his peanuts are missing. What could Clarence have done in Eli's bag?

MATERIALS

Teacher: Colored chalk; Minicomputer set
Student: None

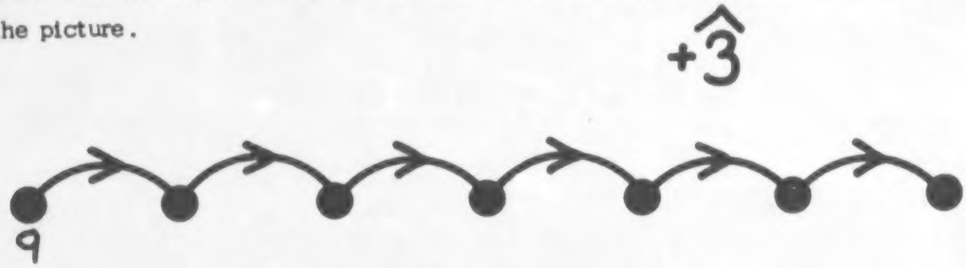
DESCRIPTION OF LESSON

Exercise 1: Detective Story

Write a large "3" on a slip of paper and fold it so that the "3" is hidden.

T: Today you are going to be detectives. You will have several clues which will help you discover a secret number.

Draw this arrow picture on the board and tell the class that the secret is in the picture.

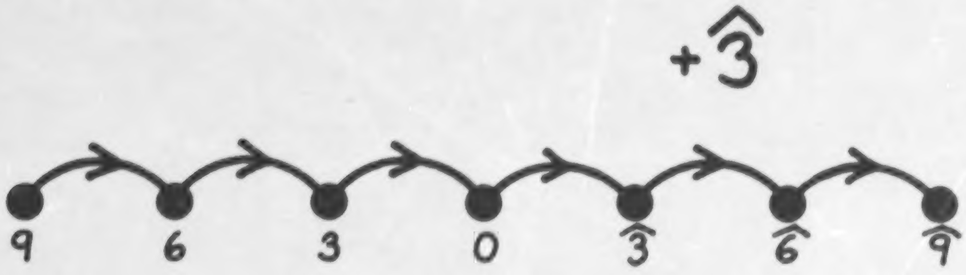


Point to "9" and trace the first red arrow as you say, "$9 + \hat{3} = \ldots$? Nine regular peanuts and three magic peanuts is . . . ?" If your class is not certain that the second dot from the left is for 6, draw Eli's bag with nine regular peanuts and three magic peanuts. Ask a student to show which peanuts would disappear. Looking at the remaining peanuts, conclude:

$$9 + \hat{3} = 6.$$

W 17

You might need to refer frequently to Eli the Elephant to help your students label the remaining dots. Always erase a peanut picture before you draw another one.



After all the dots have been labeled, make a list of the possible numbers on the board . . .

$$9, 6, 3, 0, \widehat{3}, \widehat{6}, \widehat{9}$$

and then erase the arrow picture.

T: The second clue is that the secret number is odd. Which of these numbers is odd (point to the list on the board)?

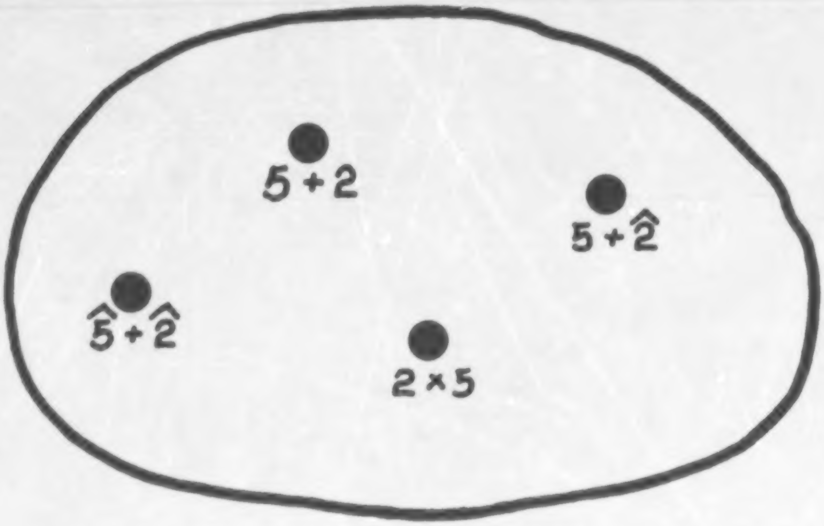
Consider with the class each number in the list and erase any numbers which are not odd.

$$9, \quad 3, \quad \widehat{3}, \quad \widehat{9}$$

T: The secret number could be 9, 3, $\widehat{3}$, or 9. I will give only one more clue.

364

Draw this string picture on the board.



T: The secret number is in this string picture. When you know which one is the secret number tell me with whispers.

When most of the students have whispered an answer to you, do each of the calculations in the string picture. If necessary, draw peanut pictures to show that $\hat{5} + \hat{2} = \hat{7}$ and $5 + \hat{2} = 3$.

T: What is the secret number?

S: 3.

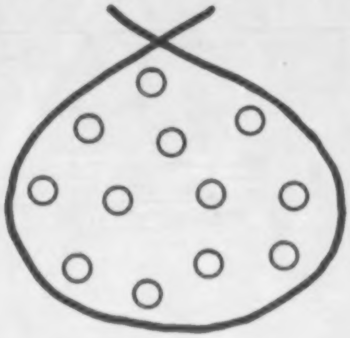
Reveal the "3" on the slip of paper to the class.

Exercise 2: Eli the Elephant

T: One day Eli was walking through the jungle. There were 12 regular peanuts in his bag.

W 17

Draw his bag of peanuts on the board.



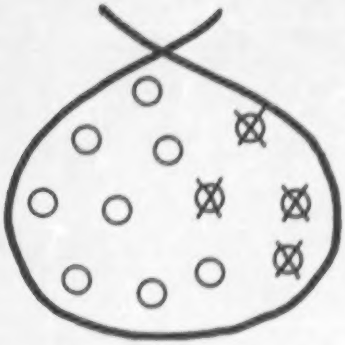
T: Eli walked a long time, he was very tired and decided to take a nap. As soon as Eli fell asleep, along came Clarence the Crafty Crocodile. Clarence saw that Eli was asleep and decided that it would be fun to play a trick on Eli. Clarence opened Eli's bag of peanuts and did something. Then he closed Eli's bag quietly and went away.

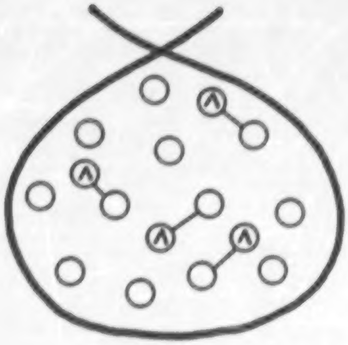
Eli felt hungry when he awoke, so he opened his peanut bag. Eli counted his peanuts and cried, "Oh no! Four peanuts are missing." What do you think Clarence the Crafty Crocodile had done?

Allow the class to discuss this problem. Two possibilities should emerge: Clarence could have removed four peanuts from Eli's bag, or Clarence could have put four magic peanuts into the bag.

NOTE: There are other more complex possibilities which might be suggested. For example, Clarence could have added two magic peanuts and removed two regular peanuts. Accept such suggestions as correct, but for the purpose of this lesson we are only interested in two possibilities: Clarence removed four regular peanuts, or Clarence added four magic peanuts.

Illustrate each of these situations as it is suggested. Write the appropriate number story under each bag of peanuts. Arrange the board so that the two pictures are side by side. (See the illustration on the next page.)





$$12 - 4 = 8 \qquad\qquad 12 + \widehat{4} = 8$$

Emphasize that we do not know what Clarence did when he opened Eli's bag, because removing four regular peanuts and adding four magic peanuts have exactly the same effect. Write this as a number story on the board.

$$12 + \widehat{4} = 12 - 4$$

T: Another day when Eli was walking in the jungle he gathered 95 regular peanuts. He became very tired and decided to take a nap. Do you know what happened while Eli was sleeping?

S: Clarence the Crafty Crocodile came by and played another trick on Eli.

T: Eli felt hungry when he awoke, so he opened his peanut bag. Eli counted his peanuts and found that 69 peanuts were missing. What do you think Clarence the Crafty Crocodile had done?

S: Clarence could have removed 69 peanuts from Eli's bag.

S: Clarence could have put 69 magic peanuts into Eli's bag.

As each possibility is suggested, write the appropriate number story on the board.

W 17

$$95 - 69 = ?$$

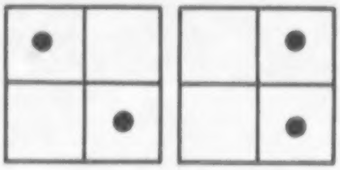
$$95 + \widehat{69} = ?$$

T: How are we going to find out how many peanuts Eli has left in his bag?

If someone suggests drawing a picture of Eli's bag, you should accept this as a good, but not a very practical, method because there are so many peanuts you would have to draw. If no one suggests using the Minicomputer, suggest it yourself. Point to the number stories on the board.

T: Which of these problems do you want to do first on the Minicomputer?

Suppose the students want to calculate "95 - 69" first. Invite someone to put 95 on the Minicomputer.



T: Can someone take 69 off the Minicomputer?

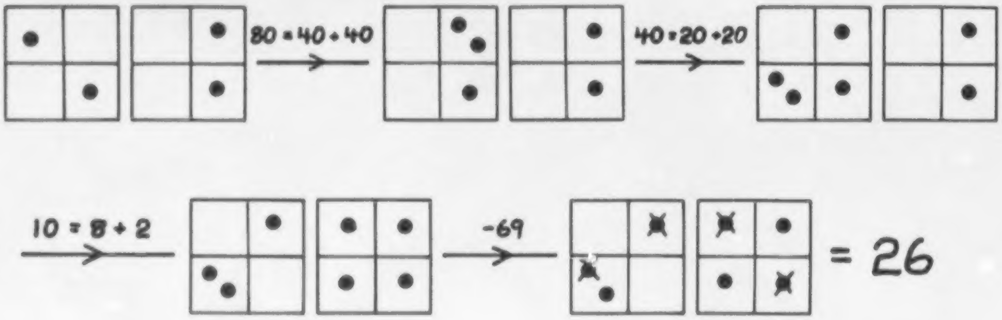
S: We do not have checkers on the right squares for 69.

Let someone point to where the checkers for 69 would be.

T: Who can make a backward play which will help us to calculate "95 - 69"?

Ask a student to identify a play before moving checkers. This will help to discourage plays which are not useful. Whenever a play is made which puts a checker in position for the subtraction, mention this to the class and indicate that they are getting closer to the goal.

Let the students make plays until 69 can be taken off the Minicomputer. A possible sequence of moves is shown below.



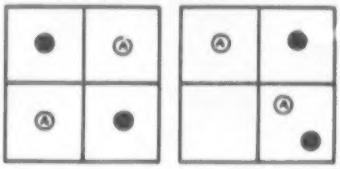
Complete the appropriate number story on the board . . .

$$95 - 69 = 26$$

and remove the checkers from the Minicomputer.

T: Now let's suppose Clarence put 69 magic peanuts into Eli's bag.

Invite a student to put 95 on the Minicomputer and another to put $\widehat{69}$ on the Minicomputer.



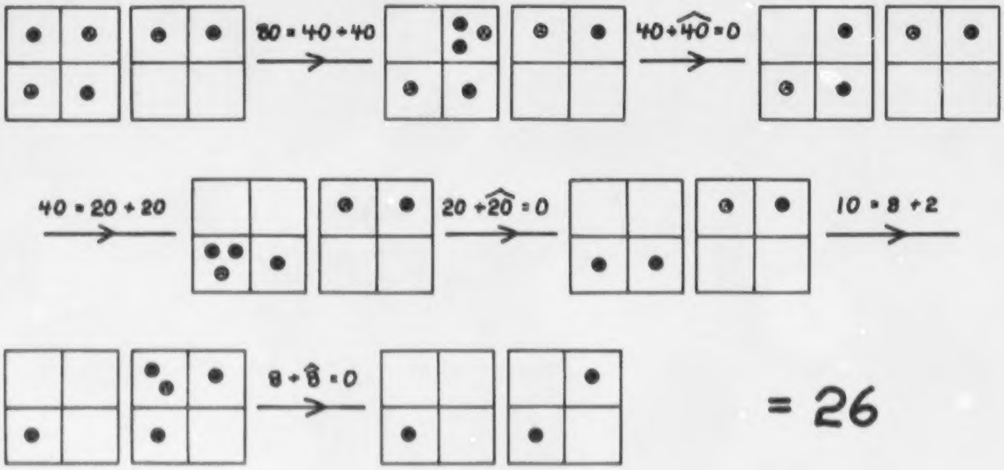
T: We want to get a regular and a negative checker on the same square so that we can remove them. Who can make a play?

W 17

Let students make plays until the checkers on the board are all of the same kind. In this case, they will all be regular.

NOTE: A student might suggest an incorrect play, for example, "$8 + \widehat{2} = 10$". In this case respond "$\widehat{8} + \widehat{2} = \widehat{10}$ and $8 + 2 = 10$, but $8 + \widehat{2} \neq 10$." You might need to illustrate the corresponding situation in Eli's bag of peanuts. For an incorrect play such as "$4 + \widehat{4} = 8$", simply correct the play, "$4 + \widehat{4} = 0$".

One sequence of plays is illustrated below.



Complete the number story on the board.

$$95 - 69 = 26$$

$$95 + \widehat{69} = 26$$

Emphasize that we do not know what Clarence did when he opened Eli's bag, because removing 69 regular peanuts and adding 69 magic peanuts have exactly the same effect. Write this as a number story.

$$95 - 69 = 26$$

$$95 + \widehat{69} = 26$$

$$95 - 69 = 95 + \widehat{69}$$

T: Which calculation did you find easier to do on the Minicomputer?

Accept either response letting students comment, if they wish, on why they thought one calculation was easier than the other.

Continue this activity with a similar situation where Eli has 83 regular peanuts and later discovers that 57 peanuts are missing. Let the students choose which calculation they will do, either "83 - 59" or "83 + $\widehat{59}$", on the Minicomputer. It is not necessary to do both calculations.